Problem Set #1 Solutions
Benchmark Exercise for Competitive Strategy Game

Problem 1:

1. Estimating demand in Market D is a simple regression, just as was explained in the first recitation. You could have attempted any number of specifications; for illustration, I have used a linear and a log-linear specification.

$$Q = \beta_0 + \beta_1 P$$
$$\ln Q = \beta_0 + \beta_1 \ln P$$

Graphs are also included. I do not see much reason to choose one specification over another. Both yield high R-squareds and are highly significant. For simplicity, I will use the linear specification; it makes the math work out a bit easier (except for the elasticities).

$$Q = 6047.18 - 3.82 * \text{Price}$$

Here are a few elasticities:

$$\text{Price} = 500 \Rightarrow Elasticity = 0.46$$
$$\text{Price} = 1000 \Rightarrow Elasticity = 1.72$$
$$\text{Price} = 1500 \Rightarrow Elasticity = 18.07$$

2. For cost data, I have used the mean costs (listed in Attachment A in the CSG Instructions and in the spreadsheets). For future reference, please do not submit spreadsheets with hundreds or thousands of rows; limit yourselves to a relatively small number of prices in the "relevant range." As a monopolist, at each particular price you will serve as many consumers as will choose to buy, i.e., the market demand. This market demand determines costs (using the firm's constant marginal cost) and revenues (using the price). Profits are maximized at a price of $892.

3. As mentioned above, price determines market demand; but how does this affect "capacity required to meet demand at that price"? You have to build at least enough capacity to meet market demand; if capacity is too low, then you ought to charge a higher price. But it makes no sense to build too much capacity; it cannot be profit-maximizing to build more capacity than you will use. Decreasing the price from p_1 to p_2 leads to a capacity increment of

$$\Delta Q = Q(p_2) - Q(p_1)$$

and a profit increment of

$$\Delta \pi = \pi(p_2) - \pi(p_1)$$

Hence, the marginal contribution to per-period profit is

$$\frac{\Delta \pi}{\Delta Q} = \frac{\pi(p_2) - \pi(p_1)}{Q(p_2) - Q(p_1)}$$

The PDV of these marginal contributions is

$$\sum_{t=2}^{5} \frac{1}{(1.02)^t} \frac{\Delta \pi}{\Delta Q}$$

because the profits come at the end of periods two through five.

Meanwhile, the economic cost of each unit of capacity is

$$\frac{p_c}{1.02} - \frac{s_c}{1.02^5} = 1046.93$$

where p_c is the price of a unit of capacity and s_c is the scrap value of capacity. The marginal contribution to profits lies above the economic cost of each unit until somewhere between the 2104^{th} and 2108^{th} units. The (discrete) profit-maximizing price to charge in each period is $1032.

Technically, I need to double check the cash flows to see if the bank balance ever drops below zero, in which case the proper discount rate is 5%. I believe the balance is actually negative until the end of period three. Using 5% as the discount rate in periods two and three, I can make a slight alteration to the formulas in the spreadsheet and refigure the optimal choices. The new economic cost of each unit of capacity is

$$\frac{p_c}{1.02} - \frac{s_c}{(1.02)^3(1.05)^2} = 1076.31$$

Therefore, the optimal capacity choice is actually (approximately) 2066.

Problem 2:

Expanding the analysis from Part 3 of Problem 1, Market D is the most profitable; one can expect to have \$6,608,380 at the end of the eight periods. Notice that I have re-evaluated the profit-maximizing capacity choice for period five; since the capacity will only be productive for three periods, it is not necessarily the same as the four-productive-period capacity choice. On the other hand, a simple no-entry strategy yields

$$\$1{,}000{,}000 * (1.02)^8 = \$1{,}171{,}659$$

Entry into Market D is clearly superior.

Problem 3:

There were two ways to interpret this problem: the hard way and the easy way. The hard way is actually what the question wanted, but it is sufficiently unclear. In terms of the decision as to which market to enter now, Market D is preferable to Market B which is preferable to Market A which is preferable to Market C. That is the easy approach. The hard approach is this: Suppose you enter Market D first, do you enter another market (having now only seven periods over which to operate) and, if so, which market? This is a long problem to undertake because it requires evaluating each market for seven, six, and five periods.

Without going through all the calculations, my gut instinct is that the relative effect of re-optimizing in each scenario is small. Hence, I think it is a good approximation to compare the period 5,6,and 7 ending balances in the spreadsheets used for part 2.

Problem 4:

The number of firms in the market can have a sizeable effect on the market demand.

$$Q = 6488.235 - 16.08 * \text{Price} + 362.53 * \#\text{Firms}$$

This would suggest that the market can handle multiple firms. Products are differentiated, allowing firms to exercise some monopoly power.

Problem 5:

With eight teams competing in four markets, the simple answer is two firms/market. But homogeneity in Market D along with the entry cost and capacity costs should lead to only one firm in the market. Markets B and C, on the other hand, should attract more than two firms due to product differentiation.

Problem 6:

You could approach this qualitatively or quantitatively. Qualitatively, you should discuss the relationship between your marginal cost and the mean. Given the normal distributions, the probability that you have the lowest marginal cost in a market is

$$P\left(z > \frac{MC - \mu}{\sigma}\right)^7$$

where z is a standard normal random variable, MC is your firm's marginal cost, μ is the mean marginal cost, and σ is the standard deviation of the marginal cost.

Problem 7:

Open-ended.