Lecture 4: Production Networks

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6.207/14.15: Networks, Spring 2022
Input-Output Analysis

This lecture covers a classic economic model of production linkages among firms in the economy.

Key questions:

► What firms are “systemically important,” meaning that some type of shock to these firms can have big consequences for the economy at large?
  
  ▶ Focus here on the real economy (production + distribution of goods and services) rather than the financial system, where issues are related but somewhat different.

► To what extent can “macro” economic fluctuations be explained by “micro” shocks to systemically important firms?

We’ll see that the answer to the first question is determined by the Leontief inverse / Katz-Bonacich centrality.

The Leontief inverse also gives clues to the answer to the second question. We’ll also discuss some recent empirical economic studies on this question.
Where do Aggregate Economic Fluctuations Come From?

Why does GDP bounce around?

Conventional wisdom: economy-wide shocks.

- policy shocks, wars, supply shocks (e.g., oil), demand shocks (e.g., financial crisis)

Idiosyncratic firm/sectoral level shocks cannot have aggregate effects, because they wash out on average.
Why Not Idiosyncratic Shocks?

An influential statement due to economist Robert Lucas (1977) says that idiosyncratic shocks wash out by the law of large numbers.

- Standard deviation of sum of $n$ independent shocks is of order $\sqrt{n}$, so the effect of independent shocks to each of $n$ firms in the economy on %GDP washes out at rate $\sqrt{n}$.
- For an economy like the US with hundreds of thousands of firms, the effect should be trivial.

However, this argument ignores “network effects”: each firm has key linkages with a small number of others, so idiosyncratic shocks can propagate.

- Input-output relationships.
- Financial relationships.
Why Not Idiosyncratic Shocks? (cntd.)

An alternative perspective:

idiosyncratic shocks + network effects → aggregate fluctuations.

▶ For example, if one firm provides a crucial input that all other firms need to produce, then an idiosyncratic supply shock to this firm can obviously have a large aggregate effect.

▶ This type of channel was long thought not to be so important, but getting revived attention after the financial crisis and pandemic-related supply chain disruptions.
“In the current crisis, we have seen that financial firms that become too interconnected to fail pose serious problems for financial stability and for regulators. Due to the complexity and interconnectivity of today’s financial markets, the failure of a major counterparty has the potential to severely disrupt many other financial institutions, their customers, and other markets.”

“If any one of the domestic companies should fail, we believe there is a strong chance that the entire industry would face severe disruption. Ours is in some significant ways an industry that is uniquely interdependent – particularly with respect to our supply base, with more than 90 percent commonality among our suppliers. Should one of the other domestic companies declare bankruptcy, the effect on Ford’s production operations would be felt within days – if not hours. Suppliers could not get financing and would stop shipments to customers. Without parts for the just-in-time inventory system, Ford plants would not be able to produce vehicles.”

Alan Mulally, President of Ford Motor Co., November 18, 2008.
Production Network (Leontief) Approach to Fluctuations

**Goal:** develop a framework for understanding how idiosyncratic shocks are transmitted by network connections and the resulting aggregate effects.

**Formulation:**
- A production economy with $n$ sectors.
- Explicitly model input-output relationships between sectors by a *production network*.
- Look at how shocks propagate through the network.

**Questions:**
- What firms/sectors are most central/important?
- What kinds of shocks propagate the most?
A Simple Model of Production Linkages

- A closed economy consisting of $n$ sectors, each of which produces a different good.

- Output of each sector is used in two ways: as an input into other sectors, and as final consumption.
  - Bread is supplied to restaurants and also directly to consumers.

- The economy is described by a matrix of weights $W = [w_{ij}]_{i,j \in N}$, with the meaning that sector $i$ requires $\alpha w_{ij}$ units of every good $j$ (including perhaps $j = i$) to produce 1 unit of good $i$. Assume that $\sum_j w_{ij} = 1$ for each $i$, so each sector needs $\alpha$ units of stuff to produce 1 unit of output.

- That is, sector $i$’s production $x_i$ is given by:

$$x_i = \frac{1}{\alpha} \min \left\{ \frac{x_{i1}}{w_{i1}}, \ldots, \frac{x_{in}}{w_{in}} \right\},$$

where $x_{ij}$ is the amount of good $j$ that sector $i$ uses as an input. This is called a Leontief production technology.

- In our simple model, final consumer demand for each good $i$ is exogenous (unmodeled) and given by $y_i$. 
Degree Distribution

- Counter-cumulative degree distribution (\(=\)fraction of sectors w/ degree \(\geq x\)).
- The tail appears roughly linear on this log-log scale. This means that the distribution is approximately power law in the tail. This is an example of a fat-tailed distribution: many sectors that link to a very large number of others. (More on power laws/fat tails in coming lectures.)
Consider a negative demand shock to sector 1 \( (y_1 \downarrow) \).

- Direct effect: sector 1’s output goes down \( (x_1 \downarrow) \).
- But this output reduction also reduces the output of everyone who supplied sector 1.
- And everyone who supplied them…
- The total effect of a shock to sector 1 on GDP (sum of all firms’ production) takes into account all indirect effects.
- This is given by sector 1’s Katz-Bonacich centrality.
Solving the Model

We wish to solve for the vector of production levels
$x = (x_1, \ldots, x_n)$ as a function of the vector of final consumer
demand $y = (y_1, \ldots, y_n)$.

Sector $i$ produces $x_i$ units of output using $\alpha w_{ij} x_i$ units of each sector $j$’s input.

Out of the $x_i$ units that sector $i$ produces, $\alpha \sum_j w_{ji} x_j$ of this is used as inputs for other sectors’ production.

This leaves $x_i - \alpha \sum_j w_{ji} x_j$ left over for consumption.

In matrix form, we have
$$y = x - \alpha W' x = (I - \alpha W') x.$$  

Supposing that $(I - \alpha W')$ is invertible, the solution is given by the Leontief inverse of $W'$ with parameter $\alpha$:
$$x = (I - \alpha W')^{-1} y = \Lambda y,$$  

where $\Lambda = (I - \alpha W')^{-1}$. 
We have \( x = (I - \alpha W')^{-1} y = \Lambda y \).

Compare to Bonacich centrality in \( W' \): \( x = (I - \alpha W')^{-1} \mathbf{1} \). If final demand for each good is 1, production of each good is equal to its Katz-Bonacich centrality in \( W' \) with decay parameter \( \alpha \).

- Sectors with high “in-centrality” (used as input by many other firms, which are used as inputs by...) produce a lot.

To understand this, rewrite as \( x = y + \alpha W'y + (\alpha W')^2 y + \ldots \).

- Production of good \( x_i \) equals demand for consumption, plus demand for inputs for consumption, plus demand for inputs for inputs for consumption, plus...
- The sum converges because less of \( x_i \) is needed for longer supply chains, since each step requires only \( \alpha \) units of inputs to create 1 unit of output.
- Higher \( \alpha \) \( \implies \) sum converges slower \( \implies \) more production required to fulfill given consumer demand.
Impact of Shocks

Now let’s ask what is the impact of a shock to demand \( y \) on production \( x \).

We have

\[
\Delta x = (I - \alpha W')^{-1} \Delta y.
\]

Consider a demand shock to sector \( i \): \((\Delta y)_i = 1, (\Delta y)_j = 0\) for all \( j \neq i \).

- We obtain \( \Delta x = (I - \alpha W')^{-1} e_i \). The effect of a demand shock to sector \( i \) on production in sector \( j \) is the sum of the value of all walks from \( j \) to \( i \), where the value of a walk is the product of the weights \( \alpha w_{kk'} \).
- The effect of this shock on total production (GDP, \( \sum_j x_j \)) is then given by \( 1' (I - \alpha W')^{-1} e_i \).
Impact of Shocks (cntd.)

Note that

\[
1' (I - \alpha W')^{-1} = \left( (I - \alpha W)^{-1} 1 \right)'.
\]

Thus, the effect of a shock to demand for sector \( i \) on GDP is equal to \( i \)'s Katz-Bonacich centrality in \( W \).

- Demand shocks to sectors with high “out-centrality” (uses the outputs of many other firms, which use the outputs of...) has the biggest impact on the system. This is because demand shocks propagate “upstream” along supply chains.

Note that the impact of a shock to demand is entirely determined by \( W \) and \( \alpha \), and in particular does **not** depend on the initial demands \( y \).

- How a demand shock propagates through the network doesn’t depend on where we start.\(^{16}\)
We have analyzed the effects of sectoral demand shocks on production in each sector and total production.

- A exogenous reduction in demand for good $i$ also reduces demand for goods that sector $i$ uses as inputs, and so on.
- Punchline: the impact of a shock to demand for sector $i$ on total production is equal to $i$’s Katz-Bonacich centrality in $W$.

However, more realistically a demand shock for good $i$ will also affect the price of good $i$, and indeed all prices and the pattern of supply and demand in the economy.

It is also not clear how to interpret demand shocks:

- People suddenly develop a taste for apples rather than bananas: OK.
- But big demand shocks usually themselves have macro causes and are correlated across sectors: e.g., a stock market crash.
Remarks/Next Steps (cntd.)

For these reasons, a more economically satisfactory model would endogenize prices and focus on shocks to supply/productivity rather than (or in addition to) demand.

- Such a model would make different predictions about what sectors are most influential: Katz-Bonacich centrality in $W'$ rather than $W$, so it’s firms that supply many others (that supply many others, that... ) that matter for GDP, not firms that demand the output of many others. (Supply shocks propagate “downstream” along supply chains, while demand shocks propagate “upstream.”)

- If you’re curious to see how such a model works, see Acemoglu, Carvalho, Ozdaglar, and Tahbaz-Salehi, “The Network Origins of Aggregate Fluctuations” (Econometrica 2012). But requires some econ background and is beyond our scope.
Leontief and Input-Output Economics

- Wassily Leontief (1906-1999) born in St. Petersburg, where his father was an economics professor.
- Migrated to New York in 1921; moved to Harvard in 1932, where he become a professor in 1946.
- Career: developing and applying his input-output analysis.
- “Leontief paradox”: In 1950s US economy, labor was thought to be scarce, capital abundant; prevailing thought was that US foreign trade involved trading capital-intensive goods for labor-intensive goods. But Leontief’s input-output tables revealed that the opposite was true!
- Major early user of computers (Harvard Mark I in 1943) and advocate for integration of theory and data in economics.
- Nobel prize for input-output analysis in 1973. His students include Paul Samuelson and Robert Solow, the two most famous MIT economists of the 20th century.
Aside: Input-Output Economics and Development

Input-output ideas are also influential in development economics.


- Analyzes production functions with a high degree of complementarity among inputs (like Leontief’s
  \[
  \min \left\{ \frac{x_{i,1}}{w_{i,1}}, \ldots, \frac{x_{i,n}}{w_{i,n}} \right\}.
  \]
- With such production functions, every step must go well.
- Development is hard because you get the minimum of development across critical sectors.

- Suppose a certain fraction $\alpha$ of output is lost or stolen at each stage of the production process.
- E.g., half the grain is stolen, half the bread is stolen, half the hamburgers are stolen.
- Low-$\alpha$ countries will specialize in simple production processes with few steps, even if these processes are less efficient in the absence of loss/stealing, because multiple rounds of stealing with low $\alpha$ is very bad. High-$\alpha$ countries will engage in more complex production processes.
Empirical Economic Analysis of Network Propagation

An active area of empirical economic research tries to measure the propagation of shocks across the input-output network.

Some of this research proceeds at a high level of aggregation, for example by constructing the Leontief inverse using sectoral output data (in the spirit of Leontief himself) and regressing current output in one sector on recent productivity growth in other sectors.

- They find evidence that upstream supply shocks and downstream demand shocks matter for sectoral output, but upstream demand shocks and downstream supply shocks do not. This is consistent with the theory.
A limitation of this type of research is that “recent productivity growth in other sectors” is not an exogenous shock: it could be correlated with current sectoral output for various reasons.

- E.g., if some connected sectors have all been investing in similar new technologies in recent years, this will increase both recent growth in connected sectors and current sectoral output in a given sector.
Empirical Economic Analysis (cntd.)

A complementary line of research takes a more “micro” approach of looking at firm-level shocks, such as localized natural disasters (which are clearly exogenous).


- Combine data on the timing and location of natural disasters in the US (blizzards, earthquakes, floods, hurricanes), the location on the physical headquarters locations of firms, and inter-firm supplier-customer linkages.

- Finding: if your supplier’s HQ is hit by a natural disaster, your sales drop by 2–3%. (They only looked at direct suppliers, not also indirect links.)
Empirical Economic Analysis (cntd.)


- Finding: output of US affiliates declines roughly 1-for-1 with decline in imports from Japan. This suggests that US affiliates cannot easily substitute for imports, so their production function is close to Leontief (at least in the short run).
Empirical Economic Analysis (cntd.)


- Look at the effects of the same earthquake, but with larger-scale data on firm-to-firm linkages across the Japanese economy.
- Data from a large Japanese credit reporting agency, which contains identities customers and suppliers of almost all Japanese firms with more than five employees.
- This richer data lets them look at indirect links, not just direct links as in the previous papers.
- They find that effects through indirect links are very important. The earthquake directly affected regions accounting for only 4.6% of Japan’s output, but they estimate that taking the indirect effects into account the earthquake caused a decrease in Japan’s GDP of 0.47%.
- This is a good example of how richer “big data” can shed light on natural and important economic questions.