

# MIT 14.13 – Problem Set 4

Please make sure to explain your answers carefully and concisely, i.e. do not simply write a numeric answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

## Part 1: Social Preferences and Workplace Incentive Schemes (40 points)

In lecture, we discussed the evidence from Bandiera et al. (2005), which studies the effect of relative pay on worker productivity. In this question, we will consider workers' effort choices for different payment schemes.

Suppose worker  $i$  can choose how many kilograms of berries to pick each day ( $q_i$ ). Picking more berries requires more strenuous physical exertion and the disutility of this exertion is  $c(q_i) = q_i^2$ . The worker works on a field with  $N - 1$  other workers (so a total of  $N$  workers work on the field). Assume that each worker independently chooses how many kilograms of berries to pick (i.e., the workers do not coordinate or collude).

1. (4 points) First, assume that workers' utility only depends on their own income and their disutility of effort. In particular, the utility of worker  $i$  with income  $y_i$  and who picks  $q_i$  kilograms of berries is

$$u_i(y_i, q_i) = y_i - c(q_i).$$

Suppose that workers are paid a *piece rate* of  $\$p$  per kilogram picked. Thus  $y_i = pq_i$ . How many kilograms of berries should worker  $i$  pick under piece rates  $q_i^{P*}$ ?

### Solution:

The utility of worker  $i$  can be written in terms of  $q_i$ ; it is

$$u_i(q_i) = pq_i - q_i^2.$$

The first-order condition w.r.t  $q_i$  is

$$p - 2q_i^{P*} = 0,$$

which implies that  $q_i^{P*} = p/2$ .

2. (4 points) Now suppose that workers receive *relative pay*; if worker  $i$  picks  $q_i$  kilograms and the other workers pick  $q_j$  kilograms, for  $j \neq i$ , then worker  $i$  is paid

$$y_i = pq_i - \gamma \sum_{j \neq i} \frac{q_j}{N-1}.$$

Assume that  $\gamma > 0$ , so that if other workers pick a lot, then worker  $i$  gets paid less. How many kilograms of berries should worker  $i$  pick under relative pay  $q_i^{R*}$ ?

**Solution:**

Since worker  $i$  does not care about other workers, and she cannot choose the number of kilograms that other workers pick, her first-order condition is the same and thus  $q_i^{P*} = q_i^{R*} = p/2$

3. (4 points) Compare your answers to the previous questions. Do you find that workers exert the same or different effort under the two payment schemes? Explain

**Solution:**

Same answers in 1 and 2. Since workers only care about their own effort and pay, they do not care about the impact of their own effort on others' pay.

4. (4 points) Now assume that workers' utility exhibits a simple form of altruism; in particular, the utility of a worker  $i$  is

$$U_i(y_i, q_i, y_{-i}, q_{-i}) = u_i(y_i, q_i) + \alpha \sum_{j \neq i} u_j(y_j, q_j),$$

where  $\alpha > 0$  and  $u_i(y_i, q_i) = y_i - c(q_i)$  (as above).

How many kilograms of berries should worker  $i$  pick under piece rates and this simple form of altruism  $q_i^{PA*}$ ? How does simple altruism affect productivity when piece rates are used?

**Solution:**

Substituting everything into the utility function gives

$$U_i(y_i, q_i, y_{-i}, q_{-i}) = pq_i - q_i^2 + \alpha \sum_{j \neq i} [pq_j - q_j^2]$$

Once again, the first-order condition w.r.t  $q_i$  is

$$p - 2q_i^{PA*} = 0,$$

and so  $q_i^{PA*} = p/2$ . Thus, this form of simple altruism has no effect on productivity when piece rates are used.

5. (4 points) How many kilograms of berries should worker  $i$  to pick under relative pay and this simple form of altruism  $q_i^{RA*}$ ?

**Solution:**

Substituting everything into the utility function gives

$$U_i(y_i, q_i, y_{-i}, q_{-i}) = pq_i - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - q_i^2 + \alpha \sum_{j \neq i} \left[ pq_j - \gamma \sum_{k \neq j} \frac{q_k}{N-1} - q_j^2 \right]$$

The first-order condition w.r.t  $q_i$  is

$$p - 2q_i^{RA*} - \alpha\gamma = 0,$$

which implies

$$q_i^{RA*} = (p - \alpha\gamma)/2 < p/2.$$

6. (4 points) Compare your answers:

- (i) Compare your answers to questions 4 and 5. Do you find different answers? Why?
- (ii) Compare your answers to questions 1 and 4. Do you find different answers? Why?
- (iii) Compare your answers to questions 2 and 5. Do you find different answers? Why?

**Solution:**

- (i) The answers to 4 and 5 are different. In both cases worker  $i$ 's is altruistic. She picks more under piece rates (question 4) than under relative pay (question 5). The reason that she picks less under relative pay is because she is altruistic and she takes into account the fact that with relative pay when she picks less she lowers her coworkers pay, which hurts them, and she does not want to do that.
- (ii) The answers to 1 and 4 are the same. In both cases, the farm uses piece rates. In question 1 the worker is not altruistic, while in question 4 she is altruistic. However, under piece rates, the quantity of berries that she picks has no effect on her coworkers and so being altruistic has no effect.
- (iii) The answers to 2 and 5 are different. In both cases, the farm uses relative pay. In question 2, the worker is not altruistic, while in question 5 she is altruistic. In question 2, she picks more than in question 5. The reason for this is because in question 2, she does not mind lowering her coworkers pay by working more because she is not altruistic. In contrast, in question 5, she works less because she does not want to lower her coworkers pay because she is altruistic.

7. (4 points) What is the difference in the aggregate quantity of kilograms picked under piece rates and relative pay when workers have simple altruism?

**Solution:** Under piece rates, the aggregate quantity picked is  $Np/2$ .  
 Under relative rates, the aggregate quantity picked is  $N(p - \alpha\gamma)N/2$ .  
 The difference is  $\alpha\gamma N/2$ . More berries are picked under piece rates.

8. (4 points) Suppose that the summer holidays begin and a lot of worker  $i$ 's friends join the farm (assume the total number of workers is still  $N$ ). In our framework, this can be modeled as increasing the  $\alpha$  parameter from  $\alpha$  to  $\tilde{\alpha} = 2\alpha$ . How might this affect the how many kilograms of berries worker  $i$  picks under the two different payment schemes from above (piece rates and relative pay)?

**Solution:** Under piece rates, altruism has no effect and worker  $i$  will still choose  $p/2$ .  
 Under relative rates, worker  $i$  will pick even fewer kilograms of berries than in question 5: specifically

$$(p - 2\alpha\gamma)/2 < (p - \alpha\gamma)/2 = q_i^{RA*}$$

where the inequality holds because  $\alpha > 0$ .

9. (4 points) Suppose that the farm uses relative pay and that the other workers will punish worker  $i$  by reducing her utility by  $D$  if she picks too much. You can think of these punishments as a tool that workers use in order to force each other to reduce output (a type of collusion). Instead of simple altruism, assume that worker  $i$ 's utility is

$$u_i(y_i, q_i) = \begin{cases} y_i - c(q_i) & \text{if } q_i \leq (p - \alpha\gamma)/2 \\ y_i - c(q_i) - D & \text{if } q_i > (p - \alpha\gamma)/2. \end{cases}$$

Assume worker  $i$ 's choice of how many kilograms to pick does not affect her coworkers quantities  $q_j$  for  $j \neq i$ . Also assume that  $D > \frac{\alpha^2\gamma^2}{4}$ .

How many kilograms of berries should worker  $i$  pick under relative pay with social enforcement  $q_i^{RS*}$ ?

How does the solution compare to question 5?

Can we distinguish between altruism or collusion simply by looking at worker  $i$ 's choice of how many berries to pick?

**Solution:** To solve for the utility maximizing  $q_i^{RS*}$ , solve for the optimal quantity for  $q_i \leq (p - \alpha\gamma)/2$  and  $q_i > (p - \alpha\gamma)/2$  and compare the maximized utilities.

First, substitute terms to get:

$$u_i(y_i, q_i) = \begin{cases} pq_i - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - q_i^2 & \text{if } q_i \leq (p - \alpha\gamma)/2 \\ pq_i - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - q_i^2 - D & \text{if } q_i > (p - \alpha\gamma)/2. \end{cases}$$

Notice that both cases are quadratic functions of  $q_i$ .

If  $q_i > (p - \alpha\gamma)/2$ , then the first-order condition is the same as question 2(ii) because  $D$  does not depend on  $q_i$ . Thus the solution is  $\hat{q}_i = p/2$ , which yields utility

$$\hat{u} = \frac{p^2}{4} - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - D.$$

If  $q_i \leq (p - \alpha\gamma)/2$ , then because the objective is a quadratic with a maximum at  $p/2$ , we know that the solution is to choose the largest quantity possible in the region. Thus the solution is  $\tilde{q}_i = (p - \alpha\gamma)/2$ , which yields utility

$$\tilde{u} = \frac{p(p - \alpha\gamma)}{2} - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - \frac{(p - \alpha\gamma)^2}{4}.$$

Expanding and simplifying yields

$$\tilde{u} = \frac{p^2}{4} - \gamma \sum_{j \neq i} \frac{q_j}{N-1} - \frac{\alpha^2 \gamma^2}{4}.$$

Since we assumed that  $D > \frac{\alpha^2 \gamma^2}{4}$ , we see that  $\tilde{u} > \hat{u}$  and thus worker  $i$  chooses

$$q_i^{RS*} = (p - \alpha\gamma)/2.$$

The solution is exactly the same as in question 5. As a consequence, we cannot distinguish between altruism or collusion if we only look at worker  $i$ 's choice of how many berries to pick.

10. (4 points) What evidence do Bandiera et al. (2005) use to distinguish between pure altruism and collusion? Explain.

**Solution:** Bandiera et al. (2005) use data from fruit with tall bushes where coworkers' effort (and output) are unobserved and short bushes where coworkers' effort is observed. The inability to observe coworkers' output has no effect if choices are driven by altruism. However if choices are affected by a desire to avoid punishment (in the collusion case), then the inability to observe coworkers' output has real effects on worker productivity. Bandiera et al. (2005) find that relative pay only has effects when output can be monitored, which provides evidence in favor of the collusion explanation.

## Part 2: Alternative Theories of Social Preferences (40 points)

In class, we played many variants of games that economists use to analyze social preferences. In this question, we consider a general utility function that can accommodate many different theories of social preferences. Suppose that Alex (player 1) and Aaron (player 2) play a two-person game with payoffs  $x_1$  (for Alex) and  $x_2$  (for Aaron). For concreteness, think of the games that were played in class during lecture on March 4th. Aaron is the second player in the game (so, for example, he may have the option to accept or reject an offer made by Alex). Aaron's utility over outcome of the game is

$$u_2(x_1, x_2) = \begin{cases} \rho x_1 + (1 - \rho)x_2 & \text{if } x_2 \geq x_1 \\ \sigma x_1 + (1 - \sigma)x_2 & \text{if } x_2 < x_1 \end{cases},$$

where  $\sigma, \rho \in \mathbb{R}$ .

1. (8 points) Describe Aaron's utility function. Why might  $\rho$  and  $\sigma$  be different? Do you think that it is more natural for  $\rho$  to be larger than  $\sigma$  or not? Why?

### Solution:

- Aaron's utility function is a linear combination of both his own and Alex's payoffs where  $\rho$  is the weight on Alex's payoff when Aaron gets more than Alex and  $\sigma$  is the weight on Alex's payoff when Alex gets more than Aaron.
- There might be a difference between  $\rho$  and  $\sigma$  if Aaron cares more or less about Alex's payoff when he gets more or less than Alex. A natural assumption might be that  $\rho$  is larger than  $\sigma$ , which means that Aaron cares more about Alex's payoff when he gets more than him. This might be natural because it represents a case where we are more generous to people who get less.

2. (8 points) For each of the following cases, describe how Aaron's utility depends on his own payoff and Alex's payoff. You might want to consult Charness and Rabin (2002) while you try to answer this question.

- (i)  $\sigma \leq \rho < 0$
- (ii)  $\sigma < 0 < \rho < 1$  (also comment on why we require  $\sigma$  and  $\rho$  to be less than 1)
- (iii)  $0 < \sigma \leq \rho \leq 1$
- (iv)  $\sigma = \rho = 0$

**Solution:** These special cases are discussed in Charness and Rabin (2002) and have the following interpretations:

- (i)  $\sigma \leq \rho \leq 0$ : simple competitive preferences. Since both weights are negative, Aaron's utility is decreasing in Alex's payoff. When Alex gets more than Aaron, Aaron's utility decreases more rapidly in Alex's payoff (because  $\sigma \leq \rho$ ).
- (ii)  $\sigma < 0 < \rho < 1$ : difference aversion. When Alex gets more than Aaron, Aaron's utility is decreasing in Alex's payoff (because  $\sigma < 0$ ), which would reduce the difference in their payoffs. In contrast, when Aaron gets more than Alex, Aaron's utility is increasing in Alex's payoff (because  $\rho > 0$ ), which also reduces the difference in their payoffs. We require  $\sigma$  and  $\rho$  to be less than 1 because otherwise, Aaron's utility is decreasing in his own payoff.
- (iii)  $1 \geq \rho \geq \sigma > 0$ : Charness and Rabin (2002) call these "social-welfare preferences". Aaron's utility is increasing in both his own payoff and Alex's payoff. When Aaron gets more than Alex he cares more about Alex's payoff (because  $\sigma \leq \rho$ ).
- (iv)  $\sigma = \rho = 0$ : simple self interest because for any payoffs  $u_2(x_1, x_2) = x_2$ .

3. (8 points) This question maintains the assumption that Aaron's utility is given by  $u_2$  and considers what we can learn about the values of  $\rho$  and  $\sigma$  using games similar to those that were played in class during lecture on March 4th. Denote payoffs  $(x_1, x_2)$  where Alex gets  $x_1$  and Aaron gets  $x_2$ . For each of the following games, state whether the given choice is consistent with each of the special cases (i) - (iv) above.

- (i) A version of a dictator game where Aaron chooses  $L = (\$4.00, \$4.00)$  instead of  $R = (\$7.50, \$4.00)$ .
- (ii) A version of a dictator game where Aaron chooses  $L = (\$2.50, \$3.50)$  instead of  $R = (\$4.00, \$4.00)$ .

In both games, Alex does not have a choice of what to do and he will always accept the amount that is offered to him.

**Solution:**

- (i) Note that under both  $L$  and  $R$ , Aaron gets the same payoff ( $x_2 = \$4.00$ ).
  - Choosing  $L$  is consistent with (i) because under (i) Aaron's utility is decreasing in  $x_1$  and  $L$  has a lower  $x_1$  than  $R$ .
  - Choosing  $L$  is consistent with (ii) because in both cases Alex gets (weakly) more and under (ii) Aaron's utility is decreasing in  $x_1$  when Alex gets more.
  - Choosing  $L$  is inconsistent with (iii) because under (iii) Aaron's utility increases in Alex's payoff.
  - Last, choosing  $L$  is consistent with (iv), because under (iv) Aaron only cares about his own payoff, which is equal for both choices.
- (ii) In this game, Aaron can reduce Alex's payoff by \$1.50, but at a cost of \$0.50.
  - Choosing  $L$  is consistent with (i) because under (i) Aaron's utility is decreasing in Alex's payoff and thus it is possible that he would pay \$0.50 to reduce Alex's payoff by \$1.50.
  - Choosing  $L$  is inconsistent with (ii) because Aaron gets (weakly) more under either option and if Aaron gets more, then his utility is increasing in both his own and Alex's payoff, so he would not choose an option that reduces both payoffs.
  - Choosing  $L$  is inconsistent with (iii) because under (iii) Aaron's utility is increasing in both payoffs.
  - Last, choosing  $L$  is inconsistent with (iv) because under (iv) Aaron maximizes  $x_2$ , which involves choosing  $R$ .

4. (8 points) Consider an ultimatum game: Alex is given \$10 and offers \$ $x$  to Aaron; Aaron can accept \$ $x$  (and then Alex gets \$10 - \$ $x$ ) or reject \$ $x$  (and then Alex and Aaron both get nothing). Suppose Alex has utility

$$u_1(x_1, x_2) = \begin{cases} \tilde{\rho}x_1 + (1 - \tilde{\rho})x_2 & \text{if } x_2 \geq x_1 \\ \tilde{\sigma}x_1 + (1 - \tilde{\sigma})x_2 & \text{if } x_2 < x_1 \end{cases}.$$

What is one reason why it is difficult to learn anything about  $\tilde{\rho}$  or  $\tilde{\sigma}$  from Alex's choice of \$ $x$ ?

**Solution:**

It is difficult to learn about  $\tilde{\rho}$  and  $\tilde{\sigma}$  because Alex's choice of \$ $x$  also depends on his beliefs about whether Aaron will accept or reject the offer. For instance, if Alex thinks that Aaron will only accept offers of  $x \geq \$5$ , then he might give Alex \$5 even if he is entirely selfish and does not care about Aaron's payoff and/or utility.

5. (8 points) Suppose Aaron has social preferences with  $0 < \rho = \sigma < 1$ , but his utility is not linear in payoffs. Specifically his utility is

$$\tilde{u}_2(x_1, x_2) = \begin{cases} \rho f(x_1) + (1 - \rho)g(x_2) & \text{if } x_2 \geq x_1 \\ \sigma f(x_1) + (1 - \sigma)g(x_2) & \text{if } x_2 < x_1 \end{cases},$$

Aaron likes chocolate, so his utility from chocolate is  $g(x_2) = x_2$ . He also knows that Alex's is lactose-intolerant; Alex does not eat chocolate, so  $f(x_1) = 0$ .

Suppose Alex and Aaron play a dictator game and Aaron is the dictator. He has 10 chocolates and he can give as many as he likes to Alex. Alex has no choices in this game. What will Aaron choose if his utility function is  $\tilde{u}_2$ ? If we instead assumed Aaron's utility function was  $u_2$  (from the beginning of the question), what will we incorrectly infer about his social preferences (because we don't know about Alex's lactose intolerance)?

**Solution:** Aaron is the dictator and he knows Alex is lactose intolerant. With utility function  $\tilde{u}_2$  Aaron incorporates Alex's lactose intolerance into his utility function ( $f(x_1) = 0$ ). Aaron also has  $0 < \sigma = \rho < 1$ , so his utility is increasing in his own payoff. Thus Aaron keeps all the chocolate for himself.

Suppose that we assumed that Aaron had utility function  $u_2$ . We see that Aaron chooses  $x_1 = 0$  and  $x_2 = 10$ . Since  $x_2 > x_1$ , we can make inferences on  $\sigma$ , but not  $\rho$ . Because of the linear utility function, all we can infer is that  $\sigma < 1/2$ , however, this is not necessarily true (there is nothing in the question to exclude the possibility that  $\sigma \geq 1/2$ ).

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