14.126 GAME THEORY

PROBLEM SET 2

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Question 1

Consider the complete information game

	α	β
α	θ, θ	$\theta - c, 0$
β	$0, \theta - c$	$0, 0$

where $c > 0$ and θ is equal to some known value $\hat{\theta} \in (0, c/2)$. Imagine now an email game scenario in which there are two possible values of θ, namely $\hat{\theta}$ and θ', with some prior probabilities p and $1-p$. Player 1 knows the value of θ, and if $\theta = \hat{\theta}$ then the email exchange takes place, where each email is lost with probability $\varepsilon \in (0, 1)$. If $\theta = \theta'$ then no emails are exchanged. For each action $a \in \{\alpha, \beta\}$, find the range of ε for which there is some email game (i.e. some choice of θ' and p) in which a is the unique rationalizable action for each type. Briefly discuss your finding.

Question 2

Let $G = (N, A, u)$ be a finite normal-form game. Suppose the players N play an infinite repetition of G, but instead of discounting, players care only about the maximum of the per-period payoffs. That is, in each period $t = 0, 1, 2, \ldots$, the stage game G is played, with each player having observed the action profile chosen at every previous period. This gives rise to an infinite history of action profiles $(a^0, a^1, a^2, \ldots)$ (which may be random, if the players are mixing). For each realization of such a history, player i's payoff in the repeated game is

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defined to be $\max_{t\geq 0} u_i(a^t)$. Prove that (a) this repeated game is in general not continuous at infinity, but (b) the single-deviation principle still holds.

Question 3

Find all (a) Nash, (b) trembling-hand perfect, (c) proper equilibria (in pure or mixed strategies) of the following normal-form game.

	L	R
U	2,2	2,2
M	3,3	1,0
D	0,0	1,1

Question 4

Give an example of a finite normal-form game G and a strategy profile σ such that for each player i, there exists a sequence $\sigma^1_{-i}, \sigma^2_{-i}, \ldots$ of independent trembles of i's opponents (i.e. each σ^k_{-i} specifies a full-support distribution over strategy profiles of players $-i$ in which the various players $j \neq i$ mix independently of each other), converging to σ_{-i}, such that σ_i is a best response to σ^k_{-i} for each k, but σ is not a perfect equilibrium of G.

Question 5

Is the following statement true or false? Give a proof or counterexample. Suppose G is a finite extensive-form game with perfect recall, and $h_x = \{x, x'\}$, $h_y = \{y, y'\}$ are two information sets, such that x is a predecessor of y, x' is a predecessor of y', and the action taken from x along the tree toward y is the same as the action taken from x' toward y'. Then there cannot be a consistent assessment (σ, μ) such that $\mu(x|h_x) = 1$ and $\mu(y|h_y) = 0$. (Note that this looks like the "no signaling what you don't know" condition, but now we do not require that x immediately precedes y; there may be other nodes in between.)

Question 6

Consider the following version of Rubinstein alternating offers bargaining game. There are three players and utility of player $i = 1, 2, 3$ from getting fraction x_i of a pie in period T is equal to $\delta^T x_i$. In the first period, player 1 proposes a partition (i.e. a vector $x = (x_1, x_2, x_3)$ with $x_1 + x_2 + x_3 = 1$), and players 2 and 3 *in turn* accept or reject this proposal. If either of them rejects it, then play passes to the next period, in which it is player 2s turn to propose a partition, to which players 3 and 1 in turn respond. If at least one of them rejects the proposal, then again play passes to the next period, in which player 3 makes a proposal, and players 1 and 2 respond. Players rotate proposals in this way until a proposal is accepted by both responders. Show that for any division of pie x if $\delta > 1/2$ then there is a subgame-perfect equilibrium in which x is agreed upon immediately.