14.123 Microeconomics III—Problem Set 2

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Instructions. You are encouraged to work in groups, but everybody must write their own solutions. Each question is 33 points. Good Luck!

1. This question is about experimental design to avoid the complications due to risk aversion. There are two subjects, named Ann and Bob, with unknown utility functions $u_A : \mathbb{R} \to \mathbb{R}$ and $u_B : \mathbb{R} \to \mathbb{R}$, respectively. The utility functions are normalized by setting $u_i(0) = 0$ and $u_i(1) = 1$ for each subject i. A bargaining problem is defined as a set L of lottery pairs (p, q) on $\mathbb{R}$. If the subjects agree on some $(p_A, p_B) \in L$, then Ann and Bob get p_A and p_B, respectively; each gets 0 otherwise. Assume that there exists $b_A \in (0, 1)$ such that for each bargaining problem L, the subjects agree on some $(p_A^*(L), p_B^*(L)) \in L$ such that $E_{p_i^*(L)}[u_i] > 0$ for each i and

$$\frac{E_{p_A^*(L)}[u_A]}{E_{p_B^*(L)}[u_B]} = \frac{b_A}{1 - b_A}$$

whenever such a pair exists. Here, b_A is called the bargaining power of Ann. Find a bargaining set L such that some $(p_A^*(L), p_B^*(L))$ as above exists for all utility functions u_A and u_B as above, and one can determine b_A from $(p_A^*(L), p_B^*(L))$ without any knowledge of u_A and u_B.

2. This question illustrates how one can introduce lotteries in Savage's subjective model. Consider the set of acts $f : S \times [0, 1] \to C$ with

$$f(s, x) = \begin{cases} 1 & \text{if } x \le p_f(s) \\ 0 & \text{otherwise} \end{cases}$$

for some $p_f : S \to [0, 1]$ where S is a finite set of "base" states and $C = \{0, 1\}$ the set of consequences. (Here, one can consider $p_f(s)$ as a lottery in which the probability of $c = 1$ is $p_f(s)$.) Consider a preference relation $\succeq$ between such acts with $1 \succ 0$, satisfying P1-P3.

   (a) Given any f and g, show that if $p_f(s) \ge p_g(s)$ for every $s \in S$, then $f \succeq g$.

   (b) For every $A \subseteq S$ and $\pi \in [0, 1]$, define acts f_A and f_π by

$$f_A(s, x) = \begin{cases} 1 & \text{if } s \in A \\ 0 & \text{otherwise} \end{cases} \quad \text{and } f_\pi(s, x) = \begin{cases} 1 & \text{if } x \le \pi \\ 0 & \text{otherwise.} \end{cases}$$

   For every $A \subseteq S$, let

$$P(A) = \sup\{\pi | f_A \succeq f_\pi\}.$$

   1. Define a continuity assumption on $\succeq$ under which $f_A \sim f_{P(A)}$.
   2. Check if P is a probability distribution on A.

(c) Under the above assumptions, check whether $\succeq$ has the following "expected utility representation":

$$f \succeq g \iff \sum_{s \in S} P(s)\, p_f(s) \geq \sum_{s \in S} P(s)\, p_g(s)$$

where $P(s) = P(\{s\})$.

3. This question is about Becker–DeGroot–Marschak mechanism, used in experimental economics to elicit beliefs. Let S and C be as in the previous question, and fix some $A \subset S$. Assuming that the subject is an expected utility maximizer, an experimenter asks a subject to submit a number $p \in [0,1]$, and then selects a random number q from a finite set $Q \subset [0,1]$ If $p < q$, then the subject receives a lottery ticket with probability q on $c = 1$. Otherwise, the subject receives $c = 1$ if $s \in A$ and $c = 0$ if $s \notin A$. The subject knows the mechanism when she submits p.

   (a) Formulate an extended state space to formulate the decision problem of the subject. (You can also extend the space of consequences if you want.) Define an act $\tilde{f}_p$ that corresponds to submitting p to the mechanism.

   (b) Formulate an assumption on the preferences $\succeq$ that states that, according to the subject, q and the outcome of the lottery ticket above are stochastically independent of A. In the remainder of the problem assume this independence assumption and the postulates P1-P3.

   (c) Define $P(A)$ using $\succeq$ in such a way that $P(A)$ corresponds to the probability the subject assigns to A.

   (d) Check whether

$$\tilde{f}_{P(A)} \succeq \tilde{f}_p \qquad (\forall p \in [0,1]).$$