

MIT 14.13 – Problem Set 4

Please make sure to explain your answers carefully and concisely, i.e. do not simply write a numeric answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

Part 1: Social Preferences and Workplace Incentive Schemes (40 points)

In lecture, we discussed the evidence from Bandiera et al. (2005), which studies the effect of relative pay on worker productivity. In this question, we will consider workers' effort choices for different payment schemes.

Suppose worker i can choose how many kilograms of berries to pick each day (q_i). Picking more berries requires more strenuous physical exertion and the disutility of this exertion is $c(q_i) = q_i^2$. The worker works on a field with $N - 1$ other workers (so a total of N workers work on the field). Assume that each worker independently chooses how many kilograms of berries to pick (i.e., the workers do not coordinate or collude).

- (4 points) First, assume that workers' utility only depends on their own income and their disutility of effort. In particular, the utility of worker i with income y_i and who picks q_i kilograms of berries is

$$u_i(y_i, q_i) = y_i - c(q_i).$$

Suppose that workers are paid a *piece rate* of $\$p$ per kilogram picked. Thus $y_i = pq_i$. How many kilograms of berries should worker i pick under piece rates q_i^{P*} ?

- (4 points) Now suppose that workers receive *relative pay*; if worker i picks q_i kilograms and the other workers pick q_j kilograms, for $j \neq i$, then worker i is paid

$$y_i = pq_i - \gamma \sum_{j \neq i} \frac{q_j}{N-1}.$$

Assume that $\gamma > 0$, so that if other workers pick a lot, then worker i gets paid less. How many kilograms of berries should worker i pick under relative pay q_i^{R*} ?

- (4 points) Compare your answers to the previous questions. Do you find that workers exert the same or different effort under the two payment schemes? Explain
- (4 points) Now assume that workers' utility exhibits a simple form of altruism; in particular, the utility of a worker i is

$$U_i(y_i, q_i, y_{-i}, q_{-i}) = u_i(y_i, q_i) + \alpha \sum_{j \neq i} u_j(y_j, q_j),$$

where $\alpha > 0$ and $u_i(y_i, q_i) = y_i - c(q_i)$ (as above).

How many kilograms of berries should worker i pick under piece rates and this simple form of altruism q_i^{PA*} ? How does simple altruism affect productivity when piece rates are used?

- (4 points) How many kilograms of berries should worker i to pick under relative pay and this simple form of altruism q_i^{RA*} ?
- (4 points) Compare your answers:

- (i) Compare your answers to questions 4 and 5. Do you find different answers? Why?
 - (ii) Compare your answers to questions 1 and 4. Do you find different answers? Why?
 - (iii) Compare your answers to questions 2 and 5. Do you find different answers? Why?
7. (4 points) What is the difference in the aggregate quantity of kilograms picked under piece rates and relative pay when workers have simple altruism?
8. (4 points) Suppose that the summer holidays begin and a lot of worker i 's friends join the farm (assume the total number of workers is still N). In our framework, this can be modeled as increasing the α parameter from α to $\tilde{\alpha} = 2\alpha$. How might this affect the how many kilograms of berries worker i picks under the two different payment schemes from above (piece rates and relative pay)?
9. (4 points) Suppose that the farm uses relative pay and that the other workers will punish worker i by reducing her utility by D if she picks too much. You can think of these punishments as a tool that workers use in order to force each other to reduce output (a type of collusion). Instead of simple altruism, assume that worker i 's utility is

$$u_i(y_i, q_i) = \begin{cases} y_i - c(q_i) & \text{if } q_i \leq (p - \alpha\gamma)/2 \\ y_i - c(q_i) - D & \text{if } q_i > (p - \alpha\gamma)/2. \end{cases}$$

Assume worker i 's choice of how many kilograms to pick does not affect her coworkers quantities q_j for $j \neq i$.

Also assume that $D > \frac{\alpha^2\gamma^2}{4}$.

How many kilograms of berries should worker i to pick under relative pay with social enforcement q_i^{RS*} ?

How does the solution compare to question 5?

Can we distinguish between altruism or collusion simply by looking at worker i 's choice of how many berries to pick?

10. (4 points) What evidence do Bandiera et al. (2005) use to distinguish between pure altruism and collusion? Explain.

Part 2: Alternative Theories of Social Preferences (40 points)

In class, we played many variants of games that economists use to analyze social preferences. In this question, we consider a general utility function that can accommodate many different theories of social preferences. Suppose that Alex (player 1) and Aaron (player 2) play a two-person game with payoffs x_1 (for Alex) and x_2 (for Aaron). For concreteness, think of the games that were played in class during lecture on March 4th. Aaron is the second player in the game (so, for example, he may have the option to accept or reject an offer made by Alex). Aaron's utility over outcome of the game is

$$u_2(x_1, x_2) = \begin{cases} \rho x_1 + (1 - \rho)x_2 & \text{if } x_2 \geq x_1 \\ \sigma x_1 + (1 - \sigma)x_2 & \text{if } x_2 < x_1 \end{cases},$$

where $\sigma, \rho \in \mathbb{R}$.

1. (8 points) Describe Aaron's utility function. Why might ρ and σ be different? Do you think that it is more natural for ρ to be larger than σ or not? Why?
2. (8 points) For each of the following cases, describe how Aaron's utility depends on his own payoff and Alex's payoff. You might want to consult Charness and Rabin (2002) while you try to answer this question.
 - (i) $\sigma \leq \rho < 0$
 - (ii) $\sigma < 0 < \rho < 1$ (also comment on why we require σ and ρ to be less than 1)
 - (iii) $0 < \sigma \leq \rho \leq 1$
 - (iv) $\sigma = \rho = 0$

3. (8 points) This question maintains the assumption that Aaron's utility is given by u_2 and considers what we can learn about the values of ρ and σ using games similar to those that were played in class during lecture on March 4th. Denote payoffs (x_1, x_2) where Alex gets x_1 and Aaron gets x_2 . For each of the following games, state whether the given choice is consistent with each of the special cases (i) - (iv) above.

(i) A version of a dictator game where Aaron chooses $L = (\$4.00, \$4.00)$ instead of $R = (\$7.50, \$4.00)$.

(ii) A version of a dictator game where Aaron chooses $L = (\$2.50, \$3.50)$ instead of $R = (\$4.00, \$4.00)$.

In both games, Alex does not have a choice of what to do and he will always accept the amount that is offered to him.

4. (8 points) Consider an ultimatum game: Alex is given \$10 and offers \$ x to Aaron; Aaron can accept \$ x (and then Alex gets \$10 - \$ x) or reject \$ x (and then Alex and Aaron both get nothing). Suppose Alex has utility

$$u_1(x_1, x_2) = \begin{cases} \tilde{\rho}x_1 + (1 - \tilde{\rho})x_2 & \text{if } x_2 \geq x_1 \\ \tilde{\sigma}x_1 + (1 - \tilde{\sigma})x_2 & \text{if } x_2 < x_1 \end{cases}$$

What is one reason why it is difficult to learn anything about $\tilde{\rho}$ or $\tilde{\sigma}$ from Alex's choice of \$ x ?

5. (8 points) Suppose Aaron has social preferences with $0 < \rho = \sigma < 1$, but his utility is not linear in payoffs. Specifically his utility is

$$\tilde{u}_2(x_1, x_2) = \begin{cases} \rho f(x_1) + (1 - \rho)g(x_2) & \text{if } x_2 \geq x_1 \\ \sigma f(x_1) + (1 - \sigma)g(x_2) & \text{if } x_2 < x_1 \end{cases}$$

Aaron likes chocolate, so his utility from chocolate is $g(x_2) = x_2$. He also knows that Alex's is lactose-intolerant; Alex does not eat chocolate, so $f(x_1) = 0$.

Suppose Alex and Aaron play a dictator game and Aaron is the dictator. He has 10 chocolates and he can give as many as he likes to Alex. Alex has no choices in this game. What will Aaron choose if his utility function is \tilde{u}_2 ? If we instead assumed Aaron's utility function was u_2 (from the beginning of the question), what will we incorrectly infer about his social preferences (because we don't know about his lactose intolerance)?

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14.13 Psychology and Economics
Spring 2021

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