MIT 14.13 – Problem Set 2

Please make sure to explain your answers carefully and concisely, i.e. do not simply write a numeric answer without an explanation of how you arrived at this answer. Answers without adequate explanation will not receive full credit.

Question: Payday Lending

Calvin is a fully naive hyperbolic discounter with $\beta = 0.5$ and $\delta = 1$ and $\hat{\beta} = 1$. Hobbes is a fully sophisticated hyperbolic discounter with $\beta = 0.5$ and $\delta = 1$ and $\hat{\beta} = \beta = 0.5$. They lives for three periods: $t = 0$, 1, and 2. They derive utility from consumption in each period. They have the same instantaneous CRRA utility from consuming an amount $c_t \geq 0$ (i.e. $c_t < 0$ is not possible) in period t :

$$u(c_t) = \sqrt{c_t} \text{ for } t = 0, 1, 2$$

Accordingly, their discounted lifetime utility from the perspective of period $t = 0, 1$ is given by

$$U_t(c_0, c_1, c_2) = \sqrt{c_t} + \beta \sum_{s=t+1}^{s=2} \sqrt{c_s}$$

and their discounted lifetime utility at $t = 2$ is simply $\sqrt{c_2}$.

We also define their long-term lifetime utility as

$$W(c_0, c_1, c_2) = \sqrt{c_0} + \sqrt{c_1} + \sqrt{c_2}$$

which for instance captures their discounted lifetime utility from a period preceding 0, without distortion from present bias.

Calvin and Hobbes start with wealth of $e_0 = \$1500$ at $t = 0$. They can keep their wealth in a checking account, which has no interest and would allow them to withdraw money at any time. That is, if they put $\$x$ into their account in period 0, they could withdraw up to $\$x$ at period 1. Similarly, if they put $\$x$ into their account in period 1, they could withdraw up to $\$x$ at period 2.

They receive a paycheck of $y = \$1200$ at $t = 2$, which is known and perfectly anticipated by both of them at all times.

Finally, they have access to a payday lending service: they can borrow up to \$600 in period 1, but they have to repay twice the borrowed amount on their payday in period 2 (i.e., they can borrow with an interest rate of 100% between these two periods).

1. Let's first consider a third fictional character, Susie, who is not present biased, and does not discount the future. Her utility at any period $t = 0, 1, 2$ is $\sum_{s=t}^{s=2} \sqrt{c_s}$ Susie also starts with \$1500 at $t = 0$, has access to the checking account, and anticipates receiving \$1200 in period 2, but has no access to payday lending. Derive Susie's consumption in period 0, 1 and 2. In particular, show that Susie does not use the checking account from period 1 to period 2.

2. Explain (with no formal derivation) why this means that neither Calvin nor Hobbes would use the checking account from period 1 to period 2.

   Given this result, we will now work under the (non-binding) assumption that the checking account is only available from period 0 to period 1, for simplicity.

3. Let $e_1 \geq 0$ denote the amount of money in Calvin's (or Hobbes') checking account when he enters period 1. Assume $e_1 \leq y$. Derive the amount that he decides to borrow from the payday lending service, b, as a function of e_1. Show that he will consume an equal amount in periods 1 and 2, i.e. $c_1 = c_2$.

4. Using the result from the previous question, derive the amount of money e_1^S that Hobbes, who is fully sophisticated, decides to put in his checking account in period 0. *Hint: Do not worry about checking corner solutions, i.e. assume that* $e_1^S \leq y$ *in order to use the answer to the previous question, and just verify that the value obtained indeed verifies this inequality.*

5. How much will Hobbes end up borrowing from the payday lending service and consuming in each period?

6. Now, let's consider Calvin, who is fully naive ($\hat{\beta} = 1$). In period 0, how much does Calvin *predict* he will borrow from the payday lending service in period 1 if he were to leave e_1 in his checking account?

7. Derive the amount e_1^N that Calvin decides to leave in his checking account in period 0. *Hint: Do not worry about checking corner solutions, i.e. assume that* $4e_1^N \leq y$ *in order to use the answer to the previous question, and just verify that the value obtained indeed verifies this inequality.*

8. How much will Calvin end up borrowing from the payday lending service and consuming in each period?

9. Discuss how Calvin's and Hobbes' consumption paths differ. Compute their long-term lifetime utilities, compare them, and discuss intuitively why they are ordered in this way.

10. Now, assume that no payday lending service is available. Derive the amounts left in the checking account in period 0 by Calvin and Hobbes.

11. Derive the full consumption paths of Calvin and Hobbes in the absence of payday lending. Compare their long-term lifetime utilities to the values found in question 9. Discuss this comparison.

12. Suppose that in period 0, a referendum is organized to ask Calvin and Hobbes whether they want the government to implement a policy that shuts down payday lending. The policy would require some administrative costs which would result in a tax of \$1 levied at the end of period 1. The two options to vote for are Yes and No. What would Calvin vote? What would Hobbes vote? (assuming they are both selfish and only care about improving their own utility). Discuss what this example suggests for the real world problem of regulating payday lending.

    For the rest of the problem, we consider a world where a shock just hurt Calvin and Hobbes before the time analyzed in the problem, so that their initial wealth is now $e_0 = \$200$. Note that the conclusions from question 2 also apply here so it's still correct to simply assume that there is no checking account from period 1 to period 2.

13. Noting that answers to questions 3 and 6 are unchanged, show that neither Calvin nor Hobbes leave anything in their checking accounts in period 0.

14. Compute Calvin and Hobbes' resulting consumption path.

15. As in question 10, derive the amounts left in the checking accounts in period 0 by Calvin and Hobbes if no payday lending service is available. Derive the resulting consumption paths.

16. Compare the long-term lifetime utilities of Calvin and Hobbes with and without access to the payday lending service now that their initial wealth e_0 is lower. Why is this comparison yielding a different conclusion than in question 11?

14.13 Psychology and Economics
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