

Midterm Exam

Instructions: You have 90 minutes to complete the exam. Please be concise and write legibly. There are two parts: part A is worth 40 points and part B is worth 60 points. Answer all questions in both parts, using a separate bluebook for each part.

Part A (True, false or uncertain)

Say whether the following statements are true, false or uncertain. Justify your answer in a couple of paragraphs (10 points per statement).

1. An economy that increases its saving rate will experience faster growth.
2. Once human capital is included, the Solow model does a good job at explaining observed per capita income differences across countries.
3. Growth theory predicts that poor countries should grow faster than rich countries, and researchers have found the evidence to be consistent with this prediction.
4. A Ramsey economy that starts above the golden rule capital stock will converge to the golden rule. But if it starts below the golden rule it will also converge to a level below the golden rule because impatient consumers are unwilling to sacrifice consumption during the transition.

Part B

Consider the following model of growth. Output is given by

$$Y(t) = K(t)^\alpha [A(t)L(t)]^{1-\alpha}.$$

Capital and knowledge accumulate according to the equations

$$\begin{aligned} \dot{K}(t) &= sY(t), & s &\in (0, 1), \\ \dot{A}(t) &= Y(t)^\phi, & \phi &\in (0, 1). \end{aligned}$$

The population grows at rate n .

1. Give an economic interpretation of the equation for the accumulation of knowledge. (5 points)
2. For what value of ϕ does this model reduce to the Solow model? (5 points)

3. Let $g_A(t) = \frac{\dot{A}(t)}{A(t)}$ and $g_K(t) = \frac{\dot{K}(t)}{K(t)}$ be the growth rates of knowledge and capital, respectively. Find expressions for $g_A(t)$ and $g_K(t)$ in terms of $A(t)$, $K(t)$ and the parameters. (5 points)
4. Differentiate to obtain expressions for $\dot{g}_A(t)$ and $\dot{g}_K(t)$ in terms of $g_A(t)$, $g_K(t)$ and the parameters. Draw the phase diagram in (g_A, g_K) space. (10 points)
5. Does the economy converge to a balanced growth path? If so, what are the growth rates of K , A and Y on the balanced growth path? (5 points)
6. How does an increase in s affect long run growth? What about an increase in n ? What happens as ϕ gets close to one? (5 points)
7. Do you think that this is a reasonable model to explain differences in growth rates across countries? (5 points)

Now endogenize savings by introducing a representative household with utility function $U = \int_{t=0}^{\infty} e^{-\rho t} \frac{C(t)^{1-\sigma}}{1-\sigma} dt$ where $\rho > 0$, $\sigma > 0$ and $\rho > \frac{1-\sigma}{1-\phi}n$. Assume that capital and labor are paid their private marginal products. Households take their initial wealth and the paths of interest rates and wages as given, and choose the path of consumption to maximize U .

8. What is the private marginal product of capital in this economy? (5 points)
9. State the Euler equation and give an economic interpretation. (5 points)
10. What are the growth rates of K , A , Y and C on the balanced growth path (don't worry about convergence to the balanced growth path)? How do these growth rates depend on σ and ρ ? Discuss your results, comparing them with the results derived for a constant savings rate. If you cannot do the algebra, provide a discussion based on your intuition. (10 points)