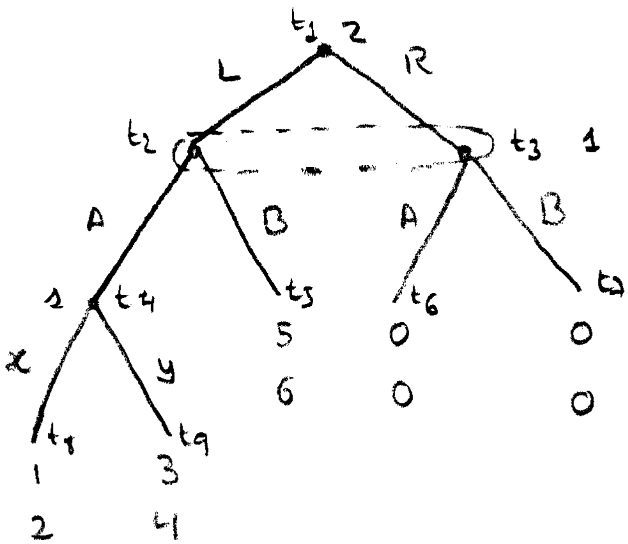
FINAL 2001

PROBLEM 1

PARTS A TO E: SEE CLASS-NOTES

PART F.



$H_1 = \{ \{t_2, t_3\}, \{t_4\} \} \Rightarrow 2^2$ possible strategies

($\{t_2, t_3\}$: 2 actions; $\{t_4\}$: 2 actions)

$H_2 = \{ \{t_1\} \}$

- Normal form game:

	L	R
Ax	1, 2	0, 0
Ay	3, 4	0, 0
Bx	5, 6	0, 0
By	5, 6	0, 0

- Subgames:

i)

ii) Whole game

PART G

		σ_A A	$1-\sigma_A$ B	C
σ_U	U	0, 5	3, 1	3, 0
$1-\sigma_U$	M	3, 1	1, 3	0, 1
	D	~~2, 0~~	~~0, 0~~	~~0, 2~~

i) ISD

$S_1^0 = \{U, M, D\}$ $\quad S_2^0 = \{A, B, C\}$

$\frac{1}{4} U + \frac{3}{4} M \succ_1 D$

$S_1^1 = \{U, M\}$ $\quad S_2^1 = \{A, B, C\}$

$B \succ_2 C$

$S_1^\infty = \{U, M\}$ $\quad S_2^\infty = \{A, B\}$

ii) NE_{PE}

$$\left.\begin{array}{l} \left.\begin{array}{l} BR_2(U) = A \\ BR_1(A) = M \end{array}\right\} \\ \left.\begin{array}{l} BR_2(M) = B \\ BR_1(B) = U \end{array}\right\} \end{array}\right\} \Rightarrow \text{No } NE_{PE}$$

iii) Indifference conditions

$$\left.\begin{array}{l} u_1(U, \sigma_2) = 3(1-\sigma_A) \\ u_1(M, \sigma_2) = 3\sigma_A + 1 - \sigma_A \end{array}\right\} \sigma_A = 2/5$$

$$\left.\begin{array}{l} u_2(\sigma_1, A) = 5\sigma_U + 1 - \sigma_U \\ u_2(\sigma_1, B) = \sigma_U + 3(1-\sigma_U) \end{array}\right\} \sigma_U = 1/3$$

iv) check NE → ok because we are considerant the whole relevant part of the support

$$NE_{ME} = \left\{ \left(\frac{2}{5} A + \frac{3}{5} B, \frac{1}{3} U + \frac{2}{3} M \right) \right\}$$

Part H

- We can solve this game using backward induction:

i) Second stage:

		σ_2 H	$1-\sigma_2$ T
σ_1	H	$1+a, -1$	$-1, 1$
$1-\sigma_1$	T	$-1, 1$	$1, -1$

- No NE_{PE}
- Indifference conditions:

$$(1+a)\sigma_2 - (1-\sigma_2) = -\sigma_2 + 1 - \sigma_2 \Rightarrow \sigma_2 = \frac{2}{4+a}$$

$$-\sigma_1 + (1-\sigma_1) = \sigma_1 - (1-\sigma_1) \Rightarrow \sigma_1 = \frac{1}{2}$$

$$NE\Big|_{G} = \left\{ \left(\frac{1}{2}H + \frac{1}{2}T,\ \frac{2}{4+a}H + \frac{2-a}{4+a}T \right) \right\} \Rightarrow \frac{a}{4+a},\ 0$$

ii) First stage:

$$\underset{\{a\}}{\text{Max}}\quad u_2 = \frac{a}{4+a} - \frac{a}{16}$$

<u>FOC</u>: $\dfrac{4+a-a}{(4+a)^2} - \dfrac{1}{16} = 0 \Rightarrow 64 = (4+a)^2 \Rightarrow \begin{cases} a = 4 \\ a = -12 \end{cases}$

<u>SOC</u>: $\left.\dfrac{-8}{(4+a)^3}\right|_{a=4} < 0 \rightarrow$ max

$\left.\dfrac{-8}{(4+a)^3}\right|_{a=-12} > 0 \rightarrow$ min

$$\Rightarrow SPE = \left\{ \left(\frac{1}{2}H + \frac{1}{2}T,\ a = 4;\ \frac{2}{4+a}H + \frac{2-a}{4+a}T \right) \right\} \Rightarrow 1/4,\ 0$$

PART I

- $BR_1(a_2) = \underset{a_1}{\text{Arg max}} \ (3-a_2)a_1 - a_1^2$

  FOC: $\quad 3 - a_2 - 2a_1 = 0 \Rightarrow BR_1(a_2) = \frac{3-a_2}{2}$

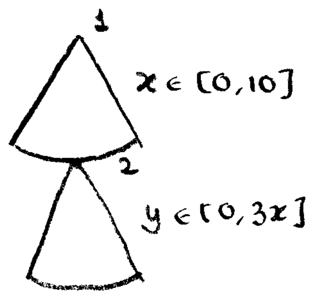
- $BR_2(a_1) = \text{Arg max} \ (4-a_1)a_2 - a_2^2$

  FOC: $\quad 4 - a_1 - 2a_2 = 0 \Rightarrow BR_2(a_1) = \frac{4-a_1}{2}$

- NE:

$$\left.\begin{aligned} a_1 &= \frac{3-a_2}{2} \\ a_2 &= \frac{4-a_1}{2} \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} a_1 &= 2/3 \\ a_2 &= 5/3 \end{aligned}\right.$$

PROBLEM 2



$u_1(x,y) = 10 - x + y$

$u_2(x,y) = 3x - y$

PART A: SPE

- Backward induction:

  i) Stage 2: Given x, Player 2 chooses y

  $$BR_2(x) = \underset{y}{\text{ArgMax}}\ 3x - y \Rightarrow BR_2(x) = 0 \quad \forall x$$

  ii) Stage 1:

  $$\underset{x}{\text{Max}}\ 10 - x - y(x) \Rightarrow x = 0$$
  $$\text{st} \quad y(x) = 0$$

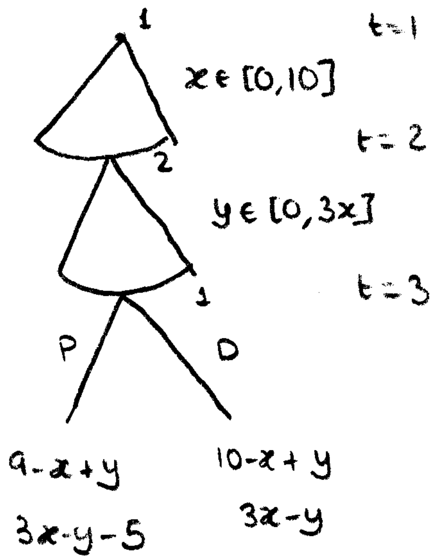
$$\Rightarrow \quad SPE = \{ x = 0,\ y(x) = 0 \quad \forall x \} \rightarrow (10, 0)$$

PART B: NE WITH BIGGER PAYOFFS

- Note that in every possible NE Player 2 should play $y(x) = 0$ along the equilibrium path. This is the worse possible punishment for player 1, so there is no noncreaible threat that induces Player 1 to not play $x = 0$ in the first stage.
- In summary, the unique NE is the SPE that we have found in the previous part.

PART C:

i) PUNCHING



$$SPE = \{ (x=0,\ s_1^3(x,y) = D \ \ \forall x,y);\ (y(x)=0 \ \ \forall x) \}$$

Nothing change because the NE of the last subgame is D

Claim: $\sigma_1 = \begin{cases} x = 2 \\ s_1^3(x,y) = \begin{cases} D & \text{if } x=2,\ y=4 \\ P & \text{otherwise} \end{cases} \end{cases}$, $\sigma_2 = \{ y(x) = \begin{cases} 4 & x=2 \\ 0 & \text{otherwise} \end{cases} \}$

is a NE

Proof:

i) No profitable deviation for player 2

$$\left.\begin{aligned} u_2(\sigma_1, y=4) &= 3\cdot 2 - 4 = 2 \\ u_2(\sigma_1, y=0) &= 3\cdot 2 - 5 = 1 \end{aligned}\right\} \Rightarrow y=4 \in BR_2(\sigma_1)$$

ii) No profitable deviation for player 1:

$$\left.\begin{aligned} u_1(x=2,\ s_1^3(x,y), y=4) &= 10-2+4 = 12 \\ u_1(x=0,\ s_1^3(x,y),\ y=4) &\leq 10 \\ u_1(x=2,\ \tilde{s}_1^3,\ y=4) &\leq 12 \end{aligned}\right\} \sigma_1 \in BR_1(y=4)$$

$\Rightarrow$ There is a NE with outcome $(12,2) > (10,0)$

ii) ADDITIONAL BOX

	A	B
A	5, 5	-5, -5
B	-5, -5	5, 5

The NE of this subgame are:

$$\{ (A,A),\ (B,B),\ (\tfrac{1}{2}A+\tfrac{1}{2}B,\ \tfrac{1}{2}A+\tfrac{1}{2}B) \}$$

$$\downarrow \quad (5,5) \qquad \downarrow \quad (5,5) \qquad \downarrow \quad (0,0)$$

<u>Claim</u>:

$$\sigma_1 = \begin{cases} x = 2 \\ s_1^3(x,y) = \begin{cases} A & \text{if } x=2,\ y=4 \\ \frac{1}{2}A+\frac{1}{2}B & \text{otherwise} \end{cases} \end{cases} \qquad \sigma_2 = \begin{cases} y(x) = \begin{cases} 4 & x=2 \\ 0 & \text{otherwise} \end{cases} \\ s_2^3(x,y) = \begin{cases} A & \text{if } x=2,\ y=4 \\ \frac{1}{2}A+\frac{1}{2}B & \text{otherwise} \end{cases} \end{cases}$$

is a SPE.

<u>Proof</u>: . $t=3 \rightarrow$ for every possible history of the game the players play a NE

- $t=2$

Eq. path $\begin{cases} u_2(\sigma_1, \sigma_2) = 3x - y + 5 = 6-4+5 = 7 \\ u_2(\sigma_1, y=0) = 6 \end{cases}$ $\quad u_2(\sigma_1,\sigma_2) \geq u_2(\sigma_1, \tilde{\sigma}_2)$

Off equilibrium path player 2 is playing the NE of the subgame

- $t=1$

$\begin{cases} u_1(x=2, \sigma_2) = 10-2+4+5 = 17 \\ u_1(x=0, \sigma_2) = 10 \end{cases}$ $\quad u_1(x=2,\sigma_2) \geq u_1(x\neq 2, \sigma_2)$

PART D:

$$\tilde{u}_1(x,y) = 10 - x + y + \alpha_1(3x - y)$$

$$\tilde{u}_2(x,y) = 3x - y + \alpha_2(10 - x + y)$$

- Backward induction:

1) Stage 2:

$BR_2(x) = \underset{y}{\text{Arg Max}} \quad \hat{u}_2(x,y) > 0 \iff \alpha_2 > 1 \rightarrow$ Player 2 cares more about Player's 1 utility than about herself.

FOC $\quad -1 + \alpha_2 = 0$

- For the last part I do not have any good intuition, but I guess that you can introduce some sort of incomplete information about the rationality of the players

PROBLEM 3

PART A:

- The payoff functions of the players are:

$$u_i(t_i,t_j) = \begin{cases} (1-\min\{t_i,t_j\})\,\mathbb{1}\{t_i > t_j\} - \min\{t_i,t_j\} & \text{if } t_i \neq t_j \text{ and } \min\{t_i,t_j\}<1 \\ -1 & \text{otherwise} \end{cases}$$

- <u>Claim</u>: $t_i = 0$, $t_j = t > 0$ is a NE

<u>Player i</u>: $u_i(0,t) = 0$

$u_i(s,t) = -s < 0 \quad \forall s < t$

$u_i(t,t) = -1 < 0$

$u_i(s,t) = (1-t) - t = 1-2t \quad \forall s > t$

Therefore $BR_i(t) = 0 \iff 1-2t \leq 0 \iff t \geq 1/2$

<u>Player j</u>: $u_j(0,s) = 1 \quad \forall s > t \Rightarrow BR_j(0) = (0,1] \Rightarrow t \in BR_j(0)$

$$\Rightarrow \quad NE_{PE} = \{(t_i = 0,\ t_j = t) \text{ st } t \geq 1/2\}$$

- <u>Claim</u>: There is no NE such that $\min\{t_i,t_j\} > 0$

<u>Proof</u>: let's assume by contradiction that $\exists$ NE st $t_i = \min\{t_i,t_j\} > 0$

$$\left.\begin{array}{l} u_i(t_i,t_j) = -t_i \\ u_i(0,t_j) = 0 > -t_i \end{array}\right\} \Rightarrow t_i \notin BR_i(t_j)$$

PART B

- **Claim:** $\{(\sigma_1 \sim \mathcal{U}(0,\bar{t}), \sigma_2 \sim \mathcal{U}(0,\bar{t}))\}$ is a NE_{NS} with $\bar{t}=1$

Proof: i) $\bar{t}=1$

From the indifference conditions, we have:

$$u_i(t,\sigma_j) = u_i(s,\sigma_j) \qquad \forall t,s \in \text{supp}\{\sigma_i\} = [0,\bar{t}]$$

In particular:

$$u_i(0,\sigma_j) = u_i(\bar{t},\sigma_j)$$

$$\left.\begin{aligned} &u_i(0,\sigma_j) = 0 \\ &u_i(\bar{t},\sigma_j) = E_{\sigma_j}[(1-\sigma_j)-\sigma_j] = 1-2E_{\sigma_j}[\sigma_j] = 1-\bar{t} \end{aligned}\right\} \Rightarrow \bar{t}=1$$

ii) $\{(\sigma_1 \sim \mathcal{U}(0,1), \sigma_2 \sim \mathcal{U}(0,1))\}$ is a NE

$$u_i(t,\sigma_j) = E_{\sigma_j}\left[(1-\min\{t,\sigma_j\})\mathbb{1}\{\sigma_j<t\} - \min\{t,\sigma_j\}\right] =$$

$$= E_{\sigma_j}\left[\mathbb{1}\{\sigma_j<t\} - (1+\mathbb{1}\{\sigma_j<t\})\min\{t,\sigma_j\}\right] =$$

$$= F_{\sigma_j}(t) - E_{\sigma_j}\left[2\sigma_j \mathbb{1}\{\sigma_j<t\} + t\,\mathbb{1}\{\sigma_j \geq t\}\right] =$$

$$= t - 2\int_0^t s\,ds - t(1-t) = t - t^2 - t(1-t) = 0$$

$\Rightarrow$ The indifference condition is satisfied

PART C

$\theta_1 = \begin{cases} 0.5 & 1/3 & S \\ -2 & 2/3 & W \end{cases}$ $\theta_2 = \begin{cases} 0.2 & 3/5 & S \\ -3 & 2/5 & W \end{cases}$

① \ ②	0	1/2
0	0, 0	0, 1
1/2	1, 0	$\frac{\theta_1}{2}$, $\frac{\theta_2}{2}$

- The strategies of each player have the form $S_i = (a_i | \theta_i = S, a_i | \theta_i = W)$

Therefore we will have the following normal form game:

①	0,0	0,1/2	1/2,0	1/2,1/2
0,0	X	X	X	(1)
0,1/2	X	X	X	X
1/2,0	X	X	(2)	X
1/2,1/2	X	X	(3)	X

- Observations:

i) $BR_i(0,0) = 1/2, 1/2$ for every type

ii) Against everything, except for $(0,0)$, $(0,1/2)$ is worse than either $(1/2,1/2)$ or $(0,0)$ for both players

iii) If $\theta_2 = 0.2$, for player 2 $t=0$ is strictly dominated by $t=1/2$

iv) $BR_1(1/2,1/2) = (0,0)$

- Possible BNE:

(1) $f_1 = (0,0)$, $f_2 = (1/2, 1/2)$

$$\left.\begin{aligned} u_1(0, f_2 \mid \theta_1 = -0.5) &= 0 \\ u_1(1/2, f_2 \mid \theta_1 = -0.5) &= -1/4 \end{aligned}\right\} \quad BR_1(f_2 \mid \theta_1 = -0.5) = 0$$

$$\left.\begin{aligned} u_1(0, f_2 \mid \theta_1 = -2) &= 0 \\ u_1(1/2, f_2 \mid \theta_1 = -2) &= -1 \end{aligned}\right\} \quad BR_1(f_2 \mid \theta_1 = -2) = 0$$

$\Rightarrow$ (f_1, f_2) is a BNE

(2) $f_1 = (1/2, 0)$, $f_2 = (1/2, 0)$

$$\left.\begin{aligned} u_1(1/2, f_2 \mid \theta_1 = -0.5) &= \frac{3}{5}\,\frac{-1}{4} + \frac{2}{5}\,1 = 1 \\ u_1(0, f_2 \mid \theta_1 = -0.5) &= 0 \end{aligned}\right\} \quad BR_1(f_2 \mid \theta_1 = -0.5) = 1/2$$

$$\left.\begin{aligned} u_1(0, f_2 \mid \theta_1 = -2) &= 0 \\ u_1(1/2, f_2 \mid \theta_1 = -2) &= -\frac{3}{5} + \frac{2}{5} = \frac{-1}{5} \end{aligned}\right\} \quad BR_1(f_2 \mid \theta_1 = -2) = 0$$

$$\left.\begin{aligned} u_2(f_1, 1/2 \mid \theta_2 = 0.2) &= -\frac{1}{3}\,\frac{1}{10} + \frac{2}{3} = \frac{19}{30} \\ u_2(f_1, 0 \mid \theta_2 = 0.2) &= 0 \end{aligned}\right\} \quad BR_2(f_1 \mid \theta_2 = 0.2) = 1/2$$

$$\left.\begin{aligned} u_2(f_1, 0 \mid \theta_2 = -3) &= 0 \\ u_2(f_1, 0 \mid \theta_2 = -3) &= \frac{-1}{3}\,\frac{3}{2} + \frac{2}{3} = \frac{1}{6} \end{aligned}\right\} \quad BR_2(f_1 \mid \theta_2 = -3) = 1/2$$

$\Rightarrow$ (f_1, f_2) is not a BNE

(3) $\ell_1 = (1/2, 1/2)$, $\ell_2 = (1/2, 0)$

$$u_1(1/2, \ell_2 \mid \theta_1 = -0.5) = \frac{3}{5}\frac{-1}{4} + \frac{2}{5} = 1$$
$$u_1(0, \ell_2 \mid \theta_1 = -0.5) = 0$$
$BR_1(\ell_2 \mid \theta_1 = -0.5) = 1/2$

$$u_1(1/2, \ell_2 \mid \theta_1 = -2) = \frac{3}{5}(-1) + \frac{2}{5} = -\frac{1}{5}$$
$$u_1(0, \ell_2 \mid \theta_1 = -2) = 0$$
$BR_1(\ell_2 \mid \theta_2 = -2) = 0$

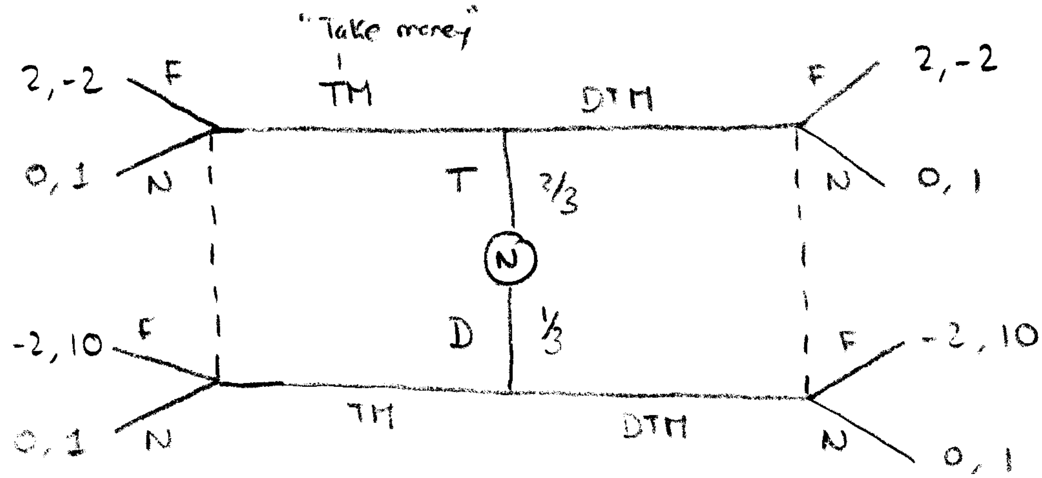
$\Rightarrow (\ell_1, \ell_2)$ is not a BNE

- In summary, the unique BNE is:

$$\{\ell_1 = (0,0),\ \ell_2 = (1/2, 1/2)\}$$

PROBLEM 4

PART A: GAME TREE



PART B: NO SEPARATING EQUILIBRIUM TM, DTM

- Beliefs:

$\mu_2(T \mid TM) = 1$

$\mu_2(T \mid DTM) = 0$

- BR_2 given beliefs:

$\left.\begin{array}{l} u_2(F \mid TM) = -2 \\ u_2(N \mid TM) = 1 \end{array}\right\} BR_2(TM) = N$

$\left.\begin{array}{l} u_2(F \mid DTM) = 10 \\ u_2(N \mid DTM) = 1 \end{array}\right\} BR_2(DTM) = F$

BR_1 given BR_2

- Type T:

$\left.\begin{array}{l} u_1(TM \mid BR_2) = 0 \\ u_1(DTM \mid BR_2) = 2 \end{array}\right\} BR_1 = DTM$ → Given Douglas' beliefs, Tyson prefers always to cheat.

PART C: POOLING EQUILIBRIUM TM, TM

- Beliefs:

$$\mu_2(T \mid TM) = 2/3$$

$$\mu_2(T \mid DTM) = \lambda$$

BR_2 given beliefs:

$$\left.\begin{array}{l} u_2(F \mid TM) = \frac{2}{3}(-2) + \frac{1}{3} 10 = 2 \\ u_2(N \mid TM) = 0 \end{array}\right\} BR_2(TM) = F$$

$$\left.\begin{array}{l} u_2(F \mid DTM) = \lambda(-2) + (1-\lambda) 10 = 10 - 12\lambda \\ u_2(N \mid DTM) = 0 \end{array}\right\} BR_2(DTM) = \begin{cases} F & \text{if } \lambda < 5/6 \\ \alpha F + (1-\alpha) N & \text{if } \lambda = 5/6 \\ N & \text{if } \lambda > 5/6 \end{cases}$$

- BR_1 given BR_2

- Type T.

$$\left.\begin{array}{l} u_1(TM \mid BR_2) = 2 \\ u_1(DTM \mid BR_2) = \begin{cases} 2 & \lambda < 5/6 \\ 2\alpha & \lambda = 5/6 \\ 0 & \lambda > 5/6 \end{cases} \end{array}\right\} \Rightarrow BR_1(BR_2) = TM$$

- Type D:

$$\left.\begin{array}{l} u_1(TM \mid BR_2) = -2 \\ u_1(DTM \mid BR_2) = \begin{cases} -2 & \lambda < 5/6 \\ -2\alpha & \lambda = 5/6 \\ 0 & \lambda > 5/6 \end{cases} \end{array}\right\} \Rightarrow BR_1(BR_2) = TM \text{ if } \lambda \le 5/6 \text{ \& } \alpha = 1$$

- PBE:

$$\mu_2(T \mid TM) = 2/3 \qquad \mu_2(T \mid DTM) \le 5/6$$

$$S_1(T) = TM \qquad S_1(D) = TM$$

$$S_2(TM, \mu_2) = F \qquad S_2(DTM, \mu_2) = F$$

PART D: SEMISEPARATING EQUILIBRIA

1) POOLING EQUILIBRIUM DTM, DTM

- Beliefs:

$$\mu_2(T \mid TM) = \lambda$$
$$\mu_2(T \mid DTM) = 2/3$$

- BR_2 given beliefs:

$$\left.\begin{aligned} u_2(F \mid TM) &= -2\lambda + 10(1-\lambda) = 10-2\lambda \\ u_2(N \mid TM) &= 0 \end{aligned}\right\} \quad BR_2(TM) = \begin{cases} F & \text{if } \lambda < 5/6 \\ \alpha F + (1-\alpha) N & \text{if } \lambda = 5/6 \\ N & \text{if } \lambda > 5/6 \end{cases}$$

$$\left.\begin{aligned} u_2(F \mid DTM) &= -2\,\frac{2}{3} + 10\,\frac{1}{3} = 2 \\ u_2(N \mid DTM) &= 0 \end{aligned}\right\} \quad BR_2(DTM) = F$$

- BR_1 given BR_1:
  - Type T:

$$\left.\begin{aligned} u_1(TM \mid BR_2) &= \begin{cases} 2 & \lambda < 5/6 \\ 2\alpha & \lambda = 5/6 \\ 0 & \lambda > 5/6 \end{cases} \\ u_1(DTM \mid BR_2) &= 2 \end{aligned}\right\} \quad BR_1(BR_2) = DTM \text{ for } \lambda = 5/6 \text{ and } \alpha \in [0,1]$$

  - Type D:

$$\left.\begin{aligned} u_1(TM \mid BR_2) &= \begin{cases} -2 & \lambda < 5/6 \\ -2\alpha & \lambda = 5/6 \\ 0 & \lambda > 5/6 \end{cases} \\ u_1(DTM \mid BR_2) &= -2 \end{aligned}\right\} \quad BR_1(BR_2) = TM \text{ for } \lambda = 5/6 \text{ \& } \alpha \in (0,1)$$

$\Rightarrow$ No semiseparating equilibrium

ii) SEMISEPARATING EQUILIBRIUM $\alpha TM+(1-\alpha)DTM,\ DTM$

- Beliefs:

$$\mu_2(T\mid TM) = 1$$

$$\mu_2(T\mid DTM) = \frac{(1-\alpha)\,2/3}{(1-\alpha)\,2/3 + 1/3} = \frac{2(1-\alpha)}{2(1-\alpha)+1} = \frac{2(1-\alpha)}{3-\alpha} = \lambda$$

- BR_2 given beliefs:

$$\left.\begin{aligned} u_2(F\mid TM) &= -2 \\ u_2(N\mid TM) &= 0 \end{aligned}\right\} \quad BR_2(TM) = N$$

$$\left.\begin{aligned} u_2(F\mid DTM) &= -2\lambda + 10(1-\lambda) = 10-12\lambda \\ u_2(N\mid DTM) &= 0 \end{aligned}\right\} \quad BR_2(DTM) = \begin{cases} F & \text{if } \lambda < 5/6 \\ pF+(1-p)N & \text{if } \lambda = 5/6 \\ N & \text{if } \lambda > 5/6 \end{cases}$$

- BR_1 given BR_2:

 - Type T:

$$\left.\begin{aligned} u_1(TM\mid BR_2) &= 2 \\ u_1(DTM\mid BR_2) &= \begin{cases} -2 \\ -2p \\ 0 \end{cases} \end{aligned}\right\} \quad u_1(TM\mid BR_2) > u_1(DTM\mid BR_2)$$

$\Rightarrow$ No semiseparating equilibrium

- Following a symmetric argument, there is not also Semiparametric Equilibrium where the first type plays a pure strategy and the second one mixes

iii) SEMISEPARATING EQUILIBRIUM $\alpha TM + (1-\alpha) DTM$, $\beta TM + (1-\beta) DTM$

- Beliefs:

$$\mu_2(T \mid TM) = \frac{\frac{2}{3}\alpha}{\frac{2}{3}\alpha + \frac{1}{3}\beta} = \frac{2\alpha}{2\alpha + \beta} = \lambda_1$$

$$\mu_2(T \mid DTM) = \frac{\frac{2}{3}(1-\alpha)}{\frac{2}{3}(1-\alpha) + \frac{1}{3}(1-\beta)} = \frac{2(1-\alpha)}{3 - 2\alpha - \beta} = \lambda_2$$

- BR_2 given beliefs:

$$\left.\begin{array}{l} u_2(F \mid TM) = -2\lambda_1 + 10(1-\lambda_1) \\ u_2(N \mid TM) = 0 \end{array}\right\} \quad BR_2(TM) = \begin{cases} F & \lambda_1 < 5/6 \\ pF + (1-p)N & \lambda_1 = 5/6 \\ N & \lambda_1 > 5/6 \end{cases}$$

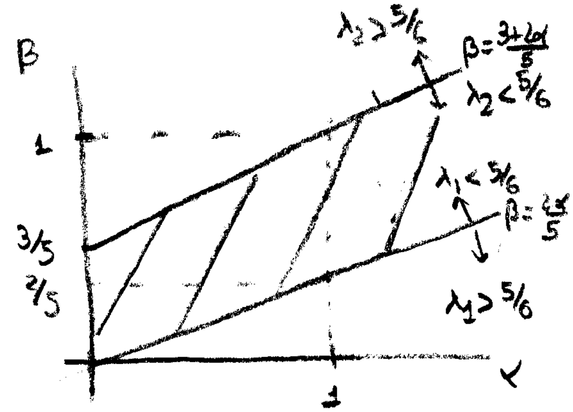
$$\left.\begin{array}{l} u_2(F \mid DTM) = -2\lambda_2 + 10(1-\lambda_2) \\ u_2(N \mid DTM) = 0 \end{array}\right\} \quad BR_2(DTM) = \begin{cases} F & \lambda_2 < 5/6 \\ qF + (1-q)N & \lambda_2 = 5/6 \\ N & \lambda_2 > 5/6 \end{cases}$$

- BR_1 given BR_2:

  - Type T:

$$u_1(TM \mid BR_2) = \begin{cases} 2 & \lambda_1 < 5/6 \\ 2p & \lambda_1 = 5/6 \\ 0 & \lambda_1 > 5/6 \end{cases}$$

$$u_1(DTM \mid BR_2) = \begin{cases} 2 & \lambda_2 < 5/6 \\ 2q & \lambda_2 = 5/6 \\ 0 & \lambda_2 > 5/6 \end{cases}$$



$u_1(TM \mid BR_2) = u_1(DTM \mid BR_2)$ (Indifference condition)

$$\lambda_1 < 5/6 \iff 12\alpha < 10\alpha + 5\beta \iff \beta > \tfrac{2}{5}\alpha$$

$$\lambda_2 < 5/6 \iff 12 - 12\alpha < 15 - 10\alpha - 5\beta \iff \beta < \frac{3 + 2\alpha}{5}$$

- Type D:

$$u_1(TM \mid BR_2) = \begin{cases} -2 & \lambda_1 < 5/6 \\ -2p & \lambda_1 = 5/6 \\ 0 & \lambda_1 > 5/6 \end{cases}$$

$$u_1(DTM \mid BR_2) = \begin{cases} -2 & \lambda_2 < 5/6 \\ -2p & \lambda_2 = 5/6 \\ 0 & \lambda_2 > 5/6 \end{cases}$$

→ same conditions as for type T.

- PBE:

$S_2(TM) = F$ $\qquad$ $S_2(DTM) = F$

$S_1(T) = \alpha\, TM + (1-\alpha)\, DTM$ $\qquad$ $S_1(D) = \beta\, TM + (1-\beta)\, DTM$

$\mu_2(T \mid TM) = \dfrac{2\alpha}{2\alpha + \beta}$ $\qquad$ $\mu_2(T \mid DTM) = \dfrac{2(1-\alpha)}{3 - 2\alpha - \beta}$

$\beta \in \left(2/5\, \alpha, \dfrac{3+2\alpha}{5} \right)$ $\qquad$ $\alpha \in (0, 1)$