MIT 14.13 – Midterm Exam Spring 2020

April 6, 2020

- There is a total of 85 points in the exam, so if you spend roughly one minute per point, you will have plenty of time to finish the exam and submit your answers in time.
- What materials can you use?
  - You can use slides and notes from lectures, recitations, and psets. You can also use a calculator.
  - You CANNOT receive help from others while taking the exam (online, in person, or any other way).
  - You CANNOT try to find answers to the questions online other than the Learning Modules website.
  - You CANNOT try to find questions or answers online other than looking at existing Piazza posts.
  - You can ask PRIVATE Piazza questions to clarify things if you think that is important and/or if you face technical difficulties, but you CANNOT ask public questions on Piazza.
  - You CANNOT watch lecture videos during the exam.
  - Support animals are fine!
- **Honor code:** We trust you to follow these rules. Question 4 asks you to type your name as an electronic signature confirming that you followed the rules given above for taking them exam.
- While taking this exam, always keep in mind that you are a wonderful person regardless of your answers in this exam. You will pass this class as long as you try your best.
- Good luck!

QUESTION 1: True, False, or Uncertain [20 points]

Please answer EXACTLY five out of the following seven questions. If more than five questions are answered, we will grade only the first five questions according to the order below.

Please state whether each of the following statements is true, false, or uncertain. Always explain your answer carefully and concisely. Your score is largely determined by the quality of your explanation. You only need to give the intuition for your answer, not a formal proof.

1. (4 points) Being fully sophisticated rather than fully naive (for the same present bias β) can make individuals worse off in their choices.

Solution: True. This is particularly likely to be the case for leisure goods. Full sophistication about one's present bias makes individuals realize they will not be patient in the future. This can lower the perceived benefits of delaying gratification and thus exacerbate short-run impatience. There are cases in which every self would be better off under full naiveté than under full sophistication (see slides from lectures 3 and 4 for an example).

2. (4 points) Suppose John is a quasi-hyperbolic discounter. Prior to this semester, he was fully naive, but attending 14.13 lectures caused him to become fully sophisticated. Taking 14.13 therefore removed John's present bias and he will not suffer from any of the negative consequences of present bias anymore.

Solution: False. Full sophistication means John is *aware* of his present bias. It does not remove his present bias or its negative consequences.

3. (4 points) People often decline small-scale gambles with positive expected value. Expected utility theory can explain such behavior but doing so requires high values of estimated risk aversion parameters (γ). This in turn leads to absurd implications when considering larger-scale choices.

Solution: True. People often decline small gambles. To explain this using expected utility theory, we require high values of risk aversion parameters. For example, in lectures 7 and 8, we saw that an individual who has a wealth of \$20,000, CRRA utility, and rejects a 50-50 bet to win \$110 or lose \$100 would have $\gamma > 18.2$. This is much higher than the $\gamma \in (0, 2)$ that is often assumed by economists based on behavior in large-scale decisions. The high values of risk aversion parameters suggested by rejection of small-scale gambles has absurd implications for larger-scale choices. We also saw in lectures 7 and 8 that an individual with $\gamma = 10$ would be indifferent between receiving $\$53,991$ with certainty and taking a 50-50 gamble for $\$50,000$ or $\$100,000$. This certainty equivalent seems very low and $\gamma > 18.2$ would imply an even lower value. See Rabin (2000) for more on this issue.

4. (4 points) Reference dependence can explain why the distribution of marathon finishing times exhibits bunching at 30-minute intervals.

Solution: True. Marathon runners may set goals of finishing within "round" times like 3:30, 4:00, or 4:30 (rather than times like 3:23, 4:04, or 4:32). This goal becomes a reference point in their utility functions, leading them to run the race so as to meet the goal. Consequently, the distribution of finishing times is not smooth and exhibits bunching just above half-hour finishing times.

5. (4 points) The Ultimatum Game allows researchers to identify whether proposers are generous or strategic.

   Solution: False. Both generous and strategic motives may lead proposers to propose more; generosity might lead them to propose more money because they want the responder to end up with more, while strategic play may lead them to propose more so that the responder does not reject the offer (in which case the proposer would receive nothing). Observing proposers propose high values therefore does not say whether proposers are generous or strategic.

6. (4 points) Many people care about what others think about them. Such social image concerns can be an important motivator of helping others, e.g. by giving them money.

   Solution: True. Many of us care about what others think of us. Helping other people can make others think more favorably of us, leading them to think we are kind, generous, etc. Our concern for what others think can therefore be an important motivation for donating money or helping other people more generally.

7. (4 points) A health insurance company exploiting its customers' loss aversion might offer insurance products with very high deductibles and low premiums.

   Solution: False. Exploiting customers' loss aversion would lead insurance companies to do the opposite: offer very *low* deductibles and *high* premiums. Loss-averse customers have a high willingness to pay to avoid losses. Insurance products with very low deductibles minimize customers' losses, generating high consumer surplus that insurance companies can extract through high premiums. Insurance companies may very well offer contracts with very high deductibles and low premiums, but these aren't contracts that exploit customers' loss aversion.

QUESTION 2: Multiple Choice [20 points]

Please select ALL of the correct answer options for each of the following questions. For each question, it is possible that none, some, or all of the options are correct.

1. (4 points) Which of the following behaviors can be consistent with quasi-hyperbolic discounting but NOT with exponential discounting?

   (a) Time inconsistency

   (b) Impatience for all time horizons

   (c) Demand for commitment

   (d) Short-run impatience and long-run patience

   Solution: (a), (c), and (d). Time consistency is an implication of exponential discounting, whereas quasi-hyperbolic discounting allows for time inconsistency. When choices are time inconsistent, individuals may demand commitment devices, meaning there can be demand for commitment in the quasi-hyperbolic model but not the exponential discounting model. The two discount factors of the quasi-hyperbolic model allow it to explain both short-run impatience and long-run patience, while the exponential discounting model cannot. Both models allow for discounting between every period and the next so both allow for impatience for all time horizons.

   Grading note: 1 point for not checking (b), 1 point each for checking (a), (c), (d).

2. (4 points) Let Maddie's utility from owning x apples be $u(x)$. Maddie currently does not own any apples. Confronted with a gamble that offers x_H apples with probability p and $x_L < x_H$ apples with probability $1 - p$, suppose

$$p \cdot u(x_H) + (1 - p) \cdot u(x_L) < u(p \cdot x_H + (1 - p) \cdot x_L).$$

   Which of the following properties do we know FOR SURE this utility function for apples exhibits?

   (a) Risk neutrality

   (b) Risk aversion

   (c) Risk seeking

   (d) Loss aversion

   Solution: (b). The inequality means the expected utility of participating in the gamble provides Maddie strictly less utility than receiving the expected value of the gamble with certainty, that is, Maddie is risk-averse. For Maddie to be risk-seeking, the opposite would need to be true, and for her to be risk-neutral, she would need to be indifferent between these two options. There is no notion of loss versus gain in the information provided, so we cannot be sure she exhibits loss aversion.

   Grading note: 4 points for checking (b) and nothing else, 2 points for checking (b) and something else, 0 points otherwise.

3. (4 points) Alex has the following utility function:

$$u(x) = \begin{cases} 5\sqrt{x} & \text{if } x \geq 0 \\ -10\sqrt{|x|} & \text{if } x < 0 \end{cases}$$

   Which of the following properties do we know FOR SURE Alex exhibits?

(a) Impatience
(b) Reference-dependence
(c) Loss aversion
(d) Diminishing sensitivity

> **Solution:** (b), (c), and (d). The utility function exhibits reference dependence because the shape of the utility changes around x = 0. The utility function exhibits loss aversion because negative deviations from x = 0 lower utility more than positive deviations increase it. The utility function also exhibits diminishing sensitivity because changes in x that are further away from 0 produce smaller changes in utility than changes closer to 0. There is no time component to the utility function so we cannot say it exhibits impatience.
>
> **Grading note:** 1 point for not checking (a), 1 point each for checking (b), (c), (d).

4. (4 points) Frank has 200 students, 100 stopwatches, and 100 baseball caps.

- Suppose none of the students own stopwatches or baseball caps to start with.
- Asking students to choose between different amounts of money and the two items, Frank finds that each student is (i) exactly indifferent between \$10 and a stopwatch, and (ii) exactly indifferent between \$10 and a baseball cap.
- Frank then randomly gives the stopwatches and baseball caps to his students, until each student receives either exactly one stopwatch or exactly one baseball cap.
- Frank then suddenly realizes that he needs lots of baseball caps and stopwatches, so he tries to purchase the items back from his students.

Once students were given a watch or a cap, which of the following valuations, i.e. minimum prices at which students are willing to sell the items back to Frank, are consistent with the endowment effect?

(a) Students given a watch are only willing to sell it back when offered at least \$3.
(b) Students given a cap are only willing to sell it back when offered at least \$5.
(c) Students given a watch are only willing to sell it back when offered at least \$9.
(d) Students given a cap are only willing to sell it back when offered at least \$10.
(e) Students given a watch are only willing to sell it back when offered at least \$15.
(f) Students given a cap are only willing to sell it back when offered at least \$22.

> **Solution:** (e) and (f). The endowment effect predicts that ownership of an item makes an individual value it more. Prior to being given a watch or a cap, all students valued each item at \$10. If subject to an endowment effect, students should value the item they received at more than \$10 when Frank tries to purchase it back. Valuations are above \$10 in options (e) and (f) but not in options (a), (b), (c), or (d).
>
> **Grading note:** 4 points for checking (e) and (f) and nothing else, 3 points for checking (e) and (f) and something else, 1 point for checking one of (e) and (f), 0 points otherwise.

5. (4 points) Evidence from various experiments shows that people give on average about 20 to 30 percent of the available money to the other person in dictator games. Such evidence could be interpreted as evidence of altruism, i.e. that people genuinely care about others. What kinds of evidence suggest that people might be motivated by more than just altruism?

(a) Experiments that show people are loss averse
(b) Experiments on moral wiggle room

(c) Experiments that allow people to exit dictator games

(d) Experiments that allow people to hide behind a computer

(e) Experiments that show people are present-biased

Solution: (b), (c), and (d). Loss aversion and present bias do not describe individuals' concern for others so showing people exhibit these tendencies would not tell us about the extent of individuals' altruism. Experiments on moral wiggle room suggest individuals are willing to be greedy if they can avoid feeling selfish about it, implying decisions in the dictator game could be motivated by avoiding guilt. Experiments that allow people to exit dictator games or to hide behind a computer suggest individuals are willing to be greedy if others will not know, implying face-saving concerns could also explain decisions in the dictator game.

Grading note: 4 points for checking (b), (c), (d), and nothing else; 3 points for checking (b), (c), (d), and something else; 2 points for checking two of (b), (c), (d); 1 point for checking one of (b), (c), (d).

QUESTION 3: Estimating Discount Factors [45 points]

Please make sure to explain your answers in this section carefully and concisely. Do not simply write an answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

You become interested in estimating the discount factors of your TAs, Maddie and Pierre-Luc, after learning about time preferences in 14.13. Through some clever interviewing of them, you obtain information about their preferences for consumption of chocolate over time (both Maddie and Pierre-Luc love chocolate!) from which you can back out their discount factors.

You start with understanding Maddie's preferences. Let the unit of time be one day and let Maddie's instantaneous utility from consuming x pieces of chocolate (on any given day) be $u(x) = \sqrt{x}$.

1. (5 points) Today (Monday), Maddie tells you she is indifferent between consuming 9 pieces of chocolate today (Monday) and 16 pieces tomorrow (Tuesday). Assuming Maddie is an exponential discounter, what daily discount factor, δ, does her statement imply?

Solution: Consuming 9 pieces of chocolate today provides Maddie a utility of $\sqrt{9}$. From the perspective of today, and assuming she is an exponential discounter, consuming 16 pieces of chocolate tomorrow provides a utility of $\delta\sqrt{16}$. Her indifference between these two options means that

$$\sqrt{9} = \delta\sqrt{16},$$

which implies $\delta = \frac{3}{4}$.

2. (5 points) Today (Monday), Maddie now tells you that she is indifferent between consuming 9 pieces of chocolate tomorrow (Tuesday) and 9 pieces of chocolate in two days (Wednesday). Still assuming that Maddie is an exponential discounter, what does this statement imply for Maddie's δ?

Solution: From the perspective of today, and assuming she is an exponential discounter, consuming 9 pieces of chocolate tomorrow provides a utility of $\delta\sqrt{9}$ while consuming 9 pieces of chocolate on Wednesday provides a utility of $\delta^2\sqrt{9}$. Her indifference between these two options means that

$$\delta\sqrt{9} = \delta^2\sqrt{9},$$

which implies $\delta = 1$.

3. (5 points) Can the exponential discounting model explain BOTH of Maddie's statements from questions 1 and 2 together?

Solution: No. In the exponential discounting model, the discount factor between any two periods should be fixed at δ. The discount factor between Monday and Tuesday implied by Maddie's statement in 1 is $\delta = \frac{3}{4}$. But the discount factor between Tuesday and Wednesday implied by her second statement is different, at $\delta = 1$. The exponential discounting model can therefore not explain the two statements together.

4. (4 points) Why might the quasi-hyperbolic discounting model be a better fit to explain Maddie's preferences?

> **Solution:** The quasi-hyperbolic discounting model has an extra parameter that allows for greater discounting in the short-run than in the long-run. This allows for more discounting between today and tomorrow than between tomorrow and the day after, which is what Maddie's statements imply.

5. (6 points) Now assume that Maddie is a quasi-hyperbolic discounter with short-term discount factor β and long-term discount factor δ. Calculate the β and δ implied by her indifference statements in parts 1 and 2.

> **Solution:** We begin by using the second indifference statement to isolate δ. In a quasi-hyperbolic discounting model, the second statement implies
> $$\beta\delta\sqrt{9} = \beta\delta^2\sqrt{9}$$
> which means $\delta = 1$. We use this and the first indifference statement to solve for β. If Maddie is a quasi-hyperbolic discounter, her first statement implies
> $$\sqrt{9} = \beta\delta\sqrt{16}.$$
> With $\delta = 1$, this means $\beta = \frac{3}{4}$.

6. (4 points) Suppose Maddie has $\hat{\beta} = \beta = \frac{3}{4}$ and $\delta = 1$. Suppose further:

   - MIT Medical has come up with a test that allows you to check whether Maddie has eaten more than x pieces of chocolate on any given day. Using this test, you offer a commitment device to Maddie on Monday: if she eats more than x pieces of chocolate on Tuesday, she needs to pay you \$100.
   - The commitment device is effective, i.e. if implemented, it will reduce Maddie's chocolate consumption on Tuesday below what she would consume if not offered the device. When offered this commitment device on Monday, Maddie's willingness to pay for the device exceeds its price p, and you implement it for her.

   On Tuesday, will Maddie wish that she had not chosen the commitment device on Monday? If yes, does this mean that she made a mistake on Monday?

> **Solution:** Yes, she will be unhappy about the commitment device on Tuesday. This is because the commitment device does not allow her to eat as many chocolates as she would like to on Tuesday. But this does not mean that Monday's Maddie made a mistake! It is precisely the point of the commitment device to constrain Tuesday's choice.

7. (6 points) Next you investigate Pierre-Luc's time preferences. Just like Maddie, Pierre-Luc's instantaneous utility from consuming x pieces of chocolate is $u(x) = \sqrt{x}$. As before, the unit of time is one day, and today is Monday.

   You know from previous investigative work that Pierre-Luc is a quasi-hyperbolic discounter with $\beta \in (0, 1)$, $\beta \leq \hat{\beta} \leq 1$, and $\delta \in (0, 1]$.

   You ask Pierre-Luc to predict his <u>future</u> choices:

   - Today, Pierre-Luc predicts that, when asked tomorrow, he will be indifferent between 16 pieces of chocolate on Tuesday and 25 pieces of chocolate on Wednesday.

- Today, Pierre-Luc also predicts that, when asked tomorrow, he will be indifferent between 16 pieces of chocolate on Tuesday and 25 pieces of chocolate on Thursday.

Show that these choices imply that $\hat{\beta} = \frac{4}{5}$ and $\delta = 1$. What can you say about Pierre-Luc's β given the choices?

Solution: Pierre-Luc's first prediction means

$$\sqrt{16} = \hat{\beta}\delta\sqrt{25}.$$

We use $\hat{\beta}$ rather than β because Pierre-Luc's statement is about how he anticipates his tomorrow self will discount future utility. Likewise, his second prediction means

$$\sqrt{16} = \hat{\beta}\delta^2\sqrt{25}.$$

For both of these to be true, it must be that $\delta = 1$. If $\delta = 1$, we have

$$\sqrt{16} = \hat{\beta}\sqrt{25}$$

which means $\hat{\beta} = \frac{4}{5}$. For his β, we can say that $0 < \beta \leq \hat{\beta} = \frac{4}{5}$.

8. (6 points) When asked tomorrow (Tuesday), Pierre-Luc is in fact indifferent between 9 pieces of chocolate tomorrow (Tuesday) and 81 pieces of chocolate on Wednesday. What is his true β? Is Pierre-Luc a fully naive, partially naive/sophisticated, or fully sophisticated quasi-hyperbolic discounter?

Solution: Pierre-Luc's indifference statement means

$$\sqrt{9} = \beta\delta\sqrt{81}.$$

We know from above that $\delta = 1$, meaning $\beta = \frac{1}{3}$.

Pierre-Luc is partially naive/sophisticated as his $\hat{\beta}$ is greater than his β but less than 1.

9. (4 points) Suppose Pierre-Luc has $\beta = \frac{1}{3}$, $\hat{\beta} = \frac{4}{5}$, and $\delta = 1$. Is it possible that Pierre-Luc is willing to pay for a commitment device (the one from above or any other) that will surely NOT reduce his future chocolate consumption?

Solution: Yes. Pierre-Luc is not fully naive so he may be willing to pay for a commitment device. However, because he is only partially sophisticated, a commitment device he purchases may not end up reducing his future consumption. He may purchase a commitment device that would work for $\beta = \frac{4}{5}$ when he thinks his β takes that value. But his future self behaves more impatiently, according to $\beta = \frac{1}{3}$, and the commitment device may not work at this true value of β.