

## 14.05 – INTERMEDIATE MACROECONOMICS

### Midterm 1, Fall 2024

*This is a closed-book exam. Read the questions carefully before starting to answer. You will be graded on the quality of your explanations as well as the accuracy of your answers. You have 85 minutes. Good luck!*

#### Exercise 1 (30 points)

For parts a)–c), decide whether the statement is true, false, or uncertain. *Briefly* explain your answer, with your explanations not exceeding two sentences for each (sub-)item.

- a) [6 points] Consider a firm with production function  $F(K, L)$ . It hires labor and rents capital on competitive spot markets, at prices  $w$  and  $r$ , respectively. Its profit maximization problem is thus

$$\Pi \equiv \max_{K,L} \{F(K, L) - rK - wL\} \quad (1)$$

If the production function is neoclassical, then  $\Pi = 0$ .

- b) Consider the standard Solow growth model, as covered in class.

- (i) [6 points] Let the production function take the form

$$Y_t = AF(K_t, L_t). \quad (2)$$

Suppose that, at some date  $t = t^*$ ,  $A$  doubles, and stays at that higher level forever. It follows from (2) that  $Y$  also doubles, at all dates  $t = t^*, t^* + 1, t^* + 2, \dots$ .

- (ii) [6 points] Suppose that the saving rate  $s$  increases. This will increase growth in the short run, but not in the long run.

- c) Consider the standard neoclassical model, as covered in class.

- (i) [6 points] The Euler equation for a benevolent planner is given by

$$u'(c_t) = \beta(1 + f'(k_{t+1}) - \delta)u'(c_{t+1}). \quad (3)$$

In the competitive decentralized equilibrium of the neoclassical growth model, the Euler equation for the representative household is instead given by

$$u'(c_t) = \beta(1 + r_{t+1}^k - \delta)u'(c_{t+1}). \quad (4)$$

(3) and (4) are different, so markets will not generate the same outcomes as a benevolent social planner.

- (ii) [6 points] In the competitive decentralized equilibrium of the model, a permanent increase in productivity  $A$  will increase the steady-state capital stock  $k^*$ , but it will leave steady-state wages  $w^*$  unchanged.

## Exercise 2 (35 points)

Consider a country that follows the Solow growth model in discrete time with savings rate  $s \in (0, 1)$  and depreciation rate  $\delta \in (0, 1)$ . Labor supply is normalized to one,  $L_t = \bar{L} = 1$ , so everything will be in per-labor units. The production function is given by

$$y_t = k_t^\alpha (A_t h_t)^{1-\alpha} \quad (5)$$

Here,  $A_t$  is a measure of productivity at time  $t$ . The variable  $h_t$  denotes the human capital index at time  $t$ .

- a) [7 points] First suppose  $A_t = A$  and  $h_t = h$  are constant. Write down the equation that determines the evolution of capital per labor. Characterize the steady-state levels of capital and output,  $k^*$  and  $y^*$ , in terms of the model's parameters, including in particular  $A$  and  $h$ . Using a graphical analysis, illustrate how the country converges to its steady state starting from some  $k_0 < k^*$ .
- b) [8 points] Suppose the country is initially at a steady state,  $k_0 = k^{*,old}$ , corresponding to some fixed parameters  $A^{old}$  and  $h^{old}$ . At date 0, thanks to an educational reform, the country's human capital index suddenly doubles and permanently remains at that higher level, i.e.,  $h_t = 2h^{old}$  for all  $t \geq 0$ . Productivity  $A_t$  however remains constant at  $A^{old}$ . How much does this change increase the country's output at date 0,  $y_0$ ? How much does it increase the country's output in the long run, after it adjusts to the new steady state? Provide an intuition for the difference between the short-run and long-run effects.

For the remaining parts, suppose the country's productivity is endogenous and evolves according to the following equation:

$$A_{t+1} - A_t = \eta h_t A^{world} - \gamma A_t \quad (6)$$

Here,  $A^{world} > 0$  can be thought of as the (constant, for simplicity) world technology frontier. The equation then says that, at every period, the country increases its technology by adopting some new ideas from the world frontier. The rate at which the country can tap into the world frontier depends on its human capital index,  $h_t$ . The rate of adoption also depends on the

parameter  $\eta > 0$ , which captures other factors that might affect research. Finally, in every period, the country “loses” a fraction  $\gamma A_t$  of its existing technology. This captures that some of the existing ideas might become obsolete over time. The parameter  $\gamma > 0$  captures the rate at which ideas “depreciate.”

- c) [10 points] Suppose  $h_t = h^{old}$  is constant over time. Characterize the steady-state level of technology,  $A^*$ , in terms of the model parameters  $\{\eta, h^{old}, A^{world}, \gamma\}$ . How does greater  $\eta$  affect  $A^*$ ?
- d) [10 points] Suppose the country is initially at a steady state,  $k_0 = k^{*,old}$  and  $A_0 = A^{*,old}$ , corresponding to the parameter  $h^{old}$ . Suppose also that the parameters are such that  $A^{*,old} = A^{old}$ , so that the initial steady state is exactly the same as that you studied in part b). The only difference is that the technology is now endogenized according to (6). Also as in part b), suppose the country’s human capital index suddenly doubles and permanently remains at that higher level, i.e.,  $h_t = 2h^{old}$  for all  $t \geq 0$ . How much does this change affect the country’s output in the long run after it adjusts to the new steady state (in capital as well as in technology)? Is your answer the same as or different than in part b) (for the long-run case)? Provide an intuition.

### Exercise 3 (35 points)

Consider the neoclassical growth model in continuous time. Output per labor is given by

$$y(t) = k(t)^\alpha h(t)^{1-\alpha}, \quad (7)$$

where  $k(t)$  denotes capital per labor,  $h(t)$  denotes human capital per labor, and  $\alpha$  denotes the share of capital in output. There is no population growth or technological progress (that is,  $n = 0$  and  $g = 0$ ). Capital per labor thus evolves according to

$$\dot{k}(t) = y(t) - \delta k(t) - c(t) \quad (8)$$

For now, suppose human capital per labor is constant,  $h(t) = h^{old}$  for all  $t$ , for some  $h^{old} > 0$ . The representative household has per-period utility  $u(c) = \frac{c^{1-\theta}-1}{1-\theta}$  for some  $\theta > 0$ , and her discount rate is  $\rho > 0$ . The depreciation rate is  $\delta \in (0, 1)$ .

- a) [5 points] Characterize steady-state capital, output, and consumption per labor,  $\{k^*, y^*, c^*\}$ .

Suppose the economy starts on this steady-state, but then there is a surprise change of government at date  $t = 0$ . The new government undertakes an educational reform. As a result of the reform, human capital per labor at  $t = 0$  jumps up from  $h^{old}$  to a higher level,

$h^{new} > h^{old}$ , and stays at that higher level forever. Furthermore suppose the reform does not cost any additional resources, e.g., because the government finds a better way of running the public school system without spending more on education.

- b) [10 points] Describe how this educational reform affects the long-run levels of capital, output, and consumption,  $\{k^*, y^*, c^*\}$ . You can use analytical derivations, or you can rely on a graphical analysis.
- c) [5 points] Using a graphical analysis, describe what happens to capital and output,  $\{k(t), y(t)\}$ , as the economy transitions to the new steady state.
- d) [5 points] Can you unambiguously tell what happens to consumption in the short-run,  $c(0)$ ? Why or why not? Your answer should describe the economic forces at play.
- e) [10 points] Now consider the more realistic scenario in which, after the new government comes into power,  $h(t)$  continuously increases from  $h^{old}$  to  $h^{new}$  over the time interval  $[0, t^*]$ . A macroeconomist analyzes the effects of the reform. Since she has taken 14.05, she decides to run a growth accounting exercise. Specifically, she considers the following accounting decomposition of the country's growth rate:

$$\underbrace{\frac{\dot{y}(t)}{y(t)}}_{g_y} = \alpha \underbrace{\frac{\dot{k}(t)}{k(t)}}_{g_k} + (1 - \alpha) \underbrace{\frac{\dot{h}(t)}{h(t)}}_{g_h} + \underbrace{x(t)}_{\text{Solow residual}} \quad (9)$$

She calculates the growth rates in this equation, as well as the Solow residual, for each year over the period  $[0, t^*]$ . She then takes averages over the entire period, and obtains the following results for the average growth rate and its decomposition:

- Average  $g_y$ : 2%
- Contribution from  $k$ : 0.66%
- Contribution from  $h$ : 1.33%
- Solow residual: 0%

Based on these results, she infers that both physical capital and human capital are important drivers of growth over this episode. In her analysis of the country's growth experience she thus decides to focus on savings policies (that might have increased the accumulation of physical capital) as well as education policies (that might have increased the accumulation of human capital). Is her interpretation of the data correct?

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