14.123 Microeconomic Theory III. 2014

Problem Set 1. Solution.

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1. See solution by Suehyun Kwon of Problem Set 1, 2010, Question 2.

2.

  2.1 i. Yes. For example, $u(x) = 3, u(y) = 2, u(z) = 0$.

  ii. No. Suppose that such expected utility representation exists. Without loss of generality, normalize $u(z) = 0$ and from the ordering of the lotteries it follows

$$\frac{u(x)}{4} + \frac{u(y)}{4} > \frac{3u(x)}{4} > \frac{5u(x)}{6} + \frac{u(y)}{6} > \frac{u(x)}{2} + \frac{u(y)}{3}.$$

  From the first inequality, $-2u(x) + u(y) > 0$, from the third inequality $2u(x) - u(y) > 0$ which contradicts the first inequality.

  2.2 i. Yes, take $u(x) = 2, u(y) = 1, u(z) = 0$.

  ii. No. Consider $p = (1, 0, 0), q = (0, 0, 1)$ which are equivalent and consider a half-half mixture of them with $r = (0, 1, 0)$. Then $p' = (1/2, 1/2, 0)$ and $q' = (0, 1/2, 1/2)$ lie on the different indifference sets, which contradicts IA.

  iii. No, as the indifference sets are not straight lines which contradicts IA.

  2.3 Consider lexicographic preferences: $p \succ q$ if and only if $p(x) > q(x)$ or $p(x) = q(x)$ and $p(y) > q(y)$. Since this preference is discontinuous, there is no representation, let alone expected utility representation.

3. In this question I refer to the second condition in Definition 3.2 in Lecture notes as (*). I will also use the following consequence of (*).

   Claim. Consider $B, C, D \in \mathcal{A}$ such that $D \subseteq B \cap C$, $D \subseteq B \cap C$. Then $B \dot{\succeq} C \iff B \backslash D \dot{\succeq} C \backslash D$.

   I refer to this claim by (**). To see that it is true, observe that $B = (B \backslash D) \cup D$ and $C = (C \backslash D) \cup D$. Since $(B \backslash D) \cap D = \emptyset$ and $(C \backslash D) \cap D = \emptyset$, it follows by (*) that $B \dot{\succeq} C \iff B \backslash D \dot{\succeq} C \backslash D$.

3.1 Let $X = A_1 \cap B_2$, $A_1' = A_1 \backslash X$, $B_2' = B_2 \backslash X$. By (**), $A_1 \cup A_2 \dot{\succeq} B_1 \cup B_2 \iff A_1' \cup A_2 \dot{\succeq} B_1 \cup B_2'$. Then

$$A_1' \cup A_2 \dot{\succeq} A_1' \cup B_2 = A_1 \cup B_2' \dot{\succeq} (B_1 \backslash B_2') \cup B_2' = B_1 \cup B_2'$$

where I used twice (*) to get inequalities and equalities are simple set manipulations.

3.2 Denote by $\dot{\geq}$ preference relation "$\dot{\succeq}$ given D". Completeness and transitivity of $\dot{\geq}$ follow from completeness and transitivity of $\dot{\succeq}$. Consider $B, C, E \in \mathcal{A}$ such that $B \cap E = C \cap E = \emptyset$. Then condition 2 in the definition of the qualitative probability is obtained by the following line of inequalities.

$$B \dot{\geq} C \iff B \cap D \dot{\succeq} C \cap D \iff (B \cap D) \cup E \dot{\succeq} (C \cap D) \cup E \iff$$

$$(B \cap D) \cup (E \cap D) \cup (E \backslash D) \dot{\succeq} (C \cap D) \cup (E \cap D) \cup (E \backslash D) \iff$$

$$(B \cap D) \cup (E \cap D) \dot{\succeq} (C \cap D) \cup (E \cap D) \iff B \cup E \dot{\geq} C \cup E,$$

where I use (*) and set manipulations to obtain the equivalence relation. $B \dot{\geq} \emptyset$ follows from the corresponding property of $\dot{\succeq}$, and $S \dot{>} \emptyset$ follows from D being non-null.

3.3 Suppose to contradiction that $A_1 \dot{\succ} B_1$. By transitivity of $\dot{\sim}$, for all $1 \leq i \leq n$, $A_i \dot{\succ} B_i$. By the argument as in part 3.1 of this question, it is possible to show that $A_1 \cup A_2 \dot{\succ} B_1 \cup B_2$ and iteratively applying this inequality I get that $S = \cup_{i=1}^n A_i \dot{\succ} \cup_{i=1}^n B_i = S$, contradiction.

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