14.129
Spring, 2025
Lecture 4

Smart Contracts as a Solution to a Coordination Problem

Commodity Space with Locations, Dates and States; The Policy Objective as Pareto Criterion with Fragmented Markets; Implementation with Privately Issued Securities as Monies; Uncoordinated Issues and Market Crises; Solution with Multi-agent Smart Contract

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Outline

- Extensions of the theory to risk, intertemporal exchange
- Policy guidance from data: Empirical tests against the efficiency benchmark
- Policy guidance from theory: High velocity private debt
- Financial market crashes, liquidity coordination, and policy goals
  - Common aspects across diverse applications
- Multi-agent smart contracts on a common ledger
- Ethereum

Extensions of the Theory to Risk and Intemporal Exchange

- The theory with dates and states made explicit

It is important to distinguish a particular *state* s_t from a *history* s_t. A history s_t is a particular sequence of states up to time t, i.e.

$$s^t = (s_1, s_2, \ldots, s_t)$$

The Endowment Economy

In its most basic form, the problem of risk-sharing occurs in an economy, where individual endowments are exogenous. In particular, we suppose that there are I agents, which receive an endowment $y^i(s^t) \in \mathscr{Y}(s^t)$ in history s^t. The utility function of individual i is given by

$$U^i = \sum_{t=1}^{\infty} \sum_{s^t \in S^{t-1}} \beta^t u^i \left(c^i \left(s^t\right)\right) Pr\left(s^t\right).$$

For simplicity we will also write this as $U^i = \sum_{t,s^t} \beta^t u^i \left(c^i \left(s^t\right)\right) Pr\left(s^t\right)$. We will also assume that u^i satisfies the usual assumptions if not otherwise stated.[1]

Given this environment, the Pareto problem is given by

$$\max_{\left\{\left[c^i(s^t)\right]_{s^t}\right\}_i} \sum_{i=1}^{I} \lambda^i \left(\sum_{t,s^t} \beta^t u^i \left(c^i \left(s^t\right)\right) Pr\left(s^t\right)\right) \tag{1}$$

$$\text{s.t.} \sum_{i=1}^{I} c^i\left(s^t\right) = \sum_{i=1}^{I} y^i\left(s^t\right) \text{ for all } s^t \in S^t,$$

where λ^i denotes the Pareto weight of individual i. The object of choice $\left\{\left[c^i\left(s^t\right)\right]_{s^t}\right\}_i$ are the consumption allocations in each state of the world, i.e. the conditional distribution of consumption for all individuals (where the appropriate distribution is of course just $Pr\left(s^t\right)$). From a conceptual point of view it is interesting to rewrite (8) as a static maximization problem, i.e. to solve it "pointwise" for each t. Doing so and defining aggregate income

$$Y\left(s^t\right) = \sum_{i=1}^{I} y\left(s^t\right) \text{ for all } s^t \in S^t$$

To fully characterize the solution $\{[c^i(s^t)]_{s^t}\}_i$, let us go back to (1). The Lagrangian for this problem is given by

$$\mathcal{L} = \sum_{i=1}^{I} \lambda^i \left(\sum_{t,s^t} \beta^t u^i\left(c^i\left(s^t\right)\right) Pr\left(s^t\right) \right) + \sum_{t,s^t} \theta\left(s^t\right) \left(Y\left(s^t\right) - \sum_i c^i\left(s^t\right) \right),$$

where $\theta(s^t)$ is the history-dependent multiplier on the resource constraint. The necessary and sufficient FOC are

$$\lambda^i \beta^t \frac{\partial u^i\left(c^i\left(s^t\right)\right)}{\partial c} Pr\left(s^t\right) = \theta\left(s^t\right) \text{ for all } i, s^t \tag{3}$$

$$Y\left(s^t\right) = \sum_i c^i\left(s^t\right) \text{ for all } s^t. \tag{4}$$

Hence, from (3) and (4), the consumption allocation in state s^t is characterized by

$$\lambda^1 \frac{\partial u^1\left(c^1\left(s^t\right)\right)}{\partial c} = \lambda^{\overset{i}{\cancel{1}}} \frac{\partial u^{\overset{i}{\cancel{1}}}\left(c^{\overset{i}{\cancel{1}}}\left(s^t\right)\right)}{\partial c} \text{ for all } i \tag{5}$$

$$Y\left(s^t\right) = \sum_i c^i\left(s^t\right). \tag{6}$$

(5) and (6) are I equations in the I unknowns ($\{c^i(s^t)\}_i$), which again shows that $c^i(s^t)$ can fully be characterized by

$$c^i\left(s^t\right) = g^i\left(Y\left(s^t\right)\right). \tag{7}$$

Clearly, (5) and (6) are just the necessary and sufficient conditions from (2).

(7) contains the main exclusion restriction, which most empirical tests of risk-sharing exploit: individual consumption is just a (individual specific) function of aggregate income. Individual income $y^i(s^t)$ however does not matter once aggregate income is controlled for. Individual preferences (e.g. risk-aversion) however determine g^i.

In the following we will now investigate in how far these insights from the simple endowment economy can be generalized to different environments.

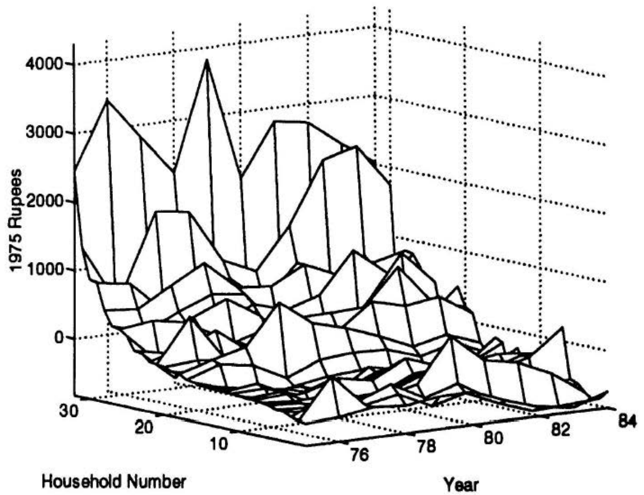
Policy Guidance from Data

- Risk and insurance in village India
- Risk premia and rates of return in village Thailand
- Gaps to be targeted, from the literature more generally

Village India: Income and Consumption Variables Over Time and Cross Section

Dynamics of the income process

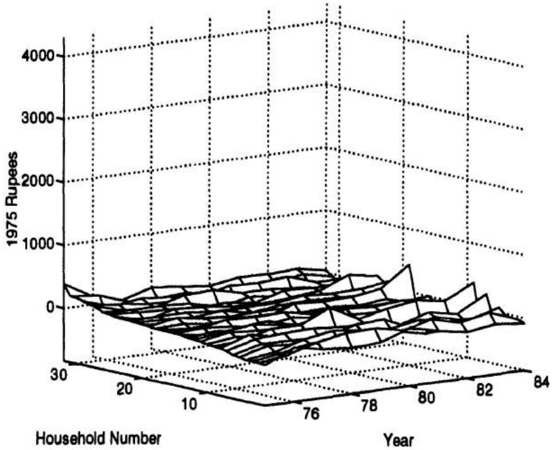
Basic picture: Inequality and uncorrelated shocks.
Recall: Households are of very different size.



(a) Comovement of household incomes (deviation from village average) Aurepalle.

Dynamics of individual consumption

Basic picture: consumption profiles much smoother than income process



(a) Comovement of household consumptions (grain only) (deviation from village average) Aurepalle.

Exclusion for idiosyncratic income: Panel

TABLE VIII

a. Panel Estimates with All Consumption[a]

	Village:	Aurepalle			Shirapur			Kanzara		
	Variable	(A) Std. ζ_w	(B) First Diff ζ_Δ	(C) 2 IV G − H ζ	(D) Std. ζ_w	(E) First Diff ζ_Δ	(F) 2 IV G − H ζ	(G) Std. ζ_w	(H) First Diff ζ_Δ	(I) 2 IV G − H ζ
1	All Income	0.0772*	0.0469		0.1169*	0.0592*		−0.0073	0.1233*	0.2177
		(0.0221)	(0.0236)	[0.768]	(0.0277)	[0.0236]	[1.290]	(0.0219)	(0.0227)	[−3.177]
2	Crop Profit	−0.0150	−0.0380		0.0825*	0.0352		0.0513*	0.0677*	−0.2545
		(0.0312)	(0.0299)	[0.380]	(0.0373)	[0.0301]	[0.609]	(0.0286)	(0.0308)	[−2.355]
3	Labor Income	0.0401	0.2597*		0.1127*	0.1925*		0.0198	0.1003*	
		(0.0647)	(0.0830)	[−1.543]	(0.0740)	[0.0655]	[−0.271]	(0.0406)	(0.0422)	[−1.058]
4	Profit from Trade and Handicrafts	0.2363*	0.1495*		0.0291	−0.1091		0.1347	0.4057*	
		(0.0352)	(0.0389)	[1.197]	(0.0671)	[0.0757]	[0.742]	(0.0895)	(0.0863)	[−1.312]
5	Profit from Animal Husbandry	0.0485	−0.0276		0.5014*	0.1994*	1.4678	0.0672	0.2252*	
		(0.0676)	(0.0689)	[−0.116]	(0.0789)	[0.0693]	[2.193]	(0.0606)	(0.0715)	[−1.387]
6	Full Income	−0.0123*	0.0016		NA	NA	NA	−0.0081	0.0058	
		(0.0027)	(0.0058)	[−1.412]				(0.0044)	(0.0043)	[−0.012]
7	Wage	−10.269	−7.1232		−41.201*	−47.7768		−116.31*	−11.7713	−297.696
		(8.4114)	(10.2640)	[0.004]	(15.4649)	[31.3120]	[−0.467]	(14.057)	(16.8668)	[−4.161]

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I. Full Risk-Sharing Benchmark - 3

- The value function of the social planner is

$$V(W;\Lambda)=\max_{k_j^{i\prime},\tau_j{}'}\left(\sum_{j=1}^{J}\lambda_j u_j\left(\sum_{i=1}^{I}\left(f_{i,j}\left(k_j^i\right)+k_j^i\right)-\sum_{i-1}^{I}k_j^{i\prime}+\tau_j\right)+\phi E\left[V\left(W';\Lambda\right)\right]\right)$$

subject to aggregate resource constraint

$$\sum_{j=1}^{J}c_j+\sum_{j=1}^{J}k_j{}'=W$$

and

$$k_j^{i\prime}\geq 0,$$

- W is the aggregate wealth of the whole economy at the beginning of the current period, i.e.

$$W=\sum_{j=1}^{J}\sum_{i=1}^{I}\left(f_{i,j}\left(k_j^i\right)+k_j^i\right).$$

I. Full Risk-Sharing Benchmark - 4

- The first-order conditions imply

$$[\tau_j]: \lambda_j u_{jc}(c_j) = \mu \qquad \text{for all } j$$

$$[k_j^{i\prime}]: -\lambda_j u_{jc}\left(c_j\right) + \phi E\left[V_W\left(W'\right)\left(1 + f_{i,j}{}'\left(k_j^{i\prime}\right)\right)\right] \leq 0 \qquad \text{for all } i \text{ and all } j$$

where μ is the shadow price of consumption in the current period

- Note that this setup assumes a closed economy
  - We can generalize and allow the economy to have external borrowing and lending, i.e. small-open economy by redefining $\tilde{W} = W - (1+r)D + D'$

Table 3 Contribution of Idiosyncratic Risk to Total Risk and Total Risk Premium

Region:	Central						Northeast					
Township (Province):	Chachoengsao			Lopburi			Buriram			Srisaket		
	p25	p50	p75	p25	p50	p75	p25	p50	p75	p25	p50	p75
Panel A: Baseline Specification												
Contribution to Total Risk (Variance)	93.9%	98.1%	99.7%	92.3%	97.6%	99.5%	84.0%	94.0%	98.2%	43.8%	65.9%	88.9%
Contribution to Total Risk Premium	4.7%	21.6%	45.4%	41.7%	61.5%	88.7%	105.6%	118.7%	152.8%	13.3%	28.8%	53.9%
Percentage of Diversified Idiosyncratic Risk	98.6%	99.6%	100.0%	92.4%	96.3%	99.9%	111.2%	135.2%	172.2%	67.4%	82.0%	90.0%
Panel B: Robustness Specification												
Contribution to Total Risk (Variance)	77.4%	84.9%	89.0%	80.2%	88.0%	91.6%	73.4%	79.7%	87.1%	40.9%	55.0%	68.9%
Contribution to Total Risk Premium	6.3%	32.6%	56.6%	21.2%	54.9%	102.2%	35.4%	88.4%	147.0%	9.1%	19.5%	33.3%
Percentage of Diversified Idiosyncratic Risk	79.4%	93.4%	100.3%	69.6%	94.9%	110.2%	75.5%	112.7%	153.6%	63.4%	79.9%	89.4%
Number of Observations	129	129	129	140	140	140	131	131	131	141	141	141

Remarks Unit of observation is household. Panel A presents the results from a baseline specification, as shown in equation (4), using the empirical results from Columns (1)-(4) of Table 1. Panel B presents the results from a full robustness specification, as shown in equation (6), using the empirical results from Columns (5)-(8) of Table 2. The numbers for each household are the average across estimates from nine different time-shifting windows.

Gaps to be Targeted from the Literature More Generally

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Network Propagation

- Kinnan, Samphantharak, Townsend and Diego Vera-Cossio (2022) "Propagation and Insurance in Village Networks"
- Townsend (1995) "Consumption Insurance: An Evaluation of Risk-Bearing Systems in Low-Income Economies"
- Guvenen, Schulhofer-Wohl, Song, and Yogo (2017) "Worker Betas: Five Facts about Systematic Earnings Risk"
- Dolls (2019) "An Unemployment Re-Insurance Scheme for the Eurozone? Stabilizing and Redistributive Effects"

Policy Guidance from Theory

- Policy guidance from theory: High velocity private debt
  - Motivation: Inland bills of exchange
  - The first welfare theorem again
  - Attempt to implement with fragmented markets
  - A coordination problem

Motivation: Bills of Exchange as High Velocity Private Debt

in the context of English inland bills of exchange (where all parties to the bill were in England), which were used as a medium of exchange during the Industrial Revolution in the north of England in the eighteenth and first half of the nineteenth centuries.

Source: Gorton (2020) "Inland Bills of Exchange: Private Money Production without Banks"

First Welfare Theorem Again: Attempt to Implement with Fragmented Markets

First Welfare Theorem

Let (x^*, y^*, p, w) be a price equilibrium with transfers. Then, if preferences are rational and locally non-satiated, then (x^*, y^*) is a Pareto Optimal allocation.

- As with Ostroy-Starr, retaining fragmented markets
- Here extended to dynamic, self-interested agents
- Economic issue focus is asset issue and ownership change
- Money again, here private issued securities with high velocity

Distributed Ledgers Solve Coordination Problems in Debt Issue Across Separated Trading Venues

- Townsend - Wallace (1987), contemporary interpretation:
  - Each trader is endowed with single good, exogenously pre-specified, but over multiple dates and varying over time.
  - Multiple locations as trading venues; segmented markets or platforms where traders meet, one location per date, different across different trader types.
  - Traders' preferences summarized by utility over end-of-each period consumption of the single good, creating possible gains to trade relative to endowment profiles. Traders wish to acquire or relinquish the good.
  - Traders are not automata but are self-interested. They explicitly maximize discounted inter-temporal utility flows, taking current and future prices of securities as given.

Table V-1. Who Meets Whom When

	Location	
Date	1	2
1	(1,2)	(3,4)
2	(1,3)	(2,4)
3	(1,2)	(3,4)
4	(1,3)	(2,4)

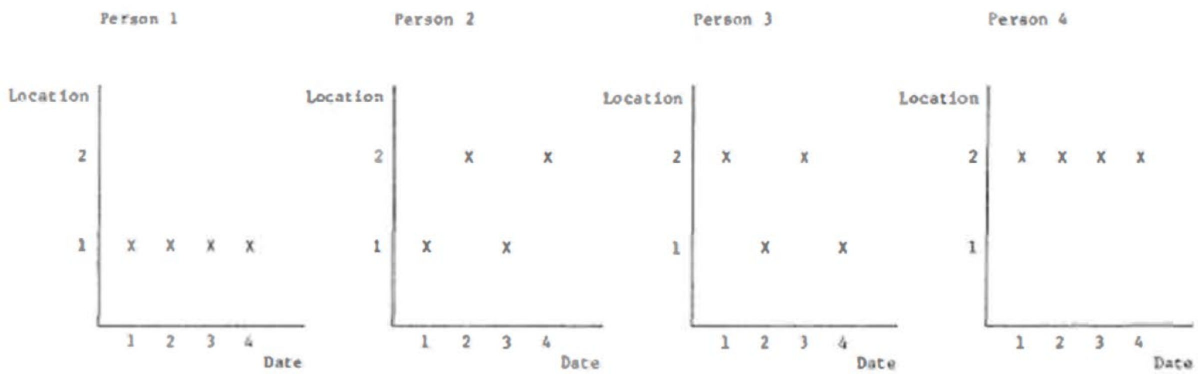


Figure V-1. Relevant commodity subspaces in the economy shown in table V-1.

Townsend-Wallace, 1987

- If $T = 2$, that is, if the economy lasts only two periods-then no trade is possible in the table V-I economy under our security trading rules.
- For example, person 1 cannot sell a promise to person 2 because person 2 can neither redeem it at date 2 nor pass it on to person 4, who has no use for it at date 2, the assumed last date.
- Absence of double coincidence: T = 2 no pair of persons has endowments and cares about a common two-dimensional subspace of the commodity space.
- In the sense that there can exist redistributions of the endowments that give rise to allocations that are Pareto-superior to the endowment allocation. Put differently, if all four people were together at some time zero and traded in complete (location and date contingent) markets, something we rule out, then the endowment would not necessarily be a competitive equilibrium.

Townsend-Wallace, 1987

More General (Cont.)

- Spatial separation limits trades in securities in what seems to be a natural way.
- Traders when they meet can issue securities, i.e., a digital asset, a smart contract programmed to be redeemed for the consumption good at pre-specified date and location. Allows both short-term and long-term debts.
- Traders can buy and sell previously acquired digital assets against the good, with value-for-delivery swaps.
- Although securities can be transported, they can move only with a person.
- Finally, the model does not allow people to renege on their debts or to counterfeit others debts. Traders can redeem or repurchase previous promises for the good, which again they must do if presented by original investors or third parties for redemption at the specified maturity date.

Townsend-Wallace, 1987

By a payments matrix we mean an N by N matrix, where N is the number of object observed in a debt equilibrium, in which the (i,j)-th element is one if object i is observed to trade for object j and is zero otherwise. Thus, for a table V-1, T = 4 economy, N equals the number of distinct consumption goods --eight--plus the number of distinct private securities issued in an equilibrium. And, if the transaction pattern is such that each consumption good gets traded for one circulating security and one noncirculating security, then there are two nonzero element in each row corresponding to a consumption good or to a noncirculating debt, and there are four in each row corresponding to a circulating debt. Note, by the way, that nontrivial spatial setup seem not to produce equilibria in which one object trade for every other object.

By the transaction velocity of an object, we mean the ratio of the average amount traded per date to the average stock, a pure number per unit time. For example, for a table V-1, T = 4 economy, the following transaction velocity pattern among objects shows up in a debt equilibrium. For a consumption good at location i, date t, the average stock outstanding may be taken to be the total endowment divided by 4 (at date other than t, the stock of this good is zero), whereas the average amount traded per date is the amount traded at t divided by 4. Thus, the transaction velocity is in the interval (0,1). Computed in a similar way, the transaction velocity of non-circulating debt in such an economy is 2/3 (such debt is outstanding for three date and the entire stock is traded at two of those dates), whereas that of circulating debt us unity (the maximum possible velocity given our choice of time unit).

Multiple Equilibria Proliferate, Generating A Coordination Problem

- Even with preferences, endowment profiles, trading venues and these trading rules all known by all (common knowledge), multiple equilibria can exist, and each equilibrium achieves the same underlying fundamentals.
  - As in Ostroy Starr, an equilibrium achieves the Walrasian outcomes as if there were initial centralized Arrow Debreu security markets that determined prices.
- However, if the distinct trading venues are not informationally connected, then this **multiplicity can lead to a coordination problem**:
  - Who is to issue the long term debts which circulate as high velocity money-like securities, i.e., digital assets as debts which trade frequently against money and other securities.

More general:

- Let J denote the number of locations and T the number of dates, the commodity space has dimension JT. We assume that each person gets utility from commodities and has positive endowments of commodities in a proper subset whose elements correspond to the location-date combinations that the person visits
- We assume an economy with G persons, each of whom lives T periods. At each time t, each person g can be paired with some other person or with no one. These pairings occur at (isolated) locations. Thus, we assume that person g is assigned to some location i at each time t and that in that location there either is or is not a single trading partner.
- One commodity for each location-date combination
- One good that is indexed by location and date
- Goods indexed by one location-date combination cannot be transformed into goods indexed by another location-date combination
- There is no transportation, production, or storage technology for goods.

Townsend-Wallace, 1987

If person g is in location i at time t then he or she is endowed with some positive number of units, w_{it}^{g} , of the consumption good at location i, date t. For other location-date combinations, the endowment is zero. Let w^{g} denote the entire JT-dimensional endowment vector for person g. Also let c_{it}^{g} denote the nonnegative number of units of location i-date t consumption of person g and let c^{g} denote the entire JT dimensional consumption vector for person g. Preference of each person g are described by a utility function $U^{g}(c^{g})$ that is continuous concave and strictly increasing in the T-dimensional sub-space that is relevant for g.

We strict attention to securities that can be redeemed. Thus, if d_{st}^{f}, which is nonnegative, denotes securities issued by person f at time s to pay d_{st}^{f} units of the consumption good where f will be at time t, we consider only triplets (f, s, t) with the property that there is a path or chain of pairing leading from where f is at s to where f is at t.

We let $p_{st}^{f}(i,u)$ be the price per unit of d_{st}^{f} at location i, date u in unit of good (i,u). However we define such a price only for pair (i,u) that potentially admit of a nontrivial trad in d_{st}^{f} . (This allows us to avoid having to determine a price for d_{st}^{f} in a market where demand and supply are identically zero and also allow us to restrict attention to positive prices.) Thus, suppose h and g meet at $(i\,u)$. We say that h is a potential demander of d_{st}^{f} at (i, u) if there is a route from h at u to f at t. We say that h is a potential supplier of d_{st}^{f} at (i, u) if there is a route from h at s to h at u. We say there is a market in d_{st}^{f} at (i, u) if and only if h is a potential demander and g is a potential supplier at (i u), or

We let $d_{st}^{fg}(i,u)$ be the excess demand by g at (i,u) for d_{st}^{f}. In terms of this notation, our debt trading rules are

$$\begin{aligned} &\sum_{u=s}^{t} d_{st}^{ff}(\cdot,u) \geq 0 \text{ for each } f \\ &\sum_{u=s}^{t'} d_{st}^{fg}(\cdot,u) \geq 0 \text{ for each } t' \geq s \text{ and } g \neq f \end{aligned} \tag{1}$$

The first inequality says that f must end up demanding as much as f issues, which expresses our no-reneging rule. The second says that $g \neq f$ cannot supply d_{st}^{f} without having previously acquired it. Finally, as a convention,

Then, as budget constraints for any person g, we may write

$$w_{iu}^{g} \geq c_{iu}^{g} + \sum d_{st}^{fg}(i,u)\, p_{st}^{f}(i,u), \tag{2}$$

there being T such constraints, one for each (i,u) that g visits. The summation in (2) is over all securities—all (f,s,t)—for which a market exists

DEFINITION. A debt equilibrium is a specification of consumption and debt demands—c^{g} and d^{g} for each $g = 1, 2, \ldots, G$—and positive security prices, $p_{st}^{f}(i,u)$, such that

(i) c^{g} and d^{g} maximize $U^{g}(c^{g})$ subject to (1) and (2)
(ii) $\Sigma_{g}\,(c_{iu}^{g} - w_{iu}^{g}) = 0$ for each (i,u) and $\Sigma_{g}\, d_{st}^{fg}(i,u) = 0$ for each (i,u) and all potentially redeemable d_{st}^{f}.

Townsend-Wallace, 1987

4. Debt Equilibria and Complete-Markets Equilibria

In this section we establish equivalence for the table V-1, T= 4 economy between the equilibrium allocations and prices of complete date-location contingent markets and the allocations and prices of debt equilibria. This proves useful in describing the transaction pattern implication of the theory and the coordination problem. We begin by showing that any complete – markets equilibrium (CME) consumption allocation can be supported by a debt equilibrium (DE).

To see how that any CME can be supported by a DE in the table V-1, T = 4 economy we start with a given CME. This we describe individual consumption excess demand $e_{it}^{g} \equiv c_{it}^{g} - w_{it}^{g}$ and associated prices, s_{it} (in terms of an abstract unit of account). These constitute a CME if they satisfy:

$$\sum_i \sum_t e_{it}^{g} s_{it} = 0 \quad \text{for each } g \qquad (3)$$

$$\sum_g e_{it}^{g} = 0 \quad \text{for each } (i\,, t) \qquad (4)$$

A corresponding DE consists of positive debt prices and nonnegative, market-clearing debt quantities such that

a) the debt quantities and the given CME e_{it}^{g}'s satisfy each person's debt budget constraints

b) the debt quantities and the given CME e_{it}^{g}'s are utility maximizing for each person given those debt prices.

Townsend-Wallace, 1987

Next 4 slides contain some technical details, punch line is equation (9)

Our first step is to produce candidate debt prices for the table V-1, T = 4 economy. This candidate is produced by matching the term of trade.

Thus, we let $p_{14}^{1}(1,1) = \frac{s_{14}}{s_{11}}$. In general, then, each debt price is taken to be a ratio of CME prices with the numerator corresponding to the redemption location-date and the denominator to the location-date of the current trade.

For noncirculating debts, then, our candidate is

$$\left(p_{13}^{1}(1,1), p_{13}^{1}(1,2), p_{24}^{2}(2,2)\right)$$
$$= \left(p_{13}^{2}(1,1), p_{13}^{4}(2,1), p_{24}^{3}(1,2), p_{24}^{4}\ (2,2)\right)$$
$$= \left(\frac{s_{13}}{s_{11}}, \frac{s_{23}}{s_{21}}, \frac{s_{14}}{s_{12}}, \frac{s_{24}}{s_{22}}\right). \qquad (5)$$

Where, for circulating debts, it is

$$(6) \qquad \begin{bmatrix} p_{14}^{1}(1,1) & p_{14}^{1}(2,2) & p_{14}^{1}(2,3) \\ p_{14}^{2}(1,1) & p_{14}^{2}(1,2) & p_{14}^{2}(2,3) \\ p_{14}^{3}(2,1) & p_{14}^{3}(2,2) & p_{14}^{3}(1,3) \\ p_{14}^{4}(2,1) & p_{14}^{4}(1,2) & p_{14}^{4}(1,3) \end{bmatrix} = \begin{bmatrix} s_{14}/s_{11} & s_{14}/s_{22} & s_{14}/s_{23} \\ s_{24}/s_{11} & s_{24}/s_{12} & s_{24}/s_{23} \\ s_{14}/s_{21} & s_{14}/s_{22} & s_{14}/s_{13} \\ s_{24}/s_{21} & s_{24}/s_{12} & s_{24}/s_{13} \end{bmatrix}$$

We can immediately indicate that this implies that satisfaction of (a) implies satisfaction of (b). To see this, multiply the debt constraint for e_{it}^{g} [equation (2)] by s_{it} and sum over i and t. Using (5) and (6), the result is (3), in which debt quantities do not appear. Thus, at prices given by (5) and (6), the debt constraints for any person are at least as constraining as (3). Therefore, if we can produce market-clearing debt quantities, d_{st}^{f}'s, which make the CME e_{it}^{g}'s feasible choices subject to the budget constraints (2), then they are certainly utility-maximizing choices. That is, (a) implies (b).

3 for excess demand

8 for location date market clearing

16 for location dates not visited 4 x 4

Two people in same island

4 people x 4 dates x 2 locations

Budget for person 1

To motivate how we produce debt quantities, recall that a CME consists of arbitrary s_{it}'s and e_{it}^{g}'s that satisfy (3), (4), and zero restrictions for those e_{it}^{g}'s that correspond to (i,t)'s that g does not visit. For the table V-1, T = 4 economy, there are 3 + 8 + 16 *independent* constraints on the 32 e_{it}^{g}'s. This leaves us free to choose 5 e_{it}^{g}'s arbitrarily, but not any 5. For example, e_{11}^{1} and e_{11}^{2} cannot both be chosen arbitrarily because (4) and the zero restrictions imply that these sum to zero. Similarly, e_{11}^{1}, e_{12}^{1}, e_{13}^{1}, e_{14}^{1} cannot each be chosen arbitrarily since (3) must be satisfied. We arrive at candidates for equilibrium debt quantities by finding some that satisfy constraints (1) and (2) and the relevant debt market clearing conditions for a set of e_{it}^{g}'s that can be chosen arbitrarily.

Townsend-Wallace, 1987

For the table V-1, T = 4 economy, the following equations are the debt budget constraints, at prices satisfying (5) and (6), for five e_{it}^{g} 's that can be chosen arbitrarily:

$$(e_{21}^{4}, e_{22}^{4}, e_{11}^{1}, e_{12}^{1}, e_{13}^{1})' = Ad \tag{7}$$

$$A = \begin{bmatrix} s_{23}/s_{21} & 0 & 0 & 0 & 0 & 0 & -s_{14}/s_{21} & s_{24}/s_{21} \\ 0 & s_{24}/s_{22} & 0 & 0 & -s_{14}/s_{22} & 0 & s_{14}/s_{22} & 0 \\ 0 & 0 & s_{13}/s_{11} & 0 & s_{14}/s_{11} & -s_{24}/s_{11} & 0 & 0 \\ 0 & 0 & 0 & s_{14}/s_{12} & 0 & s_{24}/s_{12} & 0 & -s_{24}/s_{12} \\ 0 & 0 & -1 & 0 & 0 & 0 & -s_{14}/s_{13} & s_{24}/s_{13} \end{bmatrix}$$

And $d = (d_{13}^{4} - d_{13}^{3}, d_{24}^{4} - d_{24}^{2}, d_{13}^{1} - d_{13}^{2}, d_{24}^{1} - d_{24}^{3}, d_{14}^{1}, d_{14}^{2}, d_{14}^{3}, d_{14}^{4})'$.

Note that zeros in the A matrix do not denote zero debt prices, but rather that the particular debt cannot be traded at the relevant location-date combination.

Townsend-Wallace, 1987

To see that there are nonnegative debt quantities that satisfy (7) for arbitrary s_{it}'s and an arbitrary left-hand side (LHS) of (7), consider an equivalent set of equations obtained by replacing the last equation of (7) by itself plus a multiple s_{11}/s_{13}) of the third equation, namely

$$\left[e^4_{21}, e^4_{22}, e^1_{11}, e^1_{12}, e^1_{13} + \left(\frac{s_{11}}{s_{13}}\right) e^1_{11}\right]' = [A_1, A_2, A_3, A_4, A_5 + \left(\frac{s_{11}}{s_{13}}\right) A_3]' \qquad (8)$$

where A_i denotes the i-th row of the matrix A. Note that in each of the first four equation of (8) there appear (with a nonzero coefficient) a difference between noncirculating debts that do not appear in any other equation. Thus, for any quantities of the other debts each of the first four equations can be satisfied by choosing nonnegative quantities of the noncirculating debt that appear in that equation only. This allow us to choose nonnegative quantities of the circulating debts in any way that satisfies the last equation of (8), namely

$$e^1_{13} + \left(\frac{s_{11}}{s_{13}}\right) e^1_{11} = \left(\frac{s_{14}}{s_{13}}\right)(d^1_{14} - d^3_{14}) - \left(\frac{s_{24}}{s_{13}}\right)(d^2_{14} - d^4_{14}) \qquad (9)$$

Equation (9) is easily satisfied; if the LHS is positive (negative), it can be satisfied by setting at zero all but $d^1_{14}(d^3_{14})$

Townsend-Wallace, 1987

Our coordination problem bears some resemblance to a result obtained by Ostroy (1973) and Ostroy-Starr (1974) in their study of the decentralization of exchange. In their model, knowledge of equilibrium prices of commodities is not enough to guide people to the trades that produce the equilibrium allocation in one round of bilateral trading if the trading rules are informationally decentralized. In our model knowledge of current and future equilibrium prices of securities not enough to guide people to the quantities of securities required to support an equilibrium if security transaction in other markets are not observed. Of course, both private debt in our model and money in their model alleviate a quid pro quo requirement and facilitate the attainment of equilibrium. There is a sense, though, in which the monetary exchange process is informationally centralized in their model. It requires that budget balance information be transmitted to a monetary authority or requires that there be implicit agreement about which commodity is to be used to cover budget deficits and surplus. One interpretation of our coordination problem is that a debt equilibrium also requires centralization.

Financial Market Crashes, Liquidity Coordination, and Policy Goal: Common Aspects Across Diverse Applications

- What happens in the model without coordination
- Bagehot London Money Markets, CB policy:
  - Real bills vs. the quantity theory
- LMICs, concerns about digital and currency flows
- Fragmentation of liquidity across blockchains

A Financial Crisis Emerges when Coordination Problems are not Addressed ex ante in the Regulatory Infrastructure Design

- If the coordination problem is unresolved and agents guess wrong about what is going on in other informally separate trading venues, a slow-moving financial crisis, with heterogeneous impact, would emerges (Spector and Townsend, 2020).
  - The price of the unnecessary/over-issued debt collapses quickly, as the mistake is revealed, though forward-looking agents can then mitigate the disaster by then reversing what would have been their short-term debt issues.
  - The time profile of end-of period money holdings fluctuates over time, especially for those buying and selling with the high velocity assets, as for them liquidity fluctuates at intermediate dates. They suffer relatively large welfare losses.

Spector and Townsend (2020), Notes on Townsend-Wallace --in class

- **Preference:** $U^i = \sum_{t=1}^{4} \ln(c_t^i)$
- **Endowments:** aggregate endowment is constant at $\Omega = \overline{\omega} + \underline{\omega}$, and agents alternate between the high and low states (with agents 1 and 4 starting with $\overline{\omega}$ and agents 2 and 3 starting with $\underline{\omega}$ in t = 1).
- **CME:** everyone consumes $\frac{\overline{\omega}+\underline{\omega}}{2}$ in every period, and all prices are 1.
- Suppose debts issued at t = 1 do not support the CME, and we want to find the equilibrium for t = 2 onwards (assuming everyone knows the debts that were issued at t = 1). The equilibrium is given by 10 prices and 2 quantities: the prices of the four circulating debts at t = 2 and t = 3, and the prices and quantities of the two the non-circulating debts issued at t = 2.
- We can reduce the dimensionality by noting that the price of the non-circulating debts is pinned-down by the aggregate endowment (by homotheticity), and there are arbitrage conditions between prices (e.g., agent 1 can save/borrow between periods t = 2 and t = 4 either by buying non-circulating debt or by buying and reselling circulating debts).

Spector and Townsend (2020), Notes on Townsend-Wallace, Cont'd –in class

- Suppose $\overline{\omega} = 1$, $\underline{\omega} = 0.5$, and consider two scenarios:
  - Scenario 1: CME with all circulating debt issued in location 1 (and non-circulating debts are calculated to support the CME).
  - Scenario 2: location 1 issues the same debts as in Scenario 1, but now location 2 also issues the same debts (i.e., everyone thinks, mistakenly, that all long-term debt is being issued in their location).

		Location 1 Sc.1	Location 1 Sc.2		Location 2 Sc.1	Location 2 Sc.2
$t=1$	d^1_{14}	0.50	0.50	d^3_{14}	0.00	0.00
	d^2_{14}	0.00	0.00	d^4_{14}	0.00	0.50
	d^1_{13}	-0.25	-0.25	d^4_{13}	0.25	-0.25
$t=2$	c_1	0.75	0.66	c_2	0.75	0.84
	c_3	0.75	0.84	c_4	0.75	0.66
	p^4_{14}	–	0.36	p^1_{14}	1.00	0.36
	p^1_{24}	1.00	1.00	p^4_{24}	1.00	1.00
	d^1_{24}	-0.25	-0.16	d^4_{24}	0.25	-0.16
$t=3$	c_1	0.75	1.03	c_3	0.75	0.47
	c_2	0.75	0.47	c_4	0.75	1.03
	p^4_{14}	–	0.56	p^1_{14}	1.00	0.56
$t=4$	c_1	0.75	0.66	c_2	0.75	0.84
	c_3	0.75	0.84	c_4	0.75	0.66

	Sc.1	Sc.2
U_1	-0.86	-0.80
U_2	-0.86	-1.10
U_3	-0.86	-1.10
U_4	-0.86	-0.80

Solving Coordination Problem in an Additional Multi-agent, Multi-market Smart Contract

- The above coordination problem can be resolved in an additional multi-agent multi-market smart contract implemented on a distributed ledger of digital assets shared across trading venues.
  - The contract can specify bounds on security issues and an algorithm to achieve consensus.
  - One condition that works in the Townsend-Wallace, Spector - Townsend environment is to give specified agent types to the first pass at security issues in one of the locations let the agent types in the other, second location fill in gaps, if any, each up to now-conditioned upper bounds.

Financial Market Crashes, Bagehot

- Bagehot (1873) "Lombard Street: A description of the Money Market"
  - Bills of exchange were traded on Lombard Street:
    - Bill brokers: These firms extended credit to businesses by trading in bills of exchange
    - Borrowed capital: English trade was carried on to a large extent using borrowed capital, with small traders discounting their bills
    - Discounting bills: The certainty of obtaining loans on discount of bills allowed for a steady bounty on trading with borrowed capital
- Chovanec (2006) "The Historical Roots of Modern Banking: Lessons from Walter Bagehot and London's Financial Crises" Silvercrestgroup.com
- Central bank policy: Real bills vs. the quantity theory
  - The real bills doctrine and the quantity theory of money are contrasting theories about the relationship between the money supply and inflation
  - Real bills doctrine
    - This theory states that limiting banks to issuing money backed by assets will prevent inflation. The doctrine is based on the idea that banks will automatically issue the right amount of money, and will not over-issue if they only issue money in exchange for real bills.
  - Quantity theory of money
    - This theory states that an increase in the money supply will lead to inflation. The theory suggests that the money supply and price level in an economy are directly proportional, so when the money supply changes, the price level will change proportionally.
- Sargent and Wallace (1982) "The Real-Bills Doctrine versus the Quantity Theory: A Reconsideration"

LMIC Concern About Digital Currency and Liquidity Flows

- Success of digitization in a low and middle income country (LMIC)
- Includes the use of e-switch and interoperability between bank accounts and mobile money wallets
  - Ideal system would have live feed plug-ins for deeper insights into transactions
- Regulators have some information on imbalance in flows and pre-funding of accounts
  - Credit is extended in anticipation of settlement based on projection of future flows.

Fragmentation of Liquidity Across Blockchains

Liquidity fragmentation refers to the distribution of liquidity (or available trading volume) across multiple decentralized finance (DeFi) platforms, blockchains, and networks, resulting in fragmented pools of liquidity rather than a consolidated, easily accessible market.

"What Is Liquidity Fragmentation and Why It's Killing DeFi" (2024) Medium.com.

Each blockchain has its own unique protocols, token standards, and liquidity pools, which makes assets relatively inaccessible across chains without bridging solutions.

Lim (2024) "Liquidity Aggregation in DeFi: Solving Fragmentation Across Chains" (2024) blog.web3auth.io

Multi-agent Smart Contracts on a Common Ledger

Blockchain, Ethereum, and Smart Contracts

Blockchain Basics

A Blockchain is a ***distributed ledger*** that keeps track of two-party ***transactions*** organized in ***blocks***

- Distributed ledger: A list of all transactions that is copied onto the devices of all participants
- Transaction: Movement of a cryptocurrency from one user’s address to another
- Block: a group of transactions that are all added to the ledger at once

Blockchain Basics

Blocks are ***proposed*** by a single ***node*** and added to the ledger through ***validation*** and ***consensus***

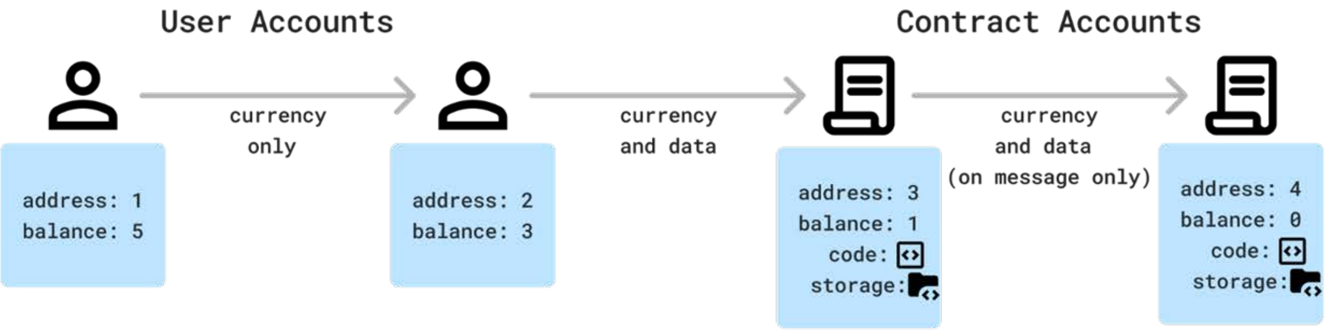
- Node: a single device in the blockchain that maintains a copy of the ledger
- Proposal: any node may offer a group of transactions to be committed as the next block
- Validation: each other node verifies the proposed block, checking that all transactions are valid
- Consensus: once enough nodes validate a block, it is added to the state of all nodes and is no longer mutable

History and Motivation

- The first blockchain was Bitcoin, and allowed a common ledger of transactions to be maintained with no central authority
- Quickly, the desire for “Smart Contracts” arose, code deployed on the blockchain that can automate transactions
- Ethereum’s goal
    - a state machine deployed on the blockchain
    - a fully turing complete language to describe state transitions
    - arbitrary functionality and complexity in transaction execution

Ethereum Structure

- Two types of accounts, externally owned (user) accounts and contract accounts
    - externally owned accounts have an address and balance
    - contract accounts also contain code and data storage
- Contract accounts receive transactions and update their state and send other transactions
    - can only initiate transactions as a response to another transaction
    - using computational resources to update state incurs "gas" fees



Transaction Structure

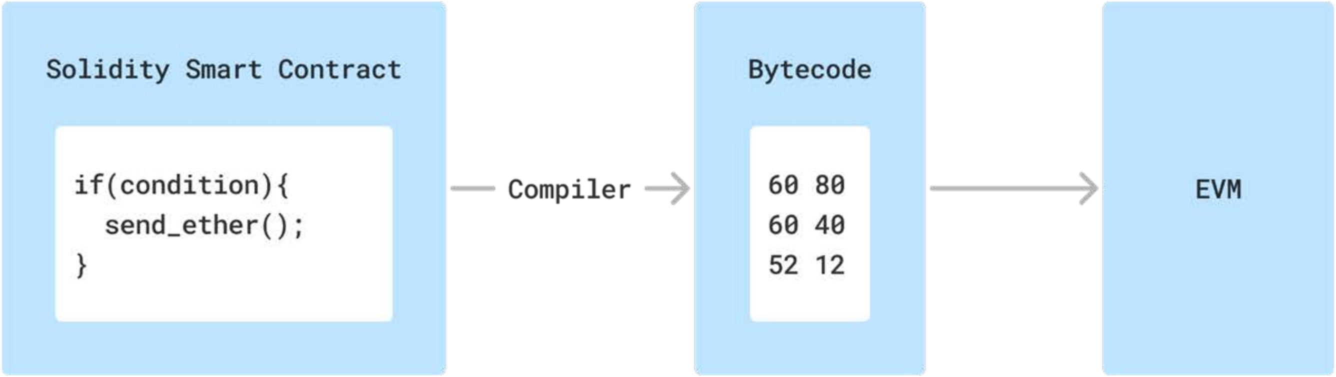
Sender	Recipient	Currency	Data	Start Gas
User address	User or contract address	Amount of Ether to transfer	key, value pairs of data to send to contract (can be empty)	extra currency to fund computation (can be empty)
0×1	0×2	3 eth	key1: value1 key2: value2 key3: value3	0.0001 eth

- All transactions have a sender, recipient, and currency field (can be 0)

- Transactions can **optionally** have data and start gas to allow message passing and computation

EVM

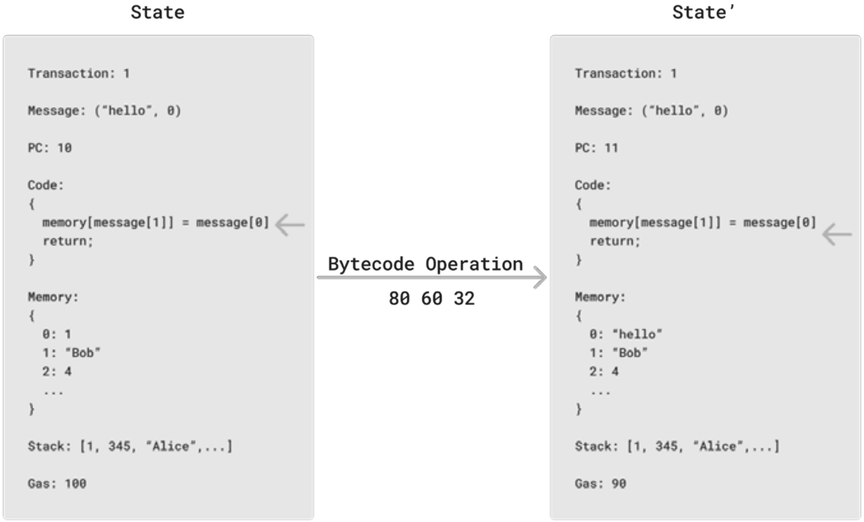
- Ethereum Smart Contracts compute on the Ethereum Virtual Machine
- Smart Contracts are written in a high level language (Solidity), then compiled down to bytecode, which is fed directly to the EVM
- The EVM uses each bytecode instruction to update the Ethereum state



EVM

- The entire EVM can be represented as a state machine, where each bytecode instruction triggers a state transition

Block State	Transaction	Message	Code	Memory	Stack	PC	Gas
Details on current block	Input transaction that triggered change	Data in the transaction	Smart contract executed by transaction	Data associated with smart contract	Data associated with smart contract	Program counter for code	Currency available for computation



Block Structure

Each block consists of a ***header***, a transaction count, and the list of transactions

- The header contains information about the block, such as the previous block ID, the timestamp, a nonce, and difficulty

- Ethereum block headers are much more complex to properly track state across contract execution
  - State tree root
  - Total gas used for the whole block
  - etc.

Bitcoin Block Structure

Block ID	Block ID	256 bits
Previous BID	Block ID of the preceding block	256 bits
Merkle Root	Merkle Tree hash of transactions in the block, for verification	256 bits
Timestamp	Time the block was created	32 bits
Nonce	Unique number used for cryptographic hashing	32 bits
Version	Block format version	32 bits
Difficulty	The difficulty rating for proposing this block	32 bits
Transaction Count	The number of transactions in the block	Variable size
Transactions	List of transactions in the block	Variable size