

14.27 - PS1 Solutions

Gaston Illanes

9/23/2014

Question 1

The price elasticity of demand is $\frac{\partial \log Q}{\partial \log P} = b$. The Lerner Index is $\frac{p-c}{p} = -\frac{1}{\epsilon} = -\frac{1}{b}$. Therefore, a firm with positive marginal cost would pick a markup of $-\frac{1}{b}$.

If the marginal cost is 0, we have a degenerate equation, as $1 \neq -\frac{1}{b}$ in general. Recall that b is a parameter of the problem, not something we can solve for. Under this demand specification, when MC is 0, we have that profits are maximized at a corner solution, and therefore the FOC approach fails, since it assumes an interior solution.

Let's look at the demand function to get a better feel for this issue. Note that the demand elasticity is constant, so regardless of the price level, the change in quantity as we change price is the same. This implies that if $|b| > 1$, lowering the price will always be profitable, regardless of the price level, as the change in quantity is greater than the change in price. This implies that if $|b| > 1$ the optimal price goes to 0. Alternatively, if $|b| < 1$, raising the price will always be profitable, regardless of the price level, as the change in quantity is lower than the change in price. Therefore, the optimal price goes to ∞ . Finally, if $|b| = 1$, a change in price will be perfectly compensated by the change in quantities, and the firm is indifferent between all prices.

The practical take away referenced in the question is that firms should never price on the inelastic portion of their demand. The reasoning is as follows. The case with $MC = 0$ should be a lower bound for marginal costs in general. Thus we can see from the above analysis that as long as demand is inelastic the firm should continue to raise its' price. Recall from the Lerner Index equation that as marginal cost rises the optimal price to charge also rises, hence even in the case of no marginal costs the firm should price above the inelastic portion of its demand, and as marginal costs rise it should perhaps even venture into the elastic portion of its demand.

Question 2

Using a linear demand specification, we have that demand is:

$$Q = 10276 - 1336 \cdot P$$

Weekday demand is:

$$P = 12835 - 1693 \cdot Q$$

Weekend demand is:

$$P = 4295 - 538.5 \cdot Q$$

The profit maximizing price is:

$$\max_P \pi = P(a - b \cdot P) \Rightarrow a - 2bP = 0 \Rightarrow P^* = \frac{a}{2b} + \frac{c}{2}$$

So the profit maximizing price is: $\frac{10276}{2 \cdot 1336} + \frac{1}{2} = 4.35$

The profit maximizing weekday price is: $\frac{12835}{2 \cdot 1693} + \frac{1}{2} = 4.29$

The profit maximizing weekend price is: $\frac{12835}{2 \cdot 1693} + \frac{1}{2} = 4.49$

If it were possible to charge different prices in different days, we would want to do that, as the optimal prices differ between the weekend and weekdays, and charging a suboptimal price loses the firm money. However, if consumers anticipate this and shift demand from weekends to weekdays, effectively eliminating the difference in demand functions between the different days, then we would not want to do so, as the weekday price would be too low and the weekend price too high.

Question 5

a)

The indifferent consumer determines demand for each firm. Their location is:

$$v_1 - p_1 - tx = v_2 - p_2 - t(1 - x)$$

Solving, $x = \frac{v_1 - v_2 + p_2 - p_1}{2t} + \frac{1}{2}$. Since everyone to the left of the indifferent consumer will purchase from firm 1, and everyone to the right will purchase from firm 2, we have that firm 1's demand is $\frac{v_1 - v_2 + p_2 - p_1}{2t} + \frac{1}{2}$, and firm 2's demand is $1 - \frac{v_1 - v_2 + p_2 - p_1}{2t} + \frac{1}{2} = \frac{v_2 - v_1 + p_1 - p_2}{2t} + \frac{1}{2}$. In general, we can write firm j 's demand function as: $\frac{v_j - v_{-j} + p_{-j} - p_j}{2t} + \frac{1}{2}$

Then firm j 's profit function is:

$$\pi_j = (p_j - c) \left(\frac{v_j - v_{-j} + p_{-j} - p_j}{2t} + \frac{1}{2} \right)$$

Maximizing, $\frac{v_j - v_{-j} + p_{-j} - 2p_j}{2t} + \frac{1}{2} = 0 \Rightarrow p_j = \frac{v_j - v_{-j} + p_{-j} + t + c}{2}$

So far, we have obtained each firm's reaction function: given rival's price, we know what each firm would charge. In a Nash Equilibrium, both firms charge prices such that no firm has an incentive to deviate. This requires both reaction functions to be met simultaneously, so in equilibrium:

$p_1 = \frac{v_1 - v_2 + \frac{v_2 - v_1 + p_1 + t + c}{2} + t + c}{2} \Rightarrow p_1^* = \frac{v_1 - v_2}{3} + t + c$. Plugging back into firm 2's reaction function, $p_2^* = \frac{v_2 - v_1}{3} + t + c$.

b)

In this case, if we have an indifferent consumer, then everyone to the right of that indifferent consumer dislikes firm 1's product more, and will choose firm 2, while everyone to the left of the indifferent consumer will like firm 1's product more, and will choose firm 1. The indifferent consumer's location is:

$$v_1 - p_1 - tx = v_2 - p_2 \Rightarrow x = \frac{v_1 - v_2 + p_2 - p_1}{t}$$

Firm 1's demand is $\frac{v_1 - v_2 + p_2 - p_1}{t}$, and firm 2's demand is $\frac{v_2 - v_1 + p_1 - p_2 + t}{t}$. Firm 1's profit function is:

$$\pi_1 = (p_1 - c) \frac{v_1 - v_2 + p_2 - p_1}{t} \Rightarrow \frac{v_1 - v_2 + p_2 - 2p_1 + c}{t} = 0 \Rightarrow p_1 = \frac{v_1 - v_2 + p_2 + c}{2}.$$

Firm 2's profit function is:

$$\pi_1 = (p_2 - c) \frac{v_2 - v_1 + p_1 - p_2 + t}{t} \Rightarrow \frac{v_2 - v_1 + p_1 - 2p_2 + t + c}{t} = 0 \Rightarrow p_2 = \frac{v_2 - v_1 + p_1 + t + c}{2}.$$

In equilibrium,

$$p_1 = \frac{v_1 - v_2 + \frac{v_2 - v_1 + p_1 + t + c}{2} + c}{2} \Rightarrow p_1 = \frac{v_1 - v_2}{3} + c + \frac{t}{3}$$

$$p_2 = \frac{v_2 - v_1 + \frac{v_1 - v_2 + t}{3} + c + t + c}{2} \Rightarrow p_2 = \frac{v_2 - v_1}{3} + c + \frac{2t}{3}$$

Firm 2 consumers do not face a travel cost, so if both firms charged the same price, everyone would purchase from Firm 2 (except the individual located at Firm 1, who would be indifferent). This is not profitable for Firm 1, who has an incentive to lower price to gain some share. Firm 2, on the other hand, does not have an incentive to match Firm 1. It makes more money with a higher price and a smaller share than with a lower price and the whole market.

Note that prices are lower in this setting than in part a. The presence of an Internet company increases competition in this case, as consumers do not face a travel cost of purchasing from it.

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14.27 Economics and E-Commerce
Fall 2014

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