

Problem Set #3

Due October 5, 2005

1. Consider a model with consumers uniformly distributed on the interval  $[0, 1]$ . Two suppliers selling the same good are located at points  $a$  and  $1 - b$  with  $0 \leq a, b \leq \frac{1}{2}$ . Their production costs per unit are  $c_1$  and  $c_2$ , respectively. Consumers buy zero or one unit of the good. They receive zero utility if they don't buy the good and utility  $v - p - tx^2$  if they buy the good from a firm at a distance of  $x$  from their location. Assume that the firms choose prices simultaneously, and that their objective is to maximize profits.

(a) Find the Nash equilibrium prices and profits in this model assuming that  $v$  is sufficiently large so that the equilibrium involves all consumers purchasing the good. How large can firm 1's cost disadvantage be if it does make positive profits in equilibrium?

(b) Suppose that before choosing prices the firms play a first period game where they simultaneously choose where to locate. Assume that the firms costs are equal,  $c_1 = c_2 = c$ . Show that in equilibrium the firms are maximally differentiated.

2. Two firms are selling two products each to a continuum of consumers of unit mass. Each firm sells a low quality good and a high quality good. Consumers are differentiated in two ways: they have a valuations for quality  $\theta$  that are uniformly distributed on  $[0, 1]$ ; and they have idiosyncratic preferences  $\epsilon_1$  and  $\epsilon_2$  for firm 1 and firm 2 that have independent extreme value distributions as in a logit model. Specifically, if a consumer of type  $(\theta, \epsilon_1, \epsilon_2)$  buys a product of quality  $k \in \{L, H\}$  from firm  $i \in \{1, 2\}$  at a price of  $p_{ik}$  her utility is

$$u_{ik} = I + v_k\theta - p_{ik} + \epsilon_i.$$

Assume throughout this question that consumers must buy exactly one of the four products and choose which product to buy to maximize their utilities.

Note: the assumption of logit errors implies that if a consumer is deciding between a product from firm 1 that gives utility  $\delta_1 + \epsilon_1$  and a product of firm 2 that gives utility  $\delta_2 + \epsilon_2$  then the probability that the consumer will buy from firm 1 is  $e^{\delta_1} / (e^{\delta_1} + e^{\delta_2})$  (this is the probability before we learn the realization of  $\epsilon_1$  and  $\epsilon_2$ ).

Assume that the marginal cost of producing a low quality good is  $c_L$  and the marginal cost of producing a high quality good is  $c_H$ . Assume that  $c_L < c_H$  and that the difference in costs is such that in equilibrium the firms make positive sales of both products.

(a) Consider a standard pricing game where the firms simultaneously announce prices  $p_{iL}$  and  $p_{iH}$  for the two products. Show that the game has a symmetric Nash equilibrium in which both firms charge  $p_L^* = c_L + 2$  for their low quality product and  $p_H^* = c_H + 2$  for their high quality product.

(b) Now consider the add-on pricing game where the firms simultaneously announce their prices for the low quality goods, consumers with rational expectations choose which one firm to visit, and

consumers then observe the price of the high quality good and must choose between buying the high quality good and the low quality good from the firm they chose to visit.

Assuming that this model has a symmetric equilibrium in which the firms both announce  $p_L$ , what must  $p_H$  be as a function of  $p_L$ ?

(c) Describe how you would use the result of part (b) to solve for the symmetric equilibrium of the add-on pricing game. Assuming that I haven't made a mistake the equilibrium turns out to be

$$p_L^* = c_L + 2 - \frac{1}{4(v_H - v_L)} ((v_H - v_L) - (c_H - c_L))^2$$
$$p_H^* = c_H + 2 + \frac{1}{4(v_H - v_L)} ((v_H - v_L) - (c_H - c_L)) ((v_H - v_L) + (c_H - c_L))$$

How do profits in this equilibrium of the add-on pricing game compare with those in part (a)? Is this answer more like the result in Lal and Matutes' "loss leader" model or Ellison's "Add-on Pricing" model? How might you change the model slightly to change the answer to this question?

3. Look at Stahl's model of price dispersion with sequential consumer search.

(a) Suppose that demand is linear. See if you can get far enough solving the equilibrium equations (analytically or numerically) to draw a picture of what the equilibrium price distribution looks like with 5 firms. Does this seem reasonable?

(b) Describe Stahl's results about the comparative statics of the equilibrium price distribution as the proportion of shoppers increases, as the search cost decreases, and as the number of firms grows. Do these seem reasonable?