

Lecture 6: The Solow Model with Exogenous Perpetual Growth + Growth Accounting

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Long-run growth in the Solow model

- Remember that we also wanted to explain growth within a country over time. So far, Solow model produces no long-run growth.
- Today we will do two things:
 1. In the Solow model: see what happens when A = productivity is growing over time. Will this be sufficient to generate persistent growth?
 - Obviously: not useful for “explaining” long-run growth
 - But: useful to see what the model needs to generate growth. Once that’s established we can look for deeper explanations.

Technical aside: will do everything in continuous time. (easier, + you get to see a different tool)

2. Growth accounting

- Repeat the production function accounting from Lecture 3, but now for a given country over time, called “growth accounting.” Useful for the paper default option.
- Why only now? Can implement growth accounting w/o Solow theory. But the additional structural Solow model insights will help us *interpret* the results.

Outline

1. Warm-up: Solow model in continuous time
2. Solow model with exogenous long-run growth
 - Aside: variants of technological progress
 - Solving the model
3. Growth accounting
 - The basic idea
 - Applications
 - Incorporating human capital

From discrete to continuous time model

- So far, the time difference between t and $t + 1$ was arbitrary.
It could be years, quarters, or even smaller.
- Let's make this difference very small, e.g., minutes. Then, we obtain a model in which time runs continuously.
- Economic insights are the same as in the discrete time version. But continuous-time version is sometimes more tractable.
 - Let's use notation $\{K(t), L(t), Y(t)\}_{t \in [0, \infty)}$ and $\{k(t), y(t)\}_{t \in [0, \infty)}$ for the time paths in continuous time.
 - To distinguish from $\{K_t, L_t, Y_t\}_{t=0}^{\infty}$ and $\{k_t, y_t\}_{t=0}^{\infty}$ in the discrete-time case.

From discrete to continuous time model

- Let's start with the population growth equation. For an arbitrary time increment Δt :

$$L_{t+\Delta t} = L_t (1 + n\Delta t)$$

- We can rearrange this as:

$$\underbrace{\frac{L_{t+\Delta t} - L_t}{\Delta t}}_{\text{change in population} \simeq \frac{dL(t)}{dt}} = n \underbrace{L_t}_{\text{level of population, } L(t)}$$

- So the continuous time analogue is a differential equation:

$$\frac{dL(t)}{dt} \equiv \dot{L}(t) = nL(t) \Rightarrow L(t) = e^{nt} L(0)$$

- We will throughout use $\dot{x}(t)$ to denote the time derivative of $x(t)$.
 - n denotes the continuous-time population growth rate. Growth rate over a short discrete time interval Δt is $\simeq n\Delta t$.
- Given n and initial $L(0)$, we can solve for $\{L(t)\}_{t \in [0, \infty)}$.

From discrete to continuous time model

- Next consider the law of motion equation for capital, K_t .
- Recall that we worked with the capital labor ratio, $k_t = K_t/L_t$. For arbitrary Δt the dynamics are described by

$$(1 + n\Delta t) k_{t+\Delta t} = (1 - \delta\Delta t)k_t + sf(k_t) \Delta t$$

- We can also rearrange this as:

$$\frac{k_{t+\Delta t} - k_t}{\Delta t} = sf(k_t) - \delta k_t - nk_{t+\Delta t}$$

- Now as before approximate this for $\Delta t \rightarrow 0$:

$$\dot{k}_t = sf(k_t) - \delta k_t - nk_t$$

Continuous time model

- Based on this heuristic derivation we would expect the law of motion for capital to become a differential equation:

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t) \quad (1)$$

- This turns out to be the right equation. If you use

$$\dot{K}(t) = sY(t) - \delta K(t) = sF(K(t), L(t)) - \delta K(t),$$

along with $\dot{L}(t) = nL(t)$, $k(t) \equiv K(t)/L(t)$, and therefore

$$\dot{k}(t) = \frac{d}{dt} \left(\frac{K(t)}{L(t)} \right) = \frac{\dot{K}(t)}{L(t)} - \frac{K(t)}{L(t)} \frac{\dot{L}(t)}{L(t)} = sf(k(t)) - (\delta + n)k(t)$$

you obtain (1).

- Given initial condition, $k(0) = K(0)/L(0)$, can solve for $[k(t)]_{t \in [0, \infty)}$.

Equilibrium as an ODE

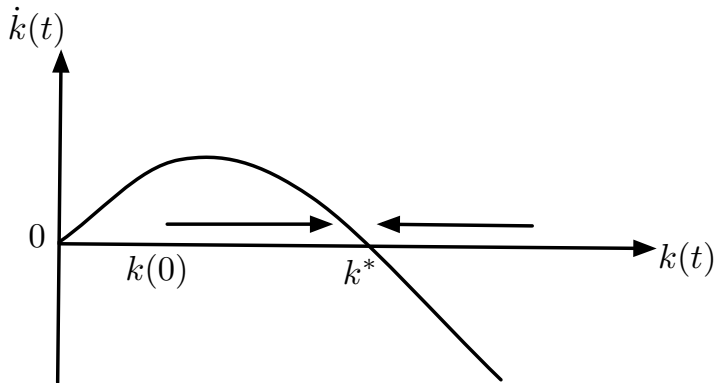
Proposition

In the continuous-time Solow model, the equilibrium path of the capital-labor ratio solves the following ordinary differential equation:

$$\dot{k}(t) = sf(k(t)) - (\delta + n)k(t), \quad (2)$$

with initial condition $k(0) = \bar{k}_0$ given by history.

Graphical representation



Can you see what happens to $k(t)$ starting from some $k(0)$?

Steady state and transitional dynamics

- Similarly to the discrete-time case, there is a unique non-zero steady state, k^* , still characterized by

$$sf(k^*) = (\delta + n)k^*.$$

- There are also the familiar transitional dynamics away from k^* .
 - Starting with $k(0) < k^*$, $k(t)$ grows towards k^* .
 - Starting with $k(0) > k^*$, $k(t)$ shrinks towards k^* .

Here it's even easier to show than in discrete time. $\dot{k}(t) > 0$ if $k(t) < k^*$ and $\dot{k}(t) < 0$ if $k(t) > k^*$, so we get convergence.

Interpreting the equilibrium

- Note that, unlike our analysis in Lecture 4, we allowed for $n > 0$.
Beyond that the analysis gave identical results, as it should be—doesn't matter whether we work in discrete or continuous time.
- We still found that per-labor variables, $k(t), y(t)$ converge to the steady state, k^*, y^* .
- When the economy is at the steady-state, our analysis implies that

$$K(t) = k^* L(t) \text{ and } Y(t) = f(k^*) L(t). \quad (3)$$

So the aggregate variables grow at the population growth rate, n .

- Thus aggregate variables actually grow in this economy, but at a rate just enough to keep the per labor variables constant.
- The other testable predictions we have obtained for the $n = 0$ case also continue to apply in this case.

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Adding exogenous technological progress

- Recall that TFP differences are important for income differences. So **TFP growth** might also be important to explain growth within a country.
- In the version of the Solow model studied so far, productivity didn't change over time (recall fixed $F(K(t), L(t))$).
- Now introduce improvements in know-how, or technological progress, by letting

$$Y(t) = F(K(t), A(t)L(t)).$$

where $A(t)$ measures how productive labor is.

- Next slides will have a little aside: how else could we model technological progress?

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Different variants of technological progress

- $A(t)$ as introduced on the previous slide is known as **labor-augmenting productivity**.
 - Technological progress (growth of $A(t)$) augments labor.
 - The expression, $A(t)L(t)$, is known as “effective labor.”
- We could alternatively consider **capital-augmenting productivity**,

$$F(A(t)K(t), L(t)).$$

- Or we could consider productivity that augments each factor equally,

$$A(t)F(K(t), L(t)) = F(A(t)K(t), A(t)L(t)).$$

This type of $A(t)$ is called factor-neutral (or Hicks-neutral) productivity, often referred to as TFP.

Relationship between the different variants

- We will work with labor-augmenting productivity. Why?
 1. It's extremely convenient analytically & allows us to generate long-run growth. Also that growth will be consistent with the patterns of growth we see in data.
 2. It's not just sufficient, but also necessary—as it turns out, everything growing at a constant rate *requires* labor-augmenting technological progress.
 3. For the Cobb-Douglas functional form, it's not at all restrictive, as all three forms are the same up to rescaling and relabeling:

$$\underbrace{K^\alpha (AL)^{1-\alpha}}_{\text{labor augmenting}} = \underbrace{A^{1-\alpha} K^\alpha L^{1-\alpha}}_{\text{Hicks neutral}} = \underbrace{\left(A^{(1-\alpha)/\alpha} K\right)^\alpha L^{1-\alpha}}_{\text{capital augmenting}}$$

We did $AK^\alpha L^{1-\alpha}$ before. Identical to $K^\alpha(\tilde{A}L)^{1-\alpha}$ up to the relabeling $A = \tilde{A}^{1-\alpha}$.

- Bottom line:
 - For general theory, work with labor-augmenting productivity, because of reasons 1. - 2.
 - Once we go to data we will go to Cobb-Douglas so it won't matter anyway.

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Labor-augmenting technological progress

- So, let's assume productivity takes the labor-augmenting form:

$$Y(t) = F(K(t), A(t)L(t))$$

- Suppose also there is population growth, $\frac{\dot{L}(t)}{L(t)} = n$, as before.
- New assumption: **labor-augmenting technological progress**,

$$\frac{\dot{A}(t)}{A(t)} = g > 0$$

- Here, g captures the growth rate of technological progress.

Solving the model with technological progress

- To analyze the equilibrium, let us define the *effective* labor as

$$L^e(t) = A(t) L(t)$$

- Note that effective labor grows at the rate $n^e = n + g$ since

$$\frac{\dot{L}^e(t)}{L^e(t)} = \frac{\dot{L}(t)}{L(t)} + \frac{\dot{A}(t)}{A(t)} = n + g$$

- In terms of $L^e(t)$, the production function can be written as,

$$Y(t) = F(K(t), L^e(t)).$$

- This looks similar to our original Solow model without technological progress, modulo the appearance of the index e in L^e and n^e .

Solving the model with technological progress

- We can leverage this similarity with the following analytical trick.
 - Imagine an economy without technological progress, $g = 0$, but with population growth, $n^e = n + g$.
 - Aggregate variables, $K(t), Y(t)$, in the imaginary economy would follow the same path as in the actual economy.
 - But we already analyzed the dynamics in the imaginary economy ...
- We can thus apply the results in the beginning of these notes for n^e .

Solving the model with technological progress

- To apply the earlier results, we need some relabeling. Let us redefine $k(t)$ as the ratio of capital to effective labor:

$$k(t) \equiv \frac{K(t)}{L^e(t)} = \frac{K(t)}{A(t)L(t)}$$

- We then immediately have that $k(t)$ obeys the following law of motion:

$$\begin{aligned}\dot{k}(t) &= sf(k(t)) - (\delta + n^e)k(t) \\ &= sf(k(t)) - (\delta + n + g)k(t)\end{aligned}$$

Recall Mankiw-Romer-Weil from last lecture. Now the g 's in their expressions hopefully make sense ...

- And that it converges to the steady-state k^* , where:

$$\begin{aligned}sf(k^*) &= (\delta + n^e)k^* \\ &= (\delta + n + g)k^*\end{aligned}$$

Solving the model with technological progress

- Recall from (3) that, when the economy is in steady-state, aggregate capital and output are given by,

$$\begin{aligned}K(t) &= k^* L^e(t) = k^* A(t) L(t), \\Y(t) &= f(k^*) L^e(t) = f(k^*) A(t) L(t).\end{aligned}$$

- So in the steady state $K(t)$ and $Y(t)$ grow at rate $n + g$.
- What does this imply for $K(t)/L(t)$ and $Y(t)/L(t)$ —capital and output *per labor* (as opposed to per effective labor)?
- They both grow at the same rate as $A(t)$, that is, g .

Steady-state: balanced growth path eq'm

- Hence, the model has a steady state in which the capital-to-effective-labor is constant and the per labor variables, $K(t)/L(t)$ and $Y(t)/L(t)$, grow at the constant rate g .
 - Recall Lecture 1: this is what we see in the data, constant growth rates for output, consumption, and investment, and constant hours worked & capital-to-output ratio.
- This kind of steady state is often also called **a balanced growth path**.

Uzawa's theorem: balanced growth path $\rightarrow$ technological progress is labor-augmenting.

Interpreting the balanced growth path eq'm

- In the baseline Solow growth model, growth stopped because of diminishing returns. Intuitively, as the economy accumulated capital, labor became the bottleneck.
- In this version, labor is getting better and better, and this contributes to a higher MPK.
- So we can sustain capital accumulation at the same rate at which labor productivity increases. Moreover, the joint growth in capital and effective labor also induces output to grow at the same rate:

$$\frac{Y(t)}{L(t)} = F \left(\underbrace{\frac{K(t)}{L(t)}}_{\text{grows at rate } g \text{ (induced by } A(t))}, \underbrace{A(t)}_{\text{grows at rate } g} \right).$$

grows at rate g due to constant returns to scale

Interpreting the balanced growth path eq'm

- This version of the model features growth and thus matches the data better. For the US, the long-run average growth rate is around $g \simeq 2\%$.
- But the model of course does not at all “explain” g . It is a black box. We will try to open this box later in the course.
- For now we have established that A being labor-augmenting and growing is indeed sufficient to generate growth that looks like what we see in data.

Transitional dynamics with growth

- Balanced growth path is a steady state in terms of normalized variables,

$$k(t) = \frac{K(t)}{A(t)L(t)} \text{ and } y(t) = \frac{Y(t)}{A(t)L(t)}.$$

- If $k(0)$ happens to be equal to k^* , then it stays there forever. That is, we have $k(t) = k^*$ for each t .
- But the economy can also start with low capital $K(0)$ (relative to its initial $L(0), A(0)$) so that $k(0) < k^*$.
- What happens in this case? Do per labor variables, $K(t)/L(t)$ and $Y(t)/L(t)$, grow faster or slower than g ?
- Does the answer change for the short run and the long run?

Transitional dynamics with growth

- If $k(0) < k^*$, then $k(t)$ increases towards k^* . This means:

$$\frac{K(t)}{L(t)} = k(t) A(t)$$

temporarily grows at a rate faster than g . Why?

- Similarly, $Y(t)/L(t)$ temporarily grows faster than g .
- But eventually the economy reaches balanced growth path and $K(t)/L(t)$ and $Y(t)/L(t)$ grow at rate g .
- So saving/capital accumulation can temporarily raise growth of per labor variables above g . But capital accumulation can't drive long-run growth, exactly as we have seen before.

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Growth accounting

- Recall that, in prior lectures, we used a production function-based calibration approach to understand determinants of cross-country income differences.
- We can use a similar method to understand the determinants of growth over time.
 - That is, use the production function approach over time, not across countries. We are thus returning to the methodology of Lecture 3.
- This method, known as **growth accounting**, was also introduced by Bob Solow.
- It is general because it is only using the production function, not any further theory.
 - In particular it does not at all use the Solow growth model ...
 - ... but the theory of that model helps us in *interpreting* the results of growth accounting.

Growth accounting with Cobb-Douglas

- To illustrate, suppose we have Cobb-Douglas production function with labor-augmenting technological progress:

$$Y(t) = K(t)^\alpha (A(t)L(t))^{1-\alpha}.$$

- Taking logs, we have:

$$\ln(Y(t)) = \alpha \ln(K(t)) + (1 - \alpha) \ln L(t) + (1 - \alpha) \ln A(t).$$

Before we stopped here, and looked at production function across countries, at a given t .

- Now let's look over time for an individual country. Differentiate both sides with respect to t to obtain:

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \frac{\dot{K}(t)}{K(t)} + (1 - \alpha) \frac{\dot{L}(t)}{L(t)} + (1 - \alpha) \frac{\dot{A}(t)}{A(t)}.$$

- Let's call the last (leftover) term Solow residual. Then:

$$\underbrace{g_Y(t)}_{\text{output growth}} = \alpha \underbrace{g_K(t)}_{K\text{-growth}} + (1 - \alpha) \underbrace{g_L(t)}_{L\text{-growth}} + \underbrace{x(t)}_{\text{Solow residual}}.$$

Growth accounting more generally

- Note that, given the Cobb-Douglas production function, this equation applies very generally:
 - It applies at steady-state or away from steady-state.
 - It applies if labor grows at constant rate, n , or not.
 - It applies if people save constant fraction, s , or not.
 - It applies for all three kinds of technological progress.
- The only additional assumption we need to take it to the data is perfect competition, implying that

$$\alpha = 1 - \text{labor share}$$

Growth accounting more generally

- With a more general production function + perfect competition we'd have

$$g_Y(t) = \alpha_K(t) g_K(t) + \alpha_L(t) g_L(t) + x(t),$$

where $\alpha_K(t)$ and $\alpha_L(t)$ are shares of capital and labor evaluated at the corresponding date according to:

$$\alpha_K(t) \equiv \frac{F_K K}{F} = \frac{r(t)K(t)}{Y(t)} \quad \text{and} \quad \alpha_L(t) \equiv \frac{F_L AL}{F} = \frac{w(t)L(t)}{Y(t)}$$

- The Cobb-Douglas case as usual is nice and easy since it implies $\alpha_K(t) = \alpha$ and $\alpha_L(t) = 1 - \alpha$ regardless of time and factor endowments.
- Why is the result so general?

Growth accounting more generally

- It is general since it's just accounting based on an aggregate production function:

$$\text{output growth} = \left[\begin{array}{l} \text{contribution from capital growth} \\ + \text{contribution from labor growth} \\ + \text{residual we cannot explain} \end{array} \right].$$

- The residual is the part we can't explain from observable inputs.
 - The leading candidate for the residual is technological progress.
 - But other things, e.g., country making reforms to increase its **productivity level** from $\underline{A}$ to $\overline{A}$ would also show up in $x(t)$.
- Growth accounting is an extremely useful **diagnostic tool**. It poses questions for economic models to address.
- Let us look at some examples and applications.

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Growth accounting for some OECD countries

TABLE 1. Selected Growth Accounting Results for Individual Countries (percent)

Economy	Share of capital in national output	GDP growth	Share contributed by		
			Capital	Labor	TFP
OECD 1947-73					
France	.40	5.40	41	4	55
Germany	.39	6.61	41	3	56
Italy	.39	5.30	34	2	64
Japan	.39	9.50	35	23	42
United Kingdom	.38	3.70	47	1	52
United States	.40	4.00	43	24	33

TFP's share is large

Capital's contribution
also seems large

What is going
on with this?

From Easterly and Levine (2001), "It is not Factor Accumulation..."

Increasing labor participation in the US



- Labor's contribution can come from population growth or from increased labor-market participation (or both).
- Why was the labor market participation increasing in the US until the early 2000's? What happened after that?

Beware the endogeneity in capital growth

- One problem for interpretation is again the endogeneity of capital.
- Recall growth in $A(t)$ and $L(t)$ **endogenously** generates g_K . We thus might misattribute the effect of $A(t)$ and/or $L(t)$ to $K(t)$.
- To get a sense of this, consider the Solow model with labor-augmenting technological progress at the growth rate g .
- Suppose the economy is on the balanced growth path. If we did growth accounting for this economy, it would look like:

$$\underbrace{\frac{\dot{Y}(t)}{Y(t)}}_{g+n} = \alpha \underbrace{\frac{\dot{K}(t)}{K(t)}}_{g+n} + (1-\alpha) \underbrace{\frac{\dot{L}(t)}{L(t)}}_n + \underbrace{x(t)}_{\text{Solow residual would be } (1-\alpha)g}$$

Beware the endogeneity in capital growth

- Interpretation
 - Capital accounts for a fraction, α , of total growth, $g + n$, even though growth ultimately comes from $L(t)$ and $A(t)$.
 - The Solow residual shows up as $(1 - \alpha)g$, even though the labor-augmenting productivity grows at rate g .
- We could correct for this, as we did with the calibration exercise in the last lecture (Hall and Jones). However, the raw version is more standard.
- Just be aware of this endogeneity as you interpret the results.

This is another illustration that growth accounting provides diagnostics but does not, by itself, identify driving forces.

Growth accounting for some OECD countries

Back to the previous table ...

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Most growth comes from TFP

Capital's contribution
seems close to its share

Growth accounting for some OECD countries

- Almost all growth comes from the Solow residual!
 - Thus results within countries over time echo our across-country conclusions. Need to focus on A .
- Capital's contribution is generally in line with the balanced growth path prediction in which all capital-growth is induced, $\alpha (g + n)$.
- Surprisingly, OECD convergence after the war seems driven by high TFP growth (high g) as opposed to high k growth.
 - So the basic Solow model might have got the right answer for the wrong reason!
 - Growth accounting has helped us see that.

Growth accounting in per labor terms

- We can also do growth accounting in intensive form. Recall:

$$\frac{\dot{Y}(t)}{Y(t)} = \alpha \frac{\dot{K}(t)}{K(t)} + (1 - \alpha) \frac{\dot{L}(t)}{L(t)} + x(t).$$

- Subtracting $\frac{\dot{L}(t)}{L(t)}$ from both sides, we obtain:

$$\frac{\dot{y}(t)}{y(t)} = \alpha \frac{\dot{k}(t)}{k(t)} + x(t),$$

where $y(t) = Y(t)/L(t)$ and $k(t) = K(t)/L(t)$.

Here, with a bit of abuse of notation, we are using the ratios of Y and K to plain, not effective, labor.

Growth accounting for the US

TABLE 6.2

Growth Accounting for the United States

	1948–2011	1948–1973	1973–1995	1995–2007	2007–2011
Output per hour, Y/L	2.5	3.3	1.5	2.7	1.9
Contribution of K/L	0.9	0.9	0.7	1.1	1.1
Contribution of labor composition	0.2	0.2	0.3	0.2	0.4
Contribution of TFP, A	1.4	2.2	0.5	1.5	0.4

The table shows the average annual growth rate (in percent) for different variables.

- There is a growth slowdown between 1973–1995. Could it be because of capital accumulation losing steam as we approached a steady state?
- Not really. Almost all of the growth slowdown in that period is due to a TFP slowdown. What could be the cause of this productivity slowdown?
- Then, there is rapid TFP growth between 1995–2007. Why?

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Extensions of growth accounting

- The growth accounting framework is quite flexible.
 - It can be done separately for individual sectors (say, manufacturing), if you have data on inputs and outputs.
 - It can also be extended to account for additional factors, or to adjust for the quality of $K(t)$ and $L(t)$.
 - One approach: treat different quality $K^i(t)$, $L^i(t)$ as different factors (with shares, α_K^i, α_L^i , that reflect their importance).
- We could also directly incorporate human capital into our analysis, as we did with the calibration exercise. Let's try to do this here.
 - Aside: I hope this extension will be useful for your paper assignment. It will give you an example of how you can go “deeper” than vanilla high-level growth accounting by having a more detailed production function.

Incorporating human capital

- Suppose that the production function is given by

$$Y(t) = K(t)^\alpha (A(t) h(t) L(t))^{1-\alpha}$$

As before, we take $h(t)$ as labor-augmenting (so that it multiplies wages).

- We now treat $A(t)$ as the labor-augmenting productivity residual of $h(t)$.
- Using the same steps as above, we obtain (in intensive form),

$$\frac{\dot{y}(t)}{y(t)} = \alpha \frac{\dot{k}(t)}{k(t)} + \underbrace{(1 - \alpha) \frac{\dot{h}(t)}{h(t)}}_{\text{contribution from human capital growth}} + x(t) \quad (4)$$

- Thus, in this model extension, $h(t)$ growth can explain some of the overall growth (i.e., it chips away from $x(t)$).

Measuring human capital

- We once again focus on the part of $h(t)$ we can measure—schooling.
- As mentioned earlier, Barro and Lee (2013) constructed an index for $h(t)$ based on micro evidence about the returns to schooling.
- This human capital index is available in the PWT data set.
 - See <https://www.rug.nl/ggdc/> or <https://cid.econ.ucdavis.edu/pwt.html> for the PWT data.
 - See also <http://www.barrolee.com> more specifically for the Barro-Lee data.
- PWT data is yearly, so use a discrete-time approximation:

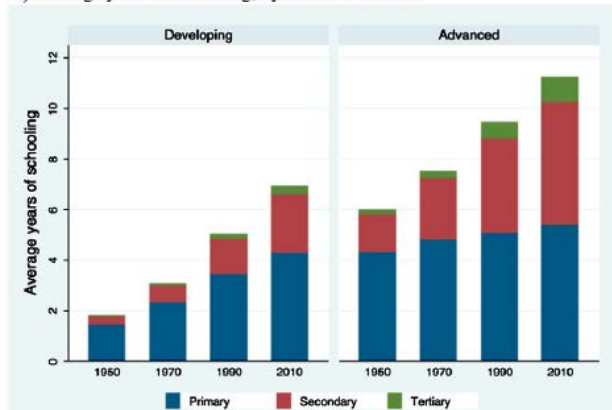
$$\text{Continuous} : \frac{\dot{y}(t)}{y(t)} = \alpha \frac{\dot{k}(t)}{k(t)} + (1 - \alpha) \frac{\dot{h}(t)}{h(t)} + x(t)$$

$$\text{Discrete} : \frac{y_{t+1} - y_t}{y_t} \simeq \alpha \frac{k_{t+1} - k_t}{k_t} + (1 - \alpha) \frac{h_{t+1} - h_t}{h_t} + x_t$$

Global trends in schooling/human capital

R.J. Barro, J.W. Lee / Journal of Development Economics 104 (2013) 184–198

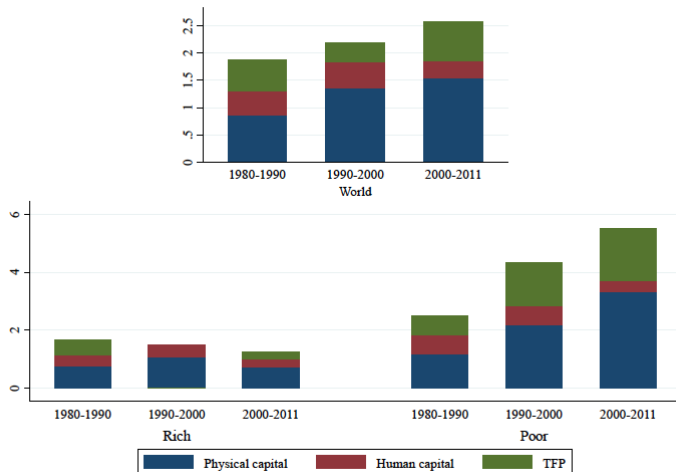
a) Average years of schooling, by educational level



Note: Advanced countries = Australia, Austria, Belgium, Canada, Denmark, Finland, France, Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, USA, United Kingdom.

Global growth accounting with human capital

Figure 8, Global labor productivity growth and its sources, 1980-2011



Poor countries are those with a level of GDP per capita below the median in 1990

Taking Stock

Taking stock: Solow model with growth

- Solow model with labor-augmenting technological progress g .
 - Counters diminishing returns, aligns per-labor outcomes with the data.
 - Continuous time formulation is useful to analyze growth.
- Growth accounting enables us to diagnose the (mechanical) sources of growth.
 - Modified growth accounting also helps to account for schooling.
 - TFP growth seems important even after accounting for $\dot{k}$ and $\dot{h}$.
 - Conclusions very much echo those from our cross-country analysis.

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