

# MIT 14.13 – From 2017 (Problem Set 2)

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## Question 1: Illiquid savings, credit card borrowing, and commitment

Alex is a fully naive hyperbolic discounter with  $\beta = 0.5$  and  $\delta = 1$  and  $\hat{\beta} = 1$ . Alex lives for three periods  $t = 0, 1$ , and  $2$ . He derives utility from consumption in each period. His instantaneous utility from consuming an amount  $c_t > 0$  (i.e.  $c_t \leq 0$  is not possible) in period  $t$  is

$$u(c_t) = \ln(c_t) \text{ for } t = 0, 1, 2$$

Accordingly, Alex's discounted lifetime utility from the perspective of period 0 is given by

$$U_0(c_0, c_1, c_2) = \ln(c_0) + \beta(\ln(c_1) + \ln(c_2))$$

Alex starts with wealth of \$60 at  $t = 0$ . He can keep his wealth in a checking account, which has no interest and would allow him to withdraw money at any time. That is, if he puts \$ $x$  into his account in period 0, he could withdraw up to \$ $x$  at period 1. Similarly, if he puts \$ $y$  into his account in period 1, he could withdraw up to \$ $y$  at period 2.

Alex also has access to a retirement account. This account requires money be deposited in period 0 and not withdrawn until period 2, but offers a  $r^r = 20\%$  interest rate. For example, if he puts \$10 into the account in period 0, he could withdraw \$0 in period 1 and \$12 in period 2.

Finally, Alex has access to a credit card in period 1. Alex can use the card to borrow money in period 1 but must repay at  $r^c = 50\%$  interest in period 2. For example, if he decides to borrow \$10 in period 1, he has to pay back \$15 in period 2.

1. When making his consumption plan in period 0, Alex has two options for transferring money to period 1. What are the two options? Which of the two options will Alex anticipate using when making his consumption plan in period 0?
2. Show that the consumption plan Alex makes in period 0 involves  $c_1 = \beta c_0$ .
3. Using the insights from parts (1) and (2), write down Alex's maximization problem in period 0 and solve for his planned  $c_0$ ,  $c_1$ , and  $c_2$ .
4. What does Alex end up doing in period 1? Comment.

5. Suppose that Alex becomes TA for 14.13 in period 0 and is now fully sophisticated, i.e.  $\hat{\beta} = \beta = 1/2$ . Argue that in period 0, Alex anticipates that in period 1 he will choose  $c_1$  and  $c_2$  such that  $c_2 = \beta(1 + r^c)c_1$ .
6. Write down Alex's maximization problem in period 0. Explain what is different from Alex's maximization problem in part (3) and why. Note that you do not need to solve it (the algebra is messy!).
7. Alex's friend Aaron offers to give (fully sophisticated) Alex a commitment device to help his with his present bias problem. Is it possible for Alex to be worse off (as measured by his discounted utility in period 0) by (voluntarily) choosing *any* commitment contract that Aaron offers to his in period 0? A concise verbal answer is sufficient.
8. Suppose now that Alex is instead partially naive (i.e. with  $\beta < \hat{\beta} < 1$ ). Can Aaron make Alex worse off by offering him a commitment device, as measured by his discounted life-time utility from the perspective of period 0? If no, explain why not. If yes, provide an example, and explain clearly why and how the commitment contract could make Alex worse off. Again, a concise verbal answer is sufficient.

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