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Lecture 7

Stochastic Financial Networks

Liquidity And The Value Of Key Players Versus Contagion Dynamics: Enhance Or Limit Markets

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Outline

- Introduction to stochastic financial networks
  - Markets vary over time in the number of participants, another source of risk
- Economic environment
- Stochastic financial network
  - Example from network models
  - With segmentation
- Liquidity injections as buffers, who is most valued in terms of market making
- Most valued – baseline environment
- Most valued for general inert environments
- Positive counterparts to financial centrality
  - Price of an Arrow Debrue individual participation bond
  - Empirical validity of key players in Thai villages
- Financial centrality and contagion
  - Systemic risk basics
  - Static cascade models

Introduction

- Goal: develop measure of financial centrality to identify key players in risk-sharing environments.
- Study Settings in which disruptions to markets take the form of shocks to market participation.
- Examples of market participation shocks:

1. Search frictions: **Duffie-Garleanu-Pederson (2005),** Armenter and Lester (2015)
2. Random Matching: Kyotaki and Wright (1989), Trejos and Wright
3. Random Market Participation: Freeman (1996) and Green (1999), Santos and Woodford (1997)

Introduction

- Formally:

$$FC_i := \text{Marginal Social Value of giving } \varepsilon > 0$$

$$\text{to agent } i \text{ whenever she can trade.}$$

- Positive contagion from risk sharing: giving resources to an agent increases her consumption, but also the consumption of agents that trade with i
- Contagion from participation: e.g. Subsidy that could make agent i more likely to trade, which in turn makes other agents more likely to trade also, etc.

Example: Bech and Atalay (2010) - Federal Funds Markets

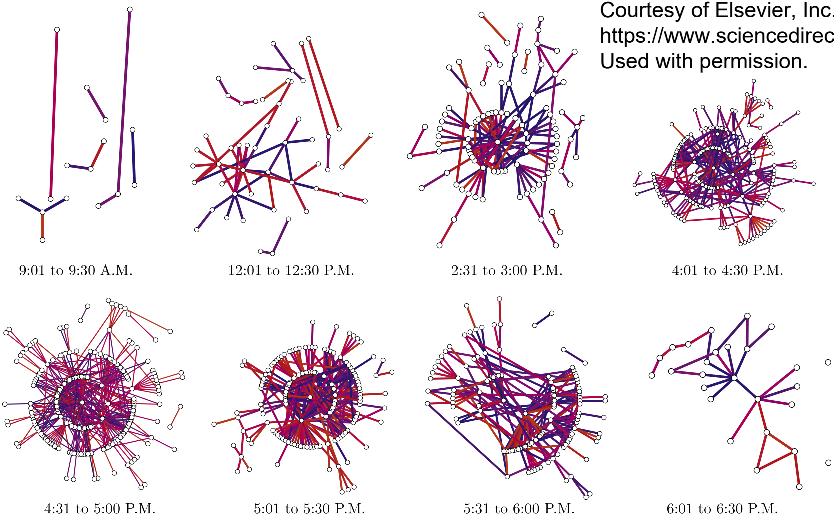

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Figure: Federal funds networks at different times September 29, 2006

Featuring Simplest Environment

- Typical risk sharing model (a la Townsend 94)
- Set of agents I, with $|I| = n$.
- Expected utility preferences $\mathbb{E}[u_i(c_i)]$ with $u' > 0, u'' < 0$
- Income risk: $y = (y_1, \ldots, y_n) \in \mathscr{Y}$, according to distribution $F(\cdot)$
- Leading example: CARA+Normal model, where

$$u_i(c_i) = -r_i^{-1} \exp(-rc_i) \text{ and } y \sim \mathscr{N}(\mu, \Sigma)$$

where $\mu \in \mathbb{R}^n_+$ and $\Sigma_{n\times n} \succeq 0$ is the covariance matrix of shocks.

Stochastic Financial Networks

- Market participation shock: only a random subset of agents can trade.
- Formally, let $\zeta \in \mathscr{M} = \{0,1\}^n$ be the description of which agents can trade, so
  - If $\zeta_i = 0 \Longrightarrow c_i = y_i$
  - If $\zeta_i = 1 \Longrightarrow c_i$ may be $\neq y_i$
- Agents that can trade can pool resources:

$$\sum_{i \in I} \zeta_i c_i \leq \sum \zeta_i y_i \text{ for all } y \in \mathscr{Y}, \zeta \in \mathscr{M}$$

- The process ζ is what we refer to as a Stochastic Financial Network
- Equivalent to Transaction Chains

Transaction Chains

Risk Sharing with Market Participation Shocks

- Relevant state $s = (y, \zeta)$
- The social planning problem we will focus on is:

$$V(\lambda) := \max_{c_i(s)} \mathbb{E}_s \left\{ \sum_{i \in I} \lambda_i u[c_i(s)] \right\} \tag{1}$$

$$s.t: \begin{cases} c_i(s) = y_i & \text{if } \zeta_i = 0 \\ \sum \zeta_i c_i(s) \leq \sum \zeta_i y_i & \text{for all } s = (\zeta, y) \in \mathscr{M} \times \mathscr{Y} \end{cases} \tag{2}$$

- This maps all possible constrained Pareto Optimal allocations

Stochastic Financial Networks

- The process ζ gives us a connected component of agents, which is what we refer to as a Stochastic Financial Network
- We refer to this model as one with *centralized markets* (i.e. either you are in the market or you are in autarky).
- Relevant state is $s = (y, \zeta)$
- Bilateral connections are enough to "complete" components (also in Bramoulle and Kranton (2007))

Transaction Chains

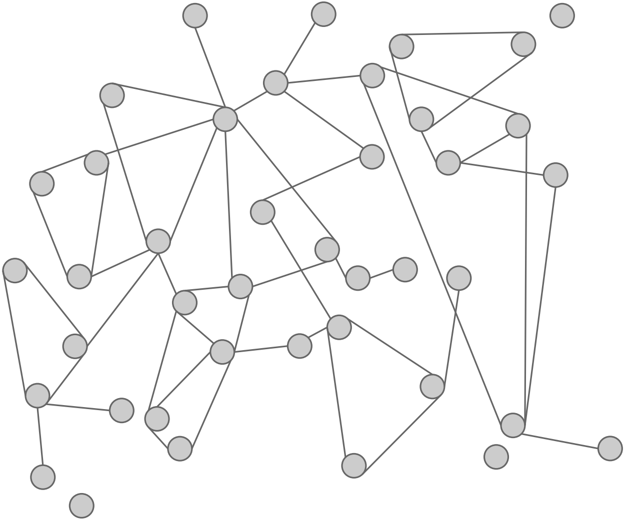
Example: Network Models

- If we have data on a social network, we can use it by modeling

$$\zeta \sim F(\cdot \mid \text{Network})$$

- Network is just a parameter in the market "matching" process
- **Salient example**: Poisson models:
    - One trader is (randomly) chosen as a market host (Prob. z_i)
    - Then sends *invitations* to trade through network to other agents.
    - Each agent gains access with probability $\text{A}(i,j)$ (ind. across agents)
    - Example: $A(i,j) = q^{\text{distance}(i,j)}$ with $q \in (0,1)$

Illustration: Market Participation



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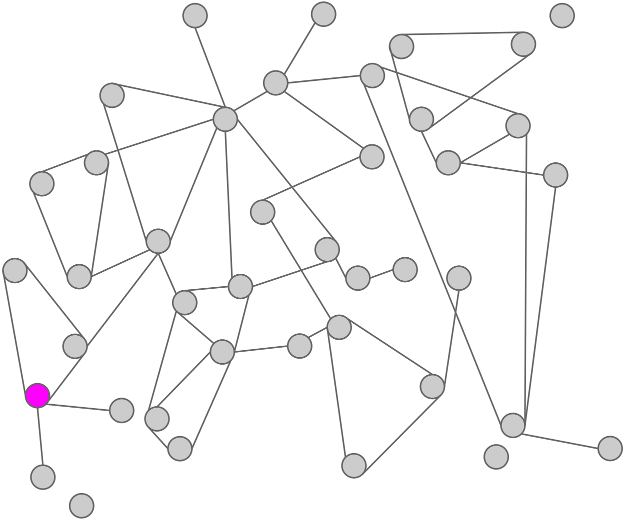

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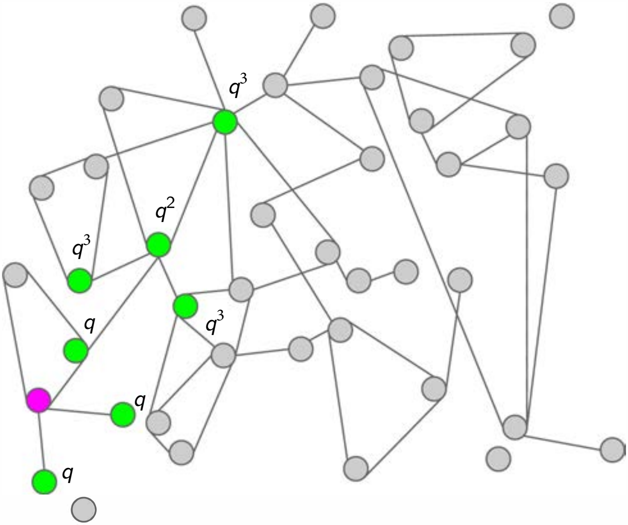
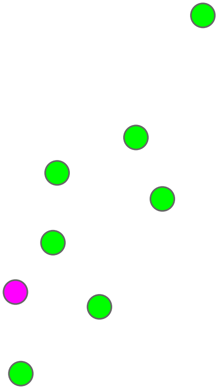


Illustration: Market Participation



Stochastic Financial Networks with Segments

- Most of our results also hold for a more general class, where market participation shock is given by a partition:

$$= \left\{ m^{(1)}, \ldots, m^{(k)} \right\} \text{ where } \begin{cases} m^{(\ell)} \subseteq 2^I & \text{for all } \ell = 1, \ldots k \\ m^{(r)} \cap m^{(\ell)} = \emptyset & \text{for } r \neq \ell \\ \bigcup_{\ell=1}^{\ell=k} m^{(\ell)} = 2^I \end{cases}$$

where is a random partition over I

- Let $\mathscr{P}$ be the set of all such partitions, and $\mu() = \Pr() \in \Delta(\mathscr{P})$

Stochastic Financial Networks with Segments

- The resource constraint is then given for each market segment (or component)

$$\sum_{j \in m} c_j \leq \sum_{j \in m} y_j \text{ for all } y \in \mathscr{Y}, m \in$$

- Relevant state for these models is $s = (y,)$
- The Stochastic Financial Network is the network formed by a collection of complete, isolated subgraphs

Network Example for Market Segments

- Suppose there is an underlying *social graph* (SG) $G = (E, V)$ (binary, undirected).
- We define a link in this network by the relationship

$$i \leftrightarrow j \iff i \text{ and } j \text{ are able to exchange resources}$$

We also write it as $G_{ij} = 1$

Network Example for Market Segments

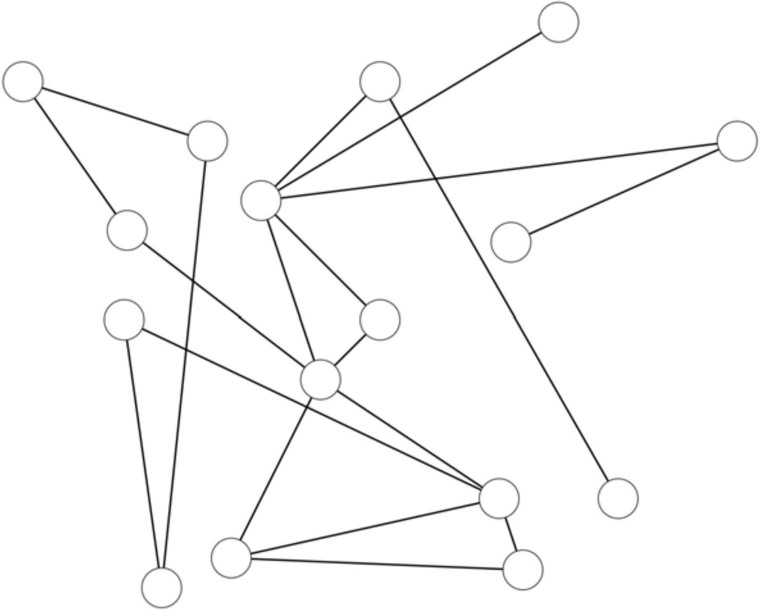
- SFN: Distribution over sub-graphs of G, capturing two facts:

1. Not all agents are active at all states (either exogenously or by choice). Let $A \subseteq I$ be the (random) subset of active traders
2. If i can only trade through her neighbours G may be split into disconnected components; i.e. the induced subgraph

$$G_A := G \mid A$$

is (potentially) disconnected.

Illustration: Segmented Markets



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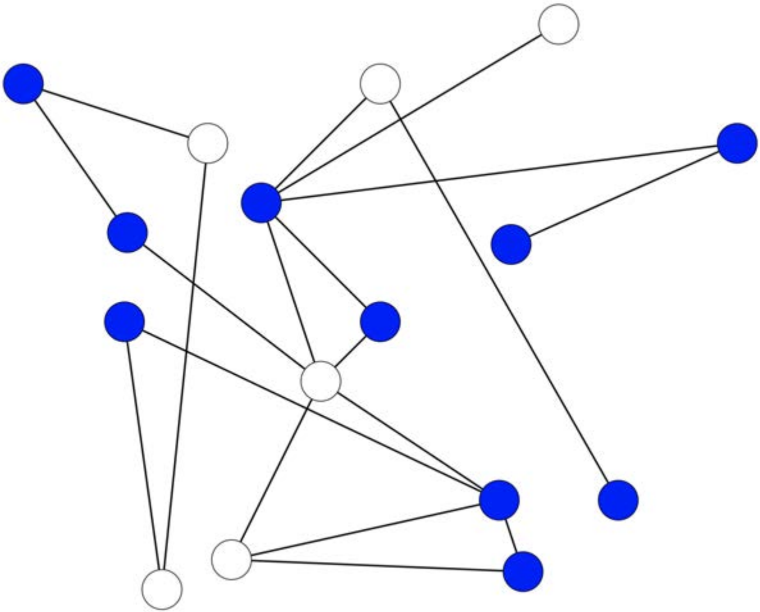

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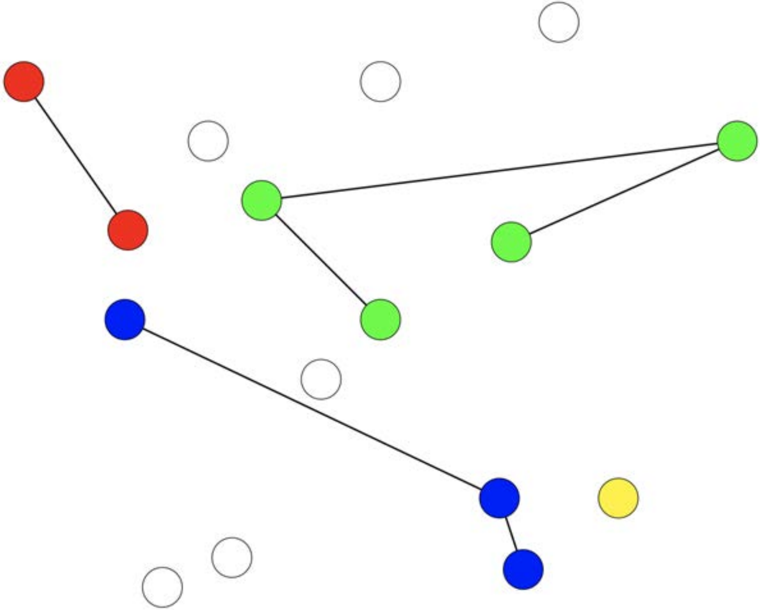
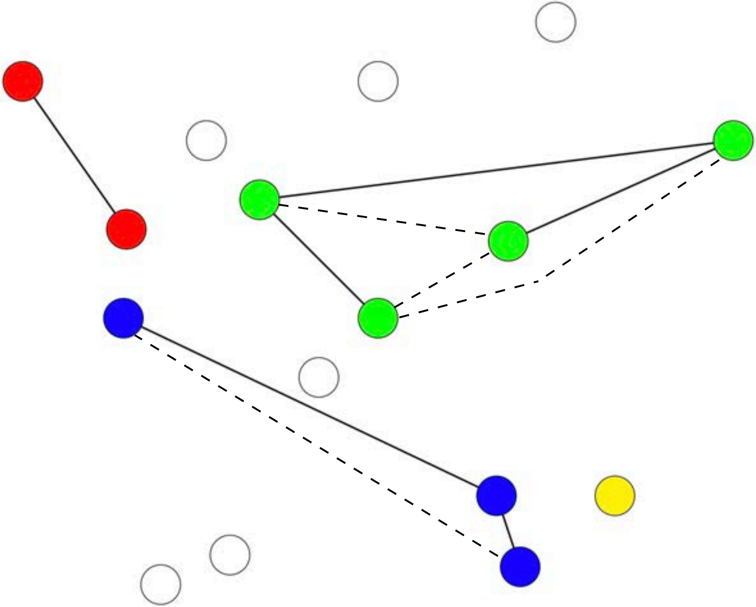


Illustration: Equivalence to Complete Components



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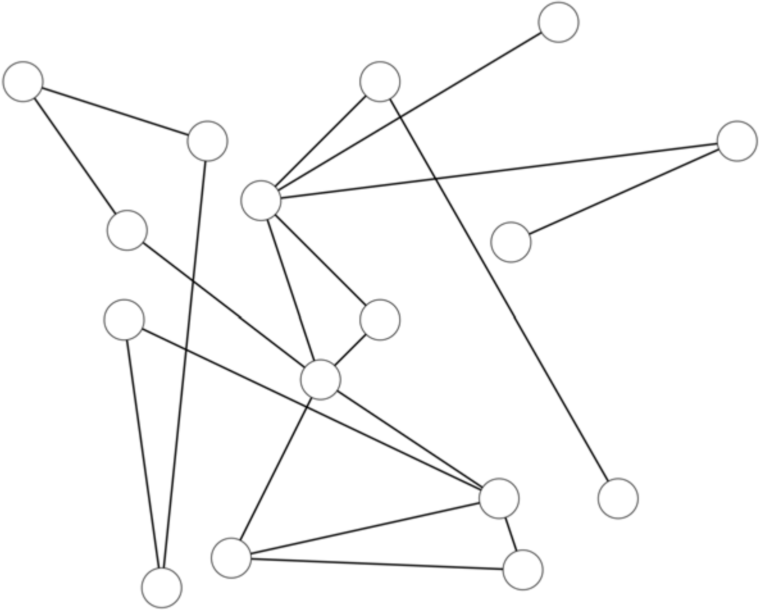

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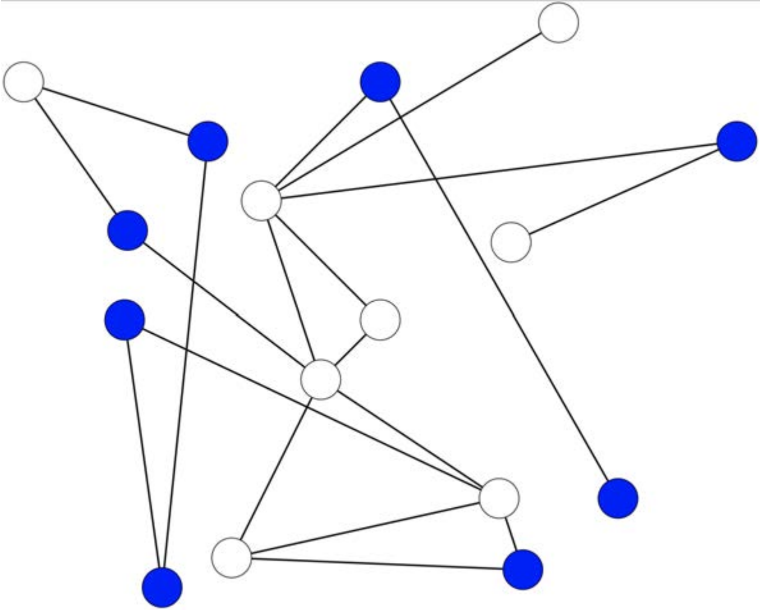
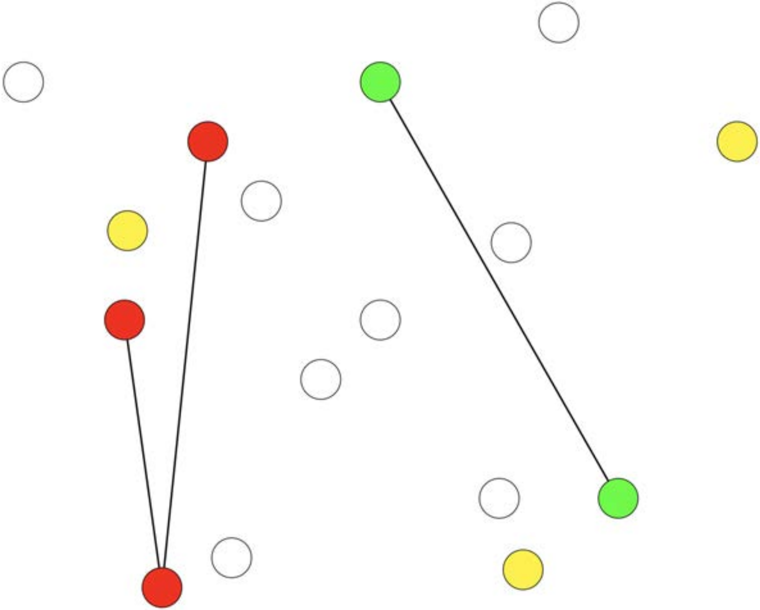


Illustration: Segmented Markets



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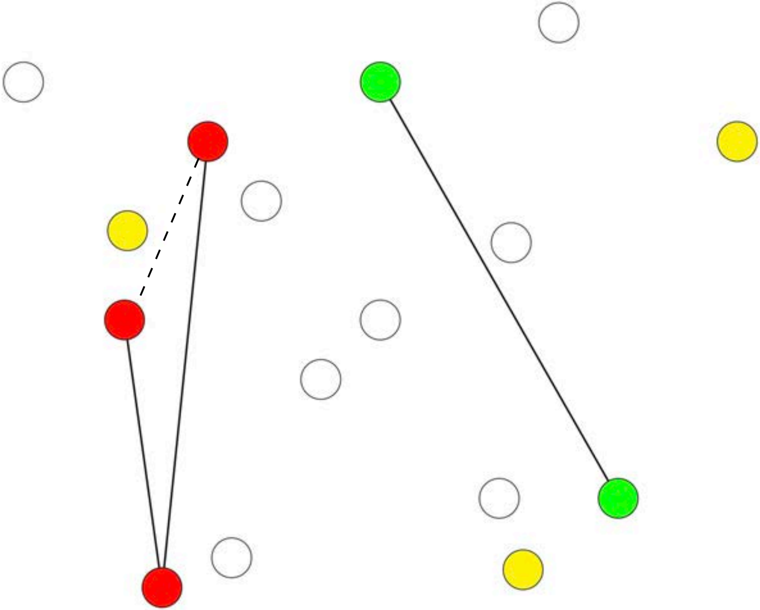

Financial Centrality: Value of liquidity

- Policy experiment: increase income y_i by $\varepsilon > 0$ in all states $s : \zeta_i = 1$
- Ex ante commitment to a named trader without knowing what situation she will be in.

Liquidity injection Interpretation: Take model where agents have

- Risky idiosyncractic income $e_i(s)$
- Endowment of "liquid asset" A_i, which can be redeemed for consumption, whenever $\zeta_i = 1$

$$y_i(s) = \begin{cases} e_i(s) + A_i & \text{if } \zeta_i = 1 \\ e_i(s) & \text{if } \zeta_i = 0 \end{cases}$$

- Equivalent experiment: $A_i \to A_i + \varepsilon$

Alternative Policy

Financial Centrality

- Let $V_{i,\varepsilon}(\lambda)$ be the value of program 1 with injection

Definition (Financial Centrality of i)

$$FC_i := \frac{\partial V_{i,\varepsilon}(\lambda)}{\partial \varepsilon}\Big|_{\varepsilon=0}$$

- Price theoretic version of "derivative w.r.t agent i"

Financial Centrality as guide to Policy

- **Policy experiment:** give income injections (i.e. liquidity) $\varepsilon_i \geq 0$ to a subset $J \subseteq I$
- **Limited resources:** $\sum_{i \in J} \varepsilon_i \leq A$
- **Objective:** choose $\varepsilon^* = (\varepsilon_i^*)_{i \in J}$ to maximize $V_\varepsilon(\lambda)$ s.t $\sum_i \varepsilon_i \leq A$ and $\varepsilon_i \geq 0$

Proposition

For any $J \subseteq I$, $\exists \overline{A}$: *if* $A \leq \overline{A}$ *then* $\varepsilon_i^* = A$ *for* $i = \underset{i \in J}{\operatorname{argmax}} FC_i$ *and* $\varepsilon_j^* = 0$ *for* $j \neq i$

Large Transfers

Responsiveness of Stochastic Financial Networks

- A marginal liquidity injection has 3 possible effects:

1. Risk Sharing effect: an additional dollar given to agent i propagates through the risk sharing contract
2. Participation Effect: injection works as a subsidy for participation.
3. Income Distribution Effect: injection changes endogenous “production” of endowment.

Financial Centrality for Inert Environments

- Write the Lagrangian of (1) as

$$\mathscr{L} = \mathbb{E}_s \Bigg\{ \Bigg(\sum_i \lambda_i u_i \left[c_i \left(s \right) \right] + q \left(s \right) \Bigg[\sum_i \zeta_i \left(y_i - c_i \right) \Bigg] \Bigg\} \Bigg($$

where $q(s) = q(\zeta, y)$ is the Lagrange multiplier of constraint 2.

Financial Centrality for Inert Environments

Proposition

If enviroment is inert to marginal liquidity injections to agent i, then

$$FC_i = \mathbb{E}_s \left[\zeta_i q(s) \right]$$

Passive Planner

Solving for FC for baseline example

- ***Goal:*** how does FC_i depend on moments of $\Pr(\zeta, y)$?
- Benchmark example: homogeneous preferences, i.i.d income shocks, and identical Pareto Weights:

1. $\lambda_i = 1$ and $u_i(c) = u(c) \forall i$
2. $y_i \sim_{i.i.d} f(y)$ with $\mu = \mathbb{E}(y_i)$ and $\sigma^2 = \mathbb{V}(y_i)$
3. $\zeta \perp y$

Approximating FC_i

- $n_\zeta := \sum_i \zeta_i$ be the market size at state ζ.
- Is easy to show that the solution to 1 is

$$c_i(s) = \overline{y}_\zeta \text{ and } q(s) = u'\left(\overline{y}_\zeta\right)$$

where $\overline{y}_\zeta = n_\zeta^{-1} \sum_{j \in I} y_j =$ average income of agents in the market

Approximating FC_i

Proposition

Mean-Value approx. yields

$$FC_i = \mathbb{E}_{\zeta,y}\left[\zeta_i q\left(s\right)\right] \approx u'\left(\mu\right)\mathbb{E}_{\zeta}\left[\zeta_i\left(1 + \gamma\frac{\sigma^2}{n(\zeta)}\right)\right]$$

where $\gamma = u'''\left(\mu\right)/2u'\left(\mu\right)$

Approximating FC_i - Segmented Markets

For the segmented markets model, we get the same formula, only that now

$$FC_i \approx u'(\mu)\mathbb{E}_s\left[\zeta_i\left(1+\gamma\frac{\sigma^2}{|m_i|}\right) \mid \zeta_i = 1\right]$$

Formula for FC: summary of key points

$$FC_i \propto \underbrace{\Pr(\zeta_i = 1)}_{\equiv \text{prob. trade}} \times \underbrace{\left[1 + \gamma \times \mathbb{E}_{\zeta}\left(\frac{\sigma^2}{n(\zeta)} \mid \zeta_i = 1\right)\right]}_{\text{inverse. comovement in market part. and size}}$$

Two parts:

- $\Pr(\zeta_i = 1) =$ likelihood of trading
- $\gamma \times \mathbb{E}\left(\frac{\sigma^2}{n(\zeta)} \mid \zeta_i = 1\right) =$ marginal welfare effect of having i trading in some market, by reducing variance of average income

Example:

- Suppose market is selected by:

1. Choose a host i (probability $1/n$)
2. All neighbours of host conform the market

- In this case

$$FC_i \propto d_i + \sum_{j \in N_i} \frac{\gamma \sigma^2}{d_j}$$

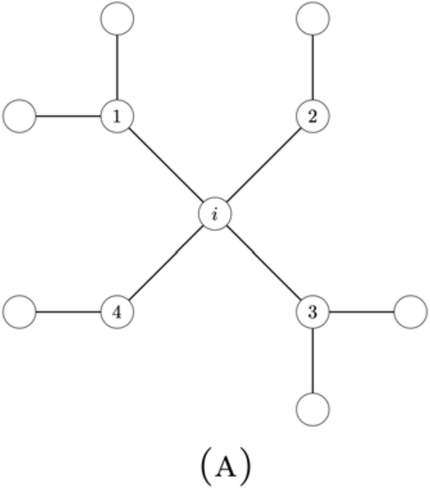

Figure: $F_i > F_j$

Centrality in General Environments

- Departures from benchmark by changing:

1. Volatility ($\text{Var}(y) = \Sigma$)
2. Pareto weights ($\lambda_i \neq 1$)
3. Expected Income ($\mathbb{E}(y_i) = \mu_i$)
4. Heterogeneous Preferences ($u_i \neq u$)
5. Shocks to fundamentals

Multidimensional Heterogeneity

- Agents are more central if
  - Market size small
  - Average volatility is high $(\sigma_i^2 \uparrow)$ and income shocks pos. correlated
  - Average trading agent is more important for the planner $(\overline{\lambda}_\zeta \uparrow)$
  - Average trading agent is poorer in expectation $(\overline{\mu}_\zeta \downarrow)$
  - Average trading agent is more risk averse $(\overline{r}_\zeta \uparrow)$

Positive Counterparts to Financial Centrality

- So far, we have considered centrality from a normative standpoint (i.e. how a planner would valuate an agent).
- Here we investigate centrality as a positive outcome of different decentralized risk sharing economies.

Summary of results

1. In a (constrained) complete markets implementation of the optimal risk sharing contract, centrality is the price of a personalized bond
2. If agents engage in ex-ante bargaining, Financial centrality determines individual fixed effects on consumption

Bargaining over Risk Sharing allocations

- With CARA preferences, Pareto efficient consumption is

$$c_i = \underbrace{\frac{1}{r}\ln\left(\frac{\lambda_i}{\overline{\lambda}_\zeta}\right)}_{\alpha_i} + \overline{y}(s)$$

  i.e. Pareto weights measure how important agents are, in terms of consumption.

- $\alpha_i =$ estimated household fixed effect

Bargaining over Risk Sharing allocations

- Suppose agents bargain over the utility possibility set

$$\mathscr{U} = \{u \in \mathbb{R}^n : \exists c \text{ sat. } (2) \text{ such that } u_i = \mathbb{E}_s[u_i(c_i(s))]\}$$

with autarky as disagreement points.

- We assume Nash Bargaining (also work with KS)

Bargaining over Risk Sharing allocations

Proposition

If preferences are CARA and $y \sim N(\mu, \Sigma)$, then NB Pareto weights solve the following fix point equations:

$$\lambda_i = \frac{r_i + FC_i(\lambda)}{p_i \mathbb{E}_{y_i}\left[u_i'(y_i)\right]}$$

where $\mathbb{E}\left[u_i'(y_i)\right] = \exp\left(-r_i\mu_i + r_i^2\sigma_i^2/2\right)$

Empirical Application: Thai villages

- **Thai Villages** - households (running SME's too). Townsend Thai monthly data on consumption, income, and all transactions.
- We know there is risk sharing among those in informal network.
- Relative Pareto weights can be estimated from Panel regression:

$$c_{i,t} = a_i + by_{it} + \delta_t + \varepsilon_{it}$$

and $a_i = \ln\left(\lambda_i/\overline{\lambda}_\zeta\right)\Bigg($

Empirical Application: Thai villages

- We study the regression

$$a_i = \beta_0 + \beta_1 \rho_i^{\zeta} + \beta_2 \rho_i^{\sigma} + u_i$$

where

$$\rho_i^{\zeta} = \mathrm{cov}_t \left(\zeta_{it}, \frac{1}{n_{vt}} \right) \Bigg($$

and

$$\rho_i^{\sigma} = \mathrm{cov}_t \left(\zeta_{it}, \overline{\sigma}_t \right)$$

with $\overline{\sigma}_t = \frac{1}{n_{vt}} \sum_{ij} \zeta_{it} \zeta_{jt} \hat{\sigma}_{i,j,t}$ the average of income correlations.

Empirical Application: Thai villages

VARIABLES	(1) α_i	(2) α_i	(3) α_i
ρ_i^{ζ}	0.095		0.112
	(0.041)		(0.045)
ρ_i^{σ}		0.103	0.118
		(0.050)	(0.051)
Observations	338	338	338

Crisis Mentality: Financial Centrality and Contagion

The title "Contagion! Systemic Risk in Financial Networks" is intended to suggest that financial contagion is analogous to the spread of disease, and that damaging financial crises may be better understood by bringing to bear ideas gained from studying the breakdown of other complex systems in our world. It also suggests that the aim of systemic risk management is similar to the primary aim of epidemiology, namely to identify situations when contagion danger is high, and then to make targeted interventions to damp out the risk.[1]

The primary goal of this book is to present a unified mathematical framework for the transmission channels for damaging shocks that can lead to instability in financial systems. Models in science and engineering can usually be described as either explanatory or predictive. In the early stages of research in a field, explanatory models may make dramatic oversimplifications or counterfactual assumptions that are only justifiable to the extent they highlight and explain the most critical mechanisms underlying the phenomenon of interest. Later, when guided by such improvements in understanding, predictive models become feasible. Certainly, predictive models will be more complex, and must be carefully calibrated to the details of the observed system in question. Since financial systemic risk is a rather new field, this book focusses on certain explanatory models developed by economists that aim to explore how disruptions can arise in large financial systems.

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Chapter 1
Systemic Risk Basics

Abstract Attempts to define systemic risk are summarized and found to be deficient in various respects. This introductory chapter, after considering some of the salient features of financial crises in the past, focusses on the key characteristics of banks, their balance sheets and how they are regulated.

Keywords Systemic risk definition · Financial stability · Balance sheet · Contagion channel · Macroprudential regulation

Chapter 2
Static Cascade Models

Abstract Network effects such as default contagion and liquidity hoarding are transmitted between banks by direct contact through their interbank exposures. During asset fire sales, shocks are transmitted indirectly from a bank selling assets to other banks via the impact on the price of their common assets. Banks maintain safety buffers in normal times, but these may be weakened or fail during a crisis. Asset prices that are relatively stable in normal times may collapse during a crisis. Banks react to such stresses by making large adjustments to their balance sheets. Such adjustments send further shocks to their counterparties both directly through their exposures and indirectly via asset price impact, creating a cascade. All these cascade mechanisms can be modelled mathematically starting from a common framework. In such models, the eventual extent of a crisis is a fixed point or equilibrium of a cascade mapping. Towards the end of the chapter, a proposal is made that the properties of cascade mappings can be most clearly understood when implemented on very large random financial networks.

Keywords Cascade mechanism · Default and liquidity buffers · Fixed point equations · Cascade equilibrium · Asset fire sales · Random financial network

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