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**GLENN  
ELLISON:**

Let me go ahead and get started. So today's topic is called strategic investment. So if you remember where we were last week is we were talking about the effect of entry on markets. And I made this big pitch that in some sense, entry is just critically important.

And if you think about, what are firms' actual profits, firms' actual profits are just a rounding error. And so it's really the amount of market power you have relative to the number of firms in the market that's going to determine profits. This means that if you could affect entry, it could have a dramatic effect on how firms are doing.

And I think in that previous lecture, it was always just that there was just fixed costs. Firms came in, paid the same-- paid some exogenous fixed costs, and then there was competition. There's a very, very large literature in economics about what can firms do to change the market structure. And that can either be actually changing the market structure to try to affect entry or just changing the market structure to try to affect what the competitive game looks like if you can't stop other firms from entering.

And if you look at Tirole's book, the chapter on strategic entry deterrence is by far the largest chapter. And it's many more things written in the 35 years since then. So anyway, many of these models are the classic three-stage game.

In the first stage, firm 1 chooses some investment  $k_1$ . It pays some sunk cost  $c$  of  $k_1$ . And. When I use the word investment, I mean investment very broadly. The important things about the investment is that it some sunk cost of making it, and it's credible that you're not going to be able to undo it.

So, for instance building capacity, if you build a new factory, you've built the new factory you're not going to unbuild the factory. And so if you build a new factory that lets you produce a lower cost you've then committed yourself to being able to produce a lower cost in the future. If you advertise, you spend money advertising, you've changed the consumer's willingness to pay for your good and willingness to pay for other goods in the market, you can't unadvertise and get the money back and then say, OK, now the other firm's done something. I'll unadvertise, get my money back, and let the preferences go back to where they were.

So anything that has that feature that you make some initial choice, it incurs a cost, and it's hard to undo, that's an investment. So for instance, like designing an iPhone with a lightning charger instead of a USB-C charger or something like that. You can't undesign the iPhone. You could design a new iPhone that costs you more money, but you've got the old ones out there that have that particular port in them.

Firm 2 observes the investment level and then decides whether to enter. If it enters, there's a cost of  $E$ . I'm going to do this in the monopoly-going-to-duopoly case. Obviously, you could do all this with two firms, three firms going to  $n + 1$  firms. And then if firm 2 stays out of the market, firm 1 becomes a monopolist. It chooses some action,  $a_1$ , like a price or a quantity, and then gets a profit that depends on the investment and on the action.

And if firm 2 enters, they simultaneously choose these actions. And they earn profits that depend on the investment and on both actions. And I'm going to assume that this game that follows in second stage has well-behaved profit functions, and it has a unique Nash equilibrium, where the Nash equilibrium depends on  $k_1$ .

And you'll typically-- if the action is an investment that firm 1 takes in reducing its costs, the prices that they set are-- firm 1's action obviously reflects its costs. But then firm 2's action is also going to reflect firm 1's costs because firm 2 knows that firm 1's investment is going to reduce its cost, leading to change its price. There can also be direct effects of the investment on both firms. So in the advertising case, the advertising can be changing the demand for firm 2's product or the valuation of firm 2's product, as well as the valuation of firm 1's product.

So in the early literature on this, Joe Bain described three different situations that you could see in an industry. The first one he described as blockaded entry. In the blockaded entry case, firm 2's entry costs are sufficiently high that firm 2 is never going to enter if firm 1 just ignores it. So in the blockaded entry case, firm 1 just is-- essentially this is the monopoly model. You just go on. You choose the optimal investment, and then you earn the profits you get with the optimal investment, and firm 2 is basically irrelevant.

Second case he talks about is the deterred entry case. So suppose entry costs are a little bit lower than in the blockaded entry case, so that if firm 1 ignores firm 2, firm 2 will enter. But then he argues that there are going to be some cases where the firm can actively deter entry, where the firm can distort its investment in some way that reduces firm 2's profits, that firm 2 will no longer want to enter, and firm 1 remains a monopolist.

And then the third case he called accommodated entry. Detering entry is either impossible or it's just so costly you wouldn't want to do it. So the monopolist chooses its investment not to keep firm 2 out, but to change the nature of competition in some way that's going to earn it higher post-entry profits.

And so we're going to focus on two main questions here. Essentially we're going to ignore the blockaded entry case except as a counterpoint and then ask, when do firms want to deter entry versus try to accommodate entry? And how do you use the investment,  $k$ , to do that?

And obviously, the answer turns out to depend on how you use your investment. It depends on what type of investment it is and what type of game it is. And Fudenberg and Tirole have this paper from 1984 that goes over a simple classification of types of games and types of investments. So I will do that before I discuss some applications.

So your paper discusses two types of games. I think I've done this earlier, mentioned these terms earlier. We say that a game has strategic complements if the best response correspondences are upward sloping. And it has strategic substitutes, actually, if there are curves and if they're downward sloping. So if you remember the Hotelling model, you graph the best responses. It looks like this. The Cornell model, you graph the best responses. They look like that.

Why was that? This is a simple theorem. You could do this more generally using increasing differences terminology. But a game is generally going to have strategic complements if the second cross partial, second derivative of profit with respect to  $a_i$  and  $a_j$  is positive and strategic substitutes if the second partial of profits with effect to  $a_i$  and  $a_j$  is negative.

And as I said, you know, we usually think of price competition as being the complements case and quantity competition as being the substitutes case. But I would note here that the strategic complements and price competition is only there, has been there in all our previous analyses, because I've always been using linear demand curves that are separable. I'm using demand curves that look like  $D_i$  of  $p_i, p_j$  is  $1/2$  plus  $p_j$  minus  $p_i$  over  $2t$  or something like that where there's no interaction between  $p_i$  and  $p_j$ , because if you look at the profit function in a price competition game, you take the-- I first differentiate with respect to  $p_i$ . And I get  $p_i$  minus  $c$ ,  $dD_i/dp_i$  plus  $D$  of  $p_i$ .

And then I differentiate this with respect to  $p_j$ . And then I get-- this doesn't depend on  $p_j$ , so I get  $p_i$  minus  $c$ , second derivative of demand with respect to  $p_i$  and  $p_j$  plus  $dD_i/dp_j$ . And we're used to this term, firm  $j$  increases its price, firm  $i$ 's demand goes up. This is the positive term.

But then there is this second term,  $p_i$  minus  $c$  times this second derivative of  $D_i$  with respect to  $p_i$  and  $p_j$ . So for instance, if I had an interaction in this demand curve, then this term would be negative. This term would be negative multiplied by this. It's no longer obvious you have strategic complements. So normally we think of price competition as complements and quantity competition as substitutes, but that's not always true.

OK, second thing we're going to have to do in this model is classify types of investments. And the way the investment classification is going to work is we're going to classify investments based on their effect on the potential entrant. So in this model, I'm going to define this reduced form profit function to be the profit that firm 1 gets when firm 1 has chosen  $k_1$ . And then we play the equilibrium action following  $k_1$ .

And so a simple definition, investment makes firm 1 tough, essentially if it hurts firm 2. If this reduced form profit function is decreasing in  $k$ , investment makes firm 1 soft if that reduced form profit is increasing in  $k$ .

And you know, sometimes it's obvious which this is going to be. So for instance, if this is an investment in cost reduction, your intuition tells you if firm 1 has lower costs, that's going to be bad for firm 2 in equilibrium when the prices shake out. And so we will have  $d\pi_2/dk_1$  less than 0, so cost reductions in investment, that makes you tough.

If the investment is lobbying to lower the tax rates in an industry, if it's just like a profits tax, a percentage tax on profits, it won't affect any of the behavior. So if you lobby to get taxes reduced, a reduction in taxes isn't going to change any of the actions. And it's going to end up being good for both firms. So lobbying to lower tax rates would be an investment that makes you soft.

When it's not obvious, it can help to write down what this profit function looks like. So if I think about differentiating, so I have  $\pi_2^*(k_1)$  is profit that firm 2 gets when the investment is  $k_1$ . And we have  $a_1^*(k_1)$ ,  $a_2^*(k_1)$ .

OK, if I differentiate this with respect to  $k_1$ , I'm going to get three terms. First, I get the derivative here, so I get the partial of  $\pi_2$  with respect to  $k_1$ . Second, I get this effect that when I change  $k_1$ ,  $a_1$  changes. And that change in  $a_1$  affects  $\pi_2$ . And so that's partial  $\pi_2$  with respect to  $a_1$  times partial  $a_1^*$  with respect to  $k_1$ .

And then there's also-- you would think there would be a third term. When I change  $k_1$ , it affects this. And that affects profits. But the third term would be  $\partial \pi_2 / \partial a_2$  with respect to  $a_2$  evaluated at  $a_2^*$  of  $k_1$  times derivative of  $a_2^*$  with respect to  $k_1$  evaluated at  $k_1$ .

And if you think about this term here, that's 0. When you evaluate  $\partial \pi_2 / \partial a_2$  at  $a_2^*$  of  $k_1$ ,  $a_2^*$  of  $k_1$  was chosen where the first order condition is 0. So there's no effect from  $k_1$  changes-- there's no third effect from  $k_1$  leads firm 2 to change  $a_2$ . And that has an effect on profits. The marginal effect of changing  $a_2$  on profits is 0.

So we really-- so this term is 0. And we just have two effects. We have what's called the direct effect where  $k_1$  directly affects firm 2's profits, and for the investment like cost reduction, this term is 0. Firm 1's cost reduction has no direct effect on firm 2's profits.

And in the cost reduction case, it comes in through this term that when I invest more in cost reduction, I lower firm 1's price. So this is negative. When I lower my price,  $\partial \pi_2 / \partial a_1$  is positive. Firm 2 benefits when firm 1 sets a higher price. So the cost reduction has a negative strategic effect. It causes firm 1 to do something that's bad for firm 2, which is cut its price.

In the lobbying to lower tax rates, there I was describing a thing that has a direct effect. If I lobbied lower tax rates and I get tax rates lowered, holding actions fixed, that raises firm 2's profits. And then if it's just a tax on profits, it doesn't affect equilibrium prices. So  $\partial a_1 / \partial k_1$  is 0, and there's no strategic effect.

And so I have some examples on the other side. You can think about-- so for instance, suppose we had a competition on a line game. And firm 1 has the option to extend the line and move even further away out in this direction. What would that do?

There, they're going to be-- and let's have it in the hotel and game. So firm 1 used to be here. And firm 1 is a chance to just move further away off the end of the beach. So moving further away, the direct effect is positive. If we hold the prices as fixed and firm 1 moves farther away, the midpoint of the line moves from here to here, so firm 2 is going to get more customers.

And then the strategic effect, again, profit of firm 2 with respect to price of firm 1 is positive. Firm 2 likes 1 to set high prices. And the further firm 1 moves away, the higher will firm 1's price be. Therefore, this is positive, this is positive. So the direct effect in that model is positive. The strategic effect is positive. Moving away from firm 2 is an investment that makes firm 1 soft.

And other things-- investments can be neither. So if you think about an advertising campaign, suppose you run an advertising campaign that will convince some number of customers to never leave your firm or to have a tremendous loyalty for your firm and not want to switch. The direct effect on your rival is negative. When those customers become captive, they're not going to consider the rival. Therefore, the rival is going to get fewer of them, holding prices fixed. That hurts the rival.

But again, price competition, firm 2 likes me charging high prices. And when I have these captive customers who won't consider switching, I'm going to price high to exploit the captive customers. I can't discriminate between the captive and non-captive people. Therefore, my price is going to be higher. So you get a negative direct effect, a positive strategic effect. It's not clear. It would depend on which of those is larger whether this is an investment that makes you tough or soft. And it could be neither. It could be sometimes soft in some ranges, tough in others. But just the tough, soft distinction is all about-- it's not really how it affects firm 1. It's how it affects firm 2.

OK, so let me now go through Bain's cases in order, so first, this blockaded entry case. So blockaded entry case is the case where firm 2 just has very, very high entry costs and will never consider entering. So I'm going to write  $k_{1m}$  and  $a_{1m}$  for what a monopolist would do if there wasn't even a firm 2. It's just going to choose the  $a_1$  and the  $k_1$  to maximize its monopoly profit function.

And the blockaded entry case is when, if you choose this optimal monopoly investment, firm 2's equilibrium profits are less than  $e$ , so firm 2 is not going to enter. And so then this is-- really, it's not an entry model. Firm 1 just ignores potential entrant. It chooses  $k_{1m}$ . That's all that happens.

I mentioned that, because now let's think about what happens if you're going to deter entry. The optimal entry deterring investment is going-- you're going to solve this problem. I'm going to try to maximize over all possible investment levels and actions the profit that I'm going to get from choosing  $k_1$  and then choosing action  $a_1$ .

But I've got to do it subject to the constraint that firm 2's equilibrium profits after I've chosen  $k_1$  are at most  $e$ , so that firm 2 doesn't enter. So I'm choosing this same maximization problem, but I just put in this constraint that I have to ensure that firm 2's profit is less than  $e$ .

And the entry deterrence case I'm looking at is suppose that  $\pi_2^*(k_{1m})$  is greater than  $e$ . So in this case, firm 2's profits, if I just behave as if firm 2 isn't there, firm 2 is going to enter. So this would be-- if this was less than  $e$ , I would be in the blockaded entry case. With this greater than  $e$ , this is when I need to be at this constraint. This constraint is binding.

So anyway, how do you distort your investment and make  $k_{1ED}$  differ from  $k_{1m}$ ? Very simple theorem says that if investment makes firm 1 tough, then the entry deterring level of investment is greater than  $k_{1m}$ . And if investment makes firm 1 soft, then the entry deterring level of investment is less than  $k_{1m}$ .

An easy way to think about this is you just have-- I put my investment on this axis. And let me graph some profits on this axis. So let this be  $e$ , the level of profits that firm 2 needs to enter to come into the market. And so imagine-- you know, imagine this is  $k_{1m}$  and this is  $\pi_2^*(k_1)$ . So this is an investment that makes firm 1 tough, so investing, investing more-- investing more hurts firm 2.

And again, if I also graph-- let's imagine  $\pi_1$  looks like this.  $\pi_1^*(k_1)$ , it's maximized at  $k_1$  monopoly. So what I would like to do if firm 2 wasn't there is I would choose this level of investment. I would receive that level of profits.

But I can't do that, because if I did, that, firm 2 would come in. So this is  $\pi_1$ . We should call this  $\pi_1$  monopoly of  $k_1$ . That's what I would like to do. I can't.

What do I have to do? I have to choose a level of investment that is sufficient to make sure that  $\pi_2$  is less than  $e$ . So what I would do in this example is this would be  $k_1$  entry deterrence. I just need to-- I need to move firm 2's-- I need to move firm 2's profits in whatever-- I need to move  $k_1$  in whatever direction moves firm 2's profits down.

And so in this case, I distort my investment going in that direction. And so I'm over-investing. We'll call that I'm over-investing in  $k_1$  in order to keep firm 2 out of the market.

So for instance, this would be the graph of I'm investing in a cost-reducing technology. And so if I-- if firm 2 wasn't there, I would just go ahead and do minimal investments in cost. Do this level of investment in cost reduction, I'd earn this profits.

But then with this, if I have the cost that this investment produces, firm 2 is going to enter. So what I do is I do more cost reduction than I need to as a monopolist. I'm distorting my monopoly investment, therefore, I'm lowering my profits. But I do that because I need to do that to keep firm 2 out of the market and remain a monopolist and not be a duopoly, which in this case, I'm assuming is even worse for you.

And if the investment was one that made firm 1 soft, if firm 2's profits instead looked like this, then obviously again, I would just distort down in this direction and take this point. So I'm always just distorting in whichever direction hurts firm 2. And so strategic entry deterrence is very simple. It's just about hurting your rival. That's all there is to it.

As I drew this, I made firm 1's monopoly profits single peaked. When firm 1's monopoly profits are single peaked, you want to go as far away from-- you want to stay as close as you can to  $k_1$  and not distort more than you need to. So you would always go just to the point where the entry cost first hits  $e$ . Any questions on that?

OK, so some examples. In the cost reduction game, I said, you know, firm 1 is going to over-invest in cost reduction to deter entry. Advertising to increase market size, so suppose you have a new product, you're advertising for the product, and advertising is going to raise demand for that product for everyone.

And so if this advertising is going to make more people want whatever this product is, if that's going to help you and that's going to help rivals, if the rivals enter, then you're going to limit that advertising to avoid attracting entry. And so that's the case where investing in something that has spillovers to you or your rivals raises-- if it raises everyone's profits, then increasing the size of the market, then it's going to attract entry and therefore that investment makes you soft, you do less of it to deter entry.

So, for instance, if I'm the Chipotle in Kendall Square and I just advertise "Burritos are great" more generally, could be a second burrito shop opens up in Kendall Square and therefore, everyone likes burritos more, whatever. So maybe it's not so plausible, but you just do less advertising, because the more you advertise, the bigger is the market, the more likely some other firm is going to come in.

Again, differentiation-- if you can have some way that you can, either by designing your product or by advertising your product, make consumers have idiosyncratic preference for your product relative to rivals' products, differentiating also will raise the-- differentiation will-- in  $c + t$ , if you increase  $t$ , it will raise the rivals' profits. Therefore, you want to avoid differentiating to deter entry.

Like in a competition on a line model, it could be that the thing I want to do is locate exactly in the middle, so that if I locate exactly in the middle, then firm 2 has to be pretty close to me, and then we don't earn profits. That keeps the firm out of the market. Any questions?

So accommodating entry, what's going to happen in this game is if you think about this, suppose I reduce  $e$ , and I make  $e$  a lot lower. If  $e$  is a lot lower, the only way I'm going to keep firm 2 out of the market is to reduce, is to increase  $k_1$  all the way up to this level. And then at this level, as I've drawn it, my profits are essentially 0.

And so I'm going to have-- this was my monopoly profits. I'm going to also have, say, like duopoly profits curve. And if the amount by which I have to raise my investment in order to keep firm 2 out of the market is so large that it's going to make my-- I should probably have made this one lower everywhere. Duopoly profits are generally going to be lower than monopoly profits. So it might look like this.

It may be that rather than go all the way out here and drive my monopoly profits to 0, I'm better off just going somewhere, doing a more moderate investment and admitting that firm 2 is going to be in the market and accommodate entry instead. So if the profits from deterring entry are sufficiently low, instead I may just go to an accommodation strategy where I instead choose  $k$  not to deter entry, but to do what's best possible for given that firm 2 is going to enter.

So let me discuss the entry accommodation solution. I'm going to call that  $k_1AC$ . That's going to be the argmax of overall  $k_1$  of  $\pi_1$  star of  $k_1$ , where  $\pi_1$  star is the duopoly profit function. So in this example, let's suppose it looks like this, it's maximized here. I could have  $k_1$  accommodation. I'm instead maximizing the duopoly profits that I'm going to get if I choose investment  $k_1$ .

And so what Fudenberg will do is give you a little bit of thinking about, you know, when are you going to over-invest or under-invest in order to accommodate entry? How do you think about that decision? I said the deterrence one was simple. You just think about I want to hurt the other firm. If it's tough, I over-invest. If it's soft, I under-invest.

How do we think about this? Well, so I need a benchmark. So as a point of comparison, they define  $k_1OL$ --  $OL$  stands for "open loop." That's a terminology that's disappeared from economics since they wrote the paper.

So we define  $k_1$  open loop to be the level of investment that firm 1 would choose if entry was going to occur and firm 2 could not observe  $k_1$ . So if firm 2 can't observe  $k_1$ , then in equilibrium it must be the case that the partial of  $\pi_1$  with respect to  $k_1$  evaluated at  $k_1$  open loop, and then the equilibrium actions that follow, is equal to 0.

OK, so think about the equilibrium-- think about the game where firm 1 chooses  $k_1$ , and then firm 2 doesn't observe  $k_1$  before it makes its entry decision. In equilibrium, it correctly anticipates what the investment was, and then the actions by both firms reflect what the investment was.

But so we get it's this partial is equal to 0 evaluated at  $k_1$  open loop, because it's almost as if firm 1 could go back and change, that  $k_1$  is not sunk, and firm 1 could go back and change  $k_1$  after firm 2 has taken its action then, because if firm 2 doesn't observe it, it's as if you're choosing it afterwards. It's the same first order condition. OK? Is that definition OK?

So anyway, we're going to think about strategic investment as being what do you do different from what you would do if  $k_1$  was unobservable. Because if  $k_1$  is unobservable, it's not affecting the other firm, the strategic use of it is doing something different because firm 2 sees it.

And so first characterization theorem just highlights the strategic effect. Firms over-invest to accommodate entry, meaning they choose  $k_1$  bigger than the open loop value if and only if  $d\pi_1 / da_2$ ,  $da_2^* / dk_1$  is greater than 0.

OK, and the proof of this is straightforward. I again, just write out here what is-- now I'm going to write out what is  $\pi_1^*$ . It's that. And then think about what is the derivative of  $\pi_1^*$  with respect to  $k_1$  when I evaluate it at  $k_1$  open loop?

First you get a partial, partial  $\pi_1$  with respect to-- partial  $\pi_1$  with respect to  $k_1$  evaluated at  $k_1$  open loop. That's 0 by the definition of  $k_1$  open loop. Second thing you get is  $d\pi_1 / da_1$ ,  $da_1^* / dk_1$  again with this  $d\pi_1 / da_1$  evaluated at  $a_1^*$  of  $k_1$ . This is 0 by the envelope theorem again.

So  $d\pi_1^* / dk_1$  is positive if this strategic effect is positive, and it's negative if the strategic effect is negative. So the strategic effect is how does the investment affect  $a_2$ , and how is your profit affected by  $a_2$ . Does  $k_1$  change the other firm's action in a way that's good or bad for you? That's the strategic effect.

And so notice that if I evaluate that derivative, if the total derivative of  $\pi_1^*$  with respect to  $k_1$  evaluated at  $k_1$  open loop is greater than 0 and profit is single peaked, then that's going to imply that  $k_1^{AC}$  is strictly greater than  $k_1$  open loop. It's just a case where we have a profit function we know evaluated at  $k_1$  open loop. This  $\pi_1^*$  is increasing, so the maximum is somewhere to the right.

So one way to think about strategic investment is just to do these calculations for yourself. How would this investment affect firm 2's action? And then what would I think of that action? So for instance, cost reduction, I'm in a price competition game, and I'm thinking of investing in cost reduction.

If I reduce my costs in a price competition game, that's going to lead the other firm to lower its equilibrium price because he knows I'm going to lower my price. And so  $da_2 / dk_1$  becomes negative, more investment, lower rival price. And then my profit is increasing in the rival's price. So this is positive, so positive times negative. This is a negative strategic effect.

So in the strategic entry accommodation game, you're going to underinvest in cost reduction, meaning you're going to do less of it than you would do if you could do secret cost reduction. So if I could do secret cost reduction, I would do more because it really does benefit me profit wise to have a lower cost. But I know that if my rival sees me investing in the cost reduction, he's just going to lower his price, so I'm going to eat up a lot of the benefits I was getting from the cost reduction.

So what we'll see firms is underinvesting in cost reduction. You could do a calculation and say, why doesn't that firm adopt this technology? That technology would clearly pay for itself.

And the answer is they don't do it for strategic reasons because if their rival knew they had that lower cost manufacturing technology, the rival would just charge a lower price. They'd be worse off. Is that example pretty clear? OK.



Fudenberg and Tirole give the second theorem noting that one way to think about strategic entry accommodation is in terms of, what's the effect of the investment on your rival? What is the rival? What does that do to you? They note that it also can be related to the type of game and the type of investment.

So here's a theorem. Let's, for instance, initially consider investments where there's no direct effect on firm 2 like a cost reduction. Let's assume that the game is at least somewhat symmetric, weakly symmetric, I'd call it. So profit-- changing the sign of  $d\pi_i / da_j$  is the same as  $d\pi_j / da_i$ .

So any game like price competition or quantity competition or whatever will have this. Just the way that firm 1's action affects firm 2 is the same as the way firm 2's action affects firm 1. If you had some bizarre game where firm 1 was choosing a price and firm 2 was choosing a quantity, then that might not be true. But most of the games we do are fully symmetric.

And so the theorem is that firms overinvest to accommodate entry if the game has in strategic complements and investment makes them soft, or if the game has strategic substitutes and investment makes firm 1 tough. So let me start with the strategic substitutes case. In the strategic substitutes case, what this says is-- so in strategic substitutes-- so I did put it over here.

So in the substitutes case, if the investment makes you tough, you overinvest. And if you have something that makes you soft, you underinvest. This is the same thing we had in-- this is the same thing we had in the entry deterrence case. When you're in deter entry, you're always trying to hurt your rivals.

So if the investment makes you tough, you do it. If it makes you soft, you don't do it. What this says in games with strategic substitutes, accommodation of entry is just like deterrence of entry. In a game with strategic substitutes, what you always want to do is just hurt your rival.

But things turn around in games with strategic complements. In games with strategic complements, you want to help your rival? So actually, when I was describing the cost reduction, I did the cost reduction here, I said in the cost reduction game, to deter entry, you don't want to do it, but to accommodate entry, you don't invest because it's just going to make your rival lower prices.

But the interesting observation here is that sometimes I describe Cournot versus Bertrand as just two different assumptions people make. They're similar models. Profits decline. This notes that a lot of the conclusions we're going to make about strategic investments would just completely flip if we had a quantity competition game versus if we had a price competition game.

So in the quantity competition game-- let me go back to my previous first order condition. What's the strategic effect of reducing my cost? In a quantity competition game, if I reduce my cost, it's a commitment that I'm going to produce a lot. And in Cournot with those downward-sloping reaction curves, if I'm going to produce a lot, my rival is going to produce less.

And so by investing in cost reduction, I make his quantity lower. So this is negative. And then if his quantity is higher, that's bad for me. So this is also negative. So I have negative times negative is a positive number, which means that I want to overinvest in cost reduction.

So if you look at firms that would be competing in a Cournot market, the conclusion you'd get is that if they were considering cost reduction-- if you look at price competing firms, we're going to expect that there will be these cost-reducing investments that we don't understand why firm 1 isn't taking it because it would be privately optimal, holding everything else fixed. In a quantity competition game, you're going to see firms having overinvested in cost reduction.

And you're going to calculate that that cost reduction technology is not paying for itself. It's making less money with the cost-reducing technology than the amount. It's actually saving on production. But it does it because of the strategic effect. That is, in a Cournot game, committing to be tough forces your rival to stay at a smaller scale. And that's good for you.

So anyway, I think the thing is in games with strategic substitutes, what you're always trying to do is take investments that hurt your rival. And what you're doing in a game with strategic complements is you're trying to help your rival. So anything you can do that's good for your rival is good for you as well or at least that's the strategic effect.

And I'm just phrasing this in the discussion of how does  $k_1$  compare to  $k_1^{OL}$ , which is the amount of investment you would be doing in the oligopoly if it was unobservable. Put a proof on the next slide. I think I won't take the time to go through this because sometimes I find it doesn't-- it's a simple calculation. I don't know that it's all that informative.

But basically, the argument is just-- You know, think about how-- think about how a sign of  $k_1$ -- what I'm trying to sign is this. I know this is the strategic effect. I'm going to overinvest if  $k_1$  moves  $a_2$  in a direction that I like. And I'm going to underinvest if  $k_1$  moves  $a_2$  in a direction I don't like.

So this was the strategic effect. And I'm just trying to think about how do I sign this strategic effect? In particular, how do I find out what  $da_2^* / dk_1$  is? Is  $k_1$  going to move  $a_2$  in a direction I like or in the direction I don't like?

And how do I do that? I note that  $a_2^*$  satisfies this for every  $k_1$ . This is the first order condition defining firm 2's best response function or defining equilibrium. That partial of  $\pi_2$  with respect to  $a_2$  evaluated at  $a_2^*$  of  $k_1$  is equal to 0.

So what I just do is I take this thing. I differentiate it with respect to  $k_1$ . It gives me these three terms. And then when I look at this expression  $d\pi_2 / dk_1$  was 0 with no direct effect. So this term drops out.  $d^2\pi_2 / da_1 da_2$ , this is the thing that's whether you have strategic complements or strategic substitutes.

This term here is negative. And so anyway, you-- I'm trying to sign  $da_2^* / dk_1$ . I move this to the other side. And the sign of  $da_2^* / dk_1$  is the sign of this. And that depends on complements or substitutes and the soft or tough.

Anyway, you can look at the calculations afterwards. But in some sense, that's-- the reason why is you're just thinking about what it is that's going to-- are you going to move the other firm's action since it hurts you or helps you? And that just turns out to depend whether if it's complements-- like it's helping-- firm 2 is helping you and vice versa. And it substitutes-- helping firm 2 is hurting you and vice versa.

But I think the main thing to take away is this intuition for what you want to do in entry accommodation games is that in Cournot games, you're always trying to still hurt your rival like you do to deter entry. But in games with strategic complements, things turn around, and you should always be thinking about I want to aid my rival and distort my investment in that direction. And I just want to do that quickly because I'll just go through now. I've got like five different examples of classic arguments to think about, what do firms do for strategic reasons?

So the first is a model of learning by doing. So we start with a firm that's a monopolist. And it has some costs  $c_0$ . And it sells some quantity in the first period,  $q_{11}$ , where I guess the upper one refers to the-- I don't know. I guess it refers to the first period. So it's firm 1 in the first period.

After firm 1 produced in the first period, firm 2 observes firm 1's production level and then chooses whether to enter at a cost of  $E$ . The critical assumption of a learning by doing model is that the firm 1's marginal cost in the second period is going to depend on how much it produced in the first period. So  $c$  of  $q_{11}$  is the period 2 marginal cost.

And so we're assuming that the more you produce in the first period, the lower is your marginal cost in the second period. So just by producing more, you get more experience producing. Every unit you produce, you figure out things that improve your production process. You make your costs lower.

So in this model, we can think of overproducing. Overproducing is just choosing-- I would normally choose  $q_{11}$  if I was-- if there was-- in the blockaded entry case, I would choose  $q_{11}$  to be the argmax over  $q_{11}$  of  $\pi_1$  of  $q_{11}$ ,  $\pi_{11}$  plus discount factor-- I don't think I do discount factors-- a  $\pi_{12}$  monopoly of  $q_{11}$   $q_{12}$ .

So my first period profit just depends on the quantity I produce in the first period. And in the second period, if I'm still a monopolist, my profit is going to depend both on what I produced in the first period and what I produced in the second period with the first one here because what I produce in the first period affects my cost in the second period.

And so this is what I would do. I think I called this  $q_1$  monopoly. This is what you would do in the blockaded entry case is just produce this level of output. And so you can think of if I choose  $q_{11}$  greater than  $q_{1m}$ , that's an investment in reducing costs. It's an investment in reducing costs that I make by just overproducing in the first period.

What's the cost of the investment? The cost of the investment is  $\pi_{11}$  of  $q_{1m}$  minus  $\pi_{11}$  of  $q_{11}$ . If I produce this-- if I produce some extra amount, it's going to lower my first period profits. The amount by which it lower-- so this is in some sense the investment cost. So producing much in the first period is a strategic investment, an investment that I spend something and lost profits. I gain something in having reduced my future costs.

So-- oh, I put a B on here. So let's suppose that there's no firm two. Then the blockaded entry level would be  $q_{1B}$ . If I need to deter entry, and I want to deter entry, then what I'm going to do is I'm going to overproduce because investment makes-- it reduces my cost. It makes me tough. I want to hurt firm two, so I'm going to choose.  $q_{11}$  greater than  $q_{1B}$ .

In the entry accommodation case. I'm either going to want to underproduce or overproduce, depending on whether I'm playing a price competition game or a quantity competition game. So if I'm playing a price competition game where now suddenly I want to be nice, in price competition games, I'm going to distort downward-- have  $q_{11}$  less than  $a_1$  open loop.

And I'm going to distort downward because I want to keep my costs high, and I want firm two to know that I'm going to have high costs, that firm two, when it enters, is going to also charge a high price because it knows I have high costs.

So in the accommodation game, I want to be nice. I do that by underinvesting in cost reduction. Note, also, that we're going to typically have--  $q_1$  open loop is going to typically be less than  $q_1$  blockaded.

So to deter entry, I'm going to produce more than I get with blockaded entry. And then to accommodate, I'm going to produce less than the open loop value. And why do I say the open loop value is typically going to be less than the blockaded value?

When I-- in the first period, I'm investing in cost reduction. I'm investing in cost reduction in part because I'm then going to have lower cost in every unit I sell in the second period. In the blockaded case, that's what I'm taking into account. How many units am I going to sell in the second period? What's my cost savings going to be?

In the open loop case, it's going to be-- I'm going to be in a duopoly in the second period, so I'm only going to be serving half the market. When I'm serving half the market, I'm going to sell fewer units in the second period. Therefore, I invest less in cost reduction in the first period.

So I do less overproduction relative to what maximizes this because I know that I'm only going to be-- I'm going to be selling fewer units of  $t$  equals 2 anyway. I don't value cost reduction as much. I go closer to just the static monopoly profit.

Now, if we were playing in a Cournot game, in a Cournot game, I'm going to choose  $q_{11}$  greater than  $a_1$  open loop. So this would be in a Bertrand kind of game or a Hotelling game.

In a Cournot game, what I'm going to know is that Cournot-- I'm going to know that  $q_1$  open loop is less than  $a_1$  blockaded less than  $q_1$  entry deterrence. And then  $q_1$  accommodation-- I don't know where it's going to be. It could be here. It could be here. We know it's bigger than  $q_1$  open loop. But  $q_1$  loop is less than  $q_1$  blockaded.

So it could be that you overinvest, but still not as much as you would do here. Or it could be that you have to produce a ton to keep the other firm out, but keeping the other firm out is really important. It could be you're producing up here, or even more-- even up here or something.

I guess this is-- it's you're investing to-- sorry. Yeah. you're investing just to reduce his profits to reduce his output, I guess. This is reducing his output all the way to zero. So if you're reducing his output all the way to zero would be the biggest benefit you can get.

So this one would have a smaller benefit than this. So I guess it would be either here or here. But in the competition game, I'm just overinvesting, but I'm overinvesting, recognizing that it's still going to be a duopoly. So I'm overinvesting relative to the somewhat lower level.

Leverage theory-- so there's this old classic literature in IO about, if you're a firm, and you're a monopolist, do you get one monopoly? It's like-- there's a big Chicago school of thought that says if you have one monopoly, you can only earn one monopoly profit from it.

And so if you can only get one-- you would never do something-- you would never use your monopoly power in one market to try to get extra profits in a second market. Because if you're trying to use your monopoly to get profit in the second market, the only way you can do that--

Like, if all I can do is sell people something they don't like as much-- like, if I'm Apple. I try to monopolize the iPhone charger market. People don't like iPhone chargers as much as they would have liked USB C chargers.

Yes, I can charge them more by making them all buy chargers for me. But anything that they-- anything that I'm making selling the chargers, I could have just charged them more for the phone to begin with and let them buy the charger they want.

So all I'm doing is just-- I'm just getting one monopoly profit. And you can't extend your monopoly power to a second market because anything people pay extra in the second market, they're going to treat as a cost of buying your initial product. And they're just going to pay less for the base product because you've tried to make them buy a second product.

So that's the Chicago school critique that says, no, you can't tie two markets together and get any more profits. And then so-- Mike Whinston has this paper, classic paper, saying sometimes you will want to try to extend profits to a second market. And this comes from a strategic investment-- either strategic entry accommodation or strategic entry deterrence motive.

So suppose you have a firm, firm one. It's a monopolist in good A. And suppose that there's some-- suppose that Samsung and whatever has disappeared. So now there's only iPhones. So firm one is an iPhone monopolist, but it does compete with firm two in producing chargers or earbuds or something like that.

Suppose we have a homogeneous-- simple, homogeneous model. All consumers get value  $v$  from good A. And then there's a demand for good B that depends on prices for good one and prices for good two.

So I'm just thinking, in this example, this has to be something where you can use good B without good A. So I guess the phone and the charger does not work here.

OK. So suppose these are-- this is taking us back in the day. Suppose that Apple's the iPhone monopolist. Good B is headphones. But suppose that headphones could also be used to listen to an MP3 player, which was a separate object that existed that you could use to listen to music on, not through your phone.

So you could just have a separate thing that looked like a phone that you carried in your pocket that you could play music on and listen to music on it. So anyway. So there's a demand for headphones even if you don't have a-- even if you can't afford a phone, you just have headphones that you use to listen to music otherwise.

Anyway. So firm one has this choice. It can design its products so that they cannot be used separately, or it can design it so that you just plug and play. You can use anybody's earphones with your iPhone.

So imagine first firm one makes a design decision. Then firm two chooses whether to enter and compete with the monopolist. And then we either get a monopoly or duopoly.

In the-- if firm two comes in, it's going to set a price for-- firm two is going to set a price for good B and doesn't sell good A. And then firm one is either going to be only selling a bundle of a phone with earbuds, or it's going to be selling the two products separately. You can buy the phone from it. You can buy the earbuds from it separately, but they're sold at two separate prices.

And we're going to think about in this model, would Apple design its product so that every phone comes with earbuds included, so that no one would then ever buy earphones from anyone else?

So absent any strategic effects, if you can't prevent entry, you can't alter  $p_2$ , firm one would never tie the products together. Because if you sell this bundle at price  $p_1 + v$ , you could instead just sell good A to everyone at price  $v$ , and then sell the bundle at price  $p_1 + v$  minus  $v$ .

And if you do this, every-- once-- with this separate pricing strategy, everybody in the world is going to buy the iPhone at price  $v$ . And sales of good B are unchanged. Because anybody who-- people are making this choice. Do I buy the Apple bundle? Or do I just not have a phone and just buy earphones to listen to my MP3 player?

Anybody-- you sell exactly the same-- anyone who values the iPhone-- anyone who values the Apple earbuds at more than this is going to buy from Apple. Anyone who buys from less than this is going to now-- is going to buy it not from Apple.

And so you sell exactly the same number of units of earphones at these unbundled prices. But you sell more phones because there are some people who would like the phone as a standalone, but they really like the outside earbuds, so that they buy-- they don't buy the phone when they can only buy the phone bundled with the earbuds. So in a nonstrategic model, you would never tie these two products together because it's always better to let people buy what they want and just have the two of them sold separately.

But think about tying the product-- tying the product-- it's a strategic investment that makes firm one tough. It hurts firm two. Why is that?

Well, because it's as if-- you've got this profit you're earning on good A--  $v$  minus  $C_A$ . But you earn that profit on good A only if you sell good B. So it's as if your opportunity cost of selling B is reduced from the cost of producing good B to the cost of good B minus this opportunity cost.

So every time I sell good B, I'm selling good A-- every time I sell good B, I'm selling good A and earning this profit. So it's like, this is my opportunity-- that's my actual cost of selling good B.

And so this is like bundling the goods together. You can think of it as a cost. It's like a cost reduction on good B. Or it's similar-- it's identical to a cost reduction algebraically.

So therefore, if you want to tie-- if you want to prevent any firm from entering market B, I tie the goods together to deter entry. Because tying the goods together is going to reduce the number of headphones that can get sold. And by reducing the number of headphones that can get sold, I may keep firm two out of the market entirely and have a monopoly on the phone-- on both of them together.

So this is a story where if you think about the entry decision, you do have an incentive to tie goods together. And by tying the goods together, you're earning higher profits than you would earn not tying them together. So in some sense, you're going beyond your monopoly profits in firm one and also gaining profits on market two.

In the entry accommodation game, this is an investment that makes you tough. So if you're in price competition in market B, you would not want to make this tie because it hurts firm two. And in a price competition game, you want to help your rivals.

So if you are competing in a price competition game in market B, then in the entry accommodate-- then you would not want to tie the products together. And the leverage theory of tying doesn't make any sense. You can't be tying them in order to get extra profits. You would always only tie-- in some sense, you're going to have an incentive to not tie them because not tying them strategically gets firm two to price higher.

Now, if somehow this was an example where you had a Cournot market for good B, if good B was being sold in a Cournot market, then you may want to also tie products in order-- so it's like, I'm doing something or other. If you buy this from me, you also have to buy your coal from me. And then coal was produced in a Cournot market, I might want to tie goods together in order-- for the same reason, to be mean to you to get you to produce less coal so that we're-- that I'm hurting you in the accommodation game.

But I think it does suggest that it's the leverage to deter entry is an easy story to tell, and leverage to accommodate entry, or the tying to accommodate entry, you're going to have to come up with strategic substitutes. I think it's a strategic substitutes game, and that's going to be-- not apply to many things that we think of.

So limit pricing. Yeah, the next two stories I'm going to say is-- again, as I've said, many things can be thought of as an investment in these previous stories. An investment is anything that's sunk. And once it's sunk, you can't easily undo it. And it's going to affect the competition after the rival enters.

Can we think of prices as an entry-deterring device? So the classic idea of limit pricing is that you have a firm facing entry. And the thought in the limit pricing literature was that way. You had this idea that firms are going to set low prices because they want to keep rivals from entering and competing with them.

And there's a natural critique of that idea, this limit pricing idea, that says, well, wait. How would that work? Sure, you can set your prices low so that everyone sees that your prices are low. But they know that if you enter, we're just going to play the static Nash equilibrium anyways.

The fact that you wasted money in all the previous periods by pricing below the monopoly price, it's just-- it's a sunk loss, but it just-- it has nothing to do with the future. If it has nothing to do with the future, that can't-- you can't deter entry by setting low prices to avoid getting people to come in and compete with you.

And so there are several different parts of the IO literature that are discussed, stories that would account for firms wanting to price below the static monopoly profit in order to keep firms out. One of them is a classic-- is Milgrom and Roberts, 1982, noted that you could have limit pricing if firm two doesn't know firm one's costs, and prices are a signal of costs.

So this is going to be a classic signaling game. Firm one has two possible cost levels,  $C$  lower bar or  $C$  upper bar. Assume that firm one knows its costs. Firm two doesn't know firm one's costs.

Firm two has some prior that there's probability  $\mu$  that it's a high-cost firm, and  $1 - \mu$  that firm one's a low-cost firm. Firm 2 is going to have some separate costs,  $C_2$ , if it comes in and competes. It knows  $C_2$ , but it doesn't know whether firm one has a better or worse technology than it is. So it doesn't know if firm one is a lower cost firm than it is, or firm one is a higher cost firm than it is.

And so we're going to think of a two-period model, where firm one chooses  $p_1$  at  $t = 1$ . Firm two observes the price that firm one is charging, and then it chooses whether to enter or not and has some fixed entry cost,  $E$ . And then we either get a monopoly, where firm one produces and earns monopoly profits that depend on its actual cost,  $C_1$ . Or we get a duopoly, where the profits depend on the costs  $C_1$  and  $C_2$ .

And in this model, Milgrom and Roberts notes there are two different ways we can think of where we can get firm one in equilibrium to set a price lower than it would set in the blockaded entry case. Or in this case, it's simply the profit-maximizing case. And why can that be?

So there are two cases. Case one is  $E$ , such that firm two won't enter if it gets no information. So imagine that firm two-- it's unsure about where the firm one's costs are. We're going to typically think of if firm one has costs  $C$  lower bar, firm two is going to get low profits and wish it hadn't entered. If  $C$  is  $C$  upper bar, then firm two is going to earn high profits and be happy that it entered.

But imagine that the balance between those two is that if firm two doesn't get any new information, so it still has this prior  $\mu$   $1 - \mu$ , it's going to stay out of the market. Well, if firm two would stay out, if it gets no new information, we can get a pooling equilibrium. And the pooling equilibrium-- firm one is setting a low price at  $t = 1$ , regardless of whether its cost is. And then firm two stays out if it sees this price and enters otherwise.

So you imagine there are these two different cost levels or two different price levels.  $P_1$  lower bar would be the argmax of firm one's profits. If it's at price  $P_1$  and its cost is  $C$  lower bar. And  $P_1$  upper bar would be the argmax of  $\pi_1$ -- these are monopoly profits-- at  $P_1$   $C$  upper bar.

So these two different profit level-- two different price levels that you would set if you had low cost or high cost. But if you imagine-- if we tried to have an equilibrium where firm one set this price when its cost was low and this price when its cost was high, if you set this price, firm two is going to realize that you have high cost. And then firm two is going to enter.

So what we can get is this pooling equilibrium, where we have  $p_1$  equals  $p_1$  lower bar for all  $C$ . So if your cost is low, you set price  $C$  lower bar. If your cost is high-- sorry. If you set-- price is low, you set price  $P_1$  lower bar. But if your cost is high, you also set price  $P_1$  lower bar.

And so you're pricing low because by pricing low, firm two-- firm two knows that firm one price is low even if its cost is high. But by doing that, it prevents firm two from learning anything. And by preventing firm two from learning anything, firm two isn't going to enter.

So kind of a familiar argument how signaling models typically work. That is, firm two is going to infer something from your equilibrium play. So we can have this pooling equilibrium where we all charge a low price regardless of our costs, and that prevents firm two from learning anything. That gives you limit pricing to keep firm two out.



The other thing you can get-- and this would be the case two. Case two is E, such that firm two enters if it gets no new info. In this case, clearly, we can't have this pooling equilibrium anymore. Because in this pooling equilibrium, if this was how people played the game, then all the-- then firm two wouldn't learn anything. But then firm two would always enter.

And so if you're a high-cost firm, why are you charging a low price when it doesn't affect entry in the future? That's, again, this here relevant entry case. But so we can get this equilibrium where we have  $P_1$ -- I'll call it the limit pricing equilibrium.  $P_1$  limit pricing of  $C_1$  is less than  $P_1$  lower bar. And  $P_1$  limit pricing of  $C_1$ -- sorry-- of  $C$  lower bar and of  $C$  upper bar equals  $P_1$  upper bar.

So we can get this equilibrium where if your cost is high, you just admit your cost is high. Set a high price. Firm two realize you have a high price and high cost and comes in and enters and competes with you.

But if your cost is low, you charge something super low, even lower than  $P_1$  lower bar, in some sense to prove to firm two that-- prove to firm two that you are the low-cost firm.

And why might you need to charge something lower than this? Because it could be that if you just charge  $P_1$  lower bar, that wouldn't be possible in equilibrium because then the  $C$  upper bar firm would pretend to be a low-cost firm. But by pricing way, way below, even below my static best response price, I set a price so low that a high-cost firm wouldn't copy me. And then it's a credible signal that my cost really is low.

So we can get these two different types of equilibria-- the pooling equilibria, where it's the high-cost guys who price below their static optimum. And then the separating equilibria, where the low-cost guys are pricing below their static optimum to signal their costs.

So that's a cost-signaling argument for why firms might want-- and it's plausible that the firm in the market knows more about their costs than the other firm does. So you price low to signal your costs. And that is why we see firms pricing low to try to keep out competition.

OK. Second kind of story I was going to tell us what Fudenberg and Tirole call a signal-jamming story. In signal-jamming models, firms one and two are both symmetrically uninformed about some state of nature. But, again, signal jamming can be motivated by a desire to deter entry or induce exit. And it, again, can look like pricing low either to try to-- pricing low either to prevent firm two from entering, or to force firm two to exit a market.

So I thought I would do this one just like-- I've been doing everything in terms of avoiding entry. I figured I'd do this one in terms of forcing exit. But some of the same stories can be told both kinds of games.

So here's my model. So firms one and two choose quantities  $q_{11}$  and  $q_{21}$  in the first period. So I'm doing this in a Cournot model. There is a market demand that determines some price. And the market demand is  $\theta$  minus  $q_1$  plus  $q_2$ , where  $\theta$  is an unknown state of the world.

So neither of the firms knows how big the market is for their product. It's a new product. They've just both invented it. They start-- they produce some number of quantities, dump them on a Cournot market, and then there's some market-clearing price for the good.

And let's assume that  $\theta$  is unknown, and it has some common prior. If  $\theta$  is distributed normal-- mean  $\mu$  variance  $\sigma^2$ . So it could be that demand for the products is very high. It could be demand for the products is very low.

And so then on the observability, we're going to assume that after firms choose their first period quantities, they know what quantity they produced. They don't know what quantity their rival produced. But they do observe what the market-clearing price was. And so they observe market-clearing price, but not the rival's quantity.

And then I'm going to assume that after they observe the market price, firm two has this option to exit the market, and by scrapping its firm earn some value,  $K$ . And then if firm two-- depending on what firm two does, we then end up with firm two exiting in their monopoly in the second period, or firm two stays in, and there's duopoly competition in the second period.

This is a misspelling. So the equilibrium is going to have  $q_{11}$  greater than the static best response to  $q_{21}$ . So the idea is going to be in this model, firm one is going to what we call signal jam and overproduce in the first period driving down the market price.

And in equilibrium-- firm two always knows what firm one is doing in equilibrium. So in equilibrium, firm two correctly anticipates  $q_{11}$ . It can always then infer that it knows that  $\theta - q_{11} + q_{21} = P_1$ . So it can always infer that  $\theta = P_1 + q_{11} - q_{21}$ .

So I guess this is the way it infers it. It's like after the first period, it sees the first period price. It knows what quantity it produced. It anticipates. It knows the equilibrium,  $q_{11}^*$ . And so it now believes that  $\theta$  equals this with probability 1.

And so result is that this equilibrium has to have overproduction at  $t = 1$  by firm one. Why is that? Well, because suppose that firm one was just playing the static best response to  $q_{21}$ . Then firm one could increase its output with no first order loss and profits. And there's a one for one relationship between the more I produce in the first period and what firm two thinks  $\theta$  is going to be.

Because if I produce-- imagine that I produce  $q_{11}$  greater than  $q_{11}^*$ . Firm two is thinking I produce  $q_{11}^*$ . So when I produce  $q_{11}$ ,  $P_1$  gets driven down. And so when  $P_1$  gets driven down,  $P_1$  is going to be actually the true  $P_1$ . Well, it's going to be the-- sorry. What I'm going to get is their estimated  $\theta$ ,  $\hat{\theta}$ , is going to be the true  $\theta - q_{11} + q_{11}^*$ .

So if I overproduce, firm two is going to miss estimate  $\theta$  and think  $\theta$  is lower than it really is because the price was lower than it should have been given this  $q_{11}^*$ . So firm two is going to think  $\theta$  is lower.

And so because firm two thinks  $\theta$  is lower than it really is, firm two is going to be more likely to exit after the first period. But then if firm two is more likely to exit in the first period, that's good for me, and so I want to do it.

So why would I not do this in equilibrium? The only reason I would not do it in equilibrium is choosing  $q_{11}$  greater than  $q_{11}^*$  has a first order cost to me. And the only way it's going to have a first order cost to me is if  $q_{11}$  is greater than the static best response to  $q_{21}$ .

So what happened is we have this equilibrium where I overproduce. He knows I'm overproducing. He correctly infers what's going to happen, given the overproduction. But the overproduction by me is necessary to have me not have an incentive to overproduce by even more.

And so we get this signal-jamming equilibrium where firm one produces more than it looks like the static best response in the first period, drives prices down. And it drives prices down in an attempt to get firm two to exit.

And notice, there is a funny efficiency thing behind that story is in equilibrium, firm two anticipates  $\theta$  correctly, undoes my signal jamming, and only exits when it would exit efficiently anyway. So this signal-jamming is, in some sense, good for social welfare relative to the world where all the cues were available-- were observable. Because if the cues were observable, you'd get exactly the same entry. Except here, we get this low price, and then we still get the same entry you'd get with observable cues. So this unobservability is just socially a good thing.

I guess, one more classic paper-- Aghion and Bolton, 1987-- discusses the question of, can firms deter entry by getting their customers to sign long-term contracts? Like, if I could get-- if I'm trying to deter entry by rival cell phone companies in Boston, could I just get everyone who's currently in Boston to sign like, say, my cell phone contracts have a five-year commitment. You sign up with me. It's a five-year commitment. You have to pay me an early termination fee of \$2,000 if you leave me before the five years. Can I use lockup long-term contracts like that to deter entry by rivals?

And, again, the Chicago school argument was always, well, no. You can't use long-term contracts to deter entry because the customer wouldn't sign the long-term contract unless it was in their best interest. And therefore, we shouldn't think that firms can use long-term contracts to deter entry because the customers wouldn't sign the contracts unless it was in their interest to sign the contracts.

Two problems with that story. One is the Chicago story may make sense for large buyers or monopoly buyers. But it certainly doesn't make sense in markets like the cell phone market, where you have a large number of infinitesimal consumers.

Because you-- you graduate student at MIT, you walk into Verizon. They say, we only offer five-year contracts. You walk to T-Mobile, and they say, we only offer five-year contracts. So you're like, I'm not-- I don't even know I'm going to be here for five years. I'm not going to sign a five-year contract. Give me a one-year contract instead. They're going to say, fine. Don't buy a phone.

And you're going to say. I want to buy the one-year contract because I know that if all of us buy one-year contracts, some third firm is going to enter the Boston market. Well, the answer is you, as a graduate student, that's not happening.

Like, if there was a monopoly buyer who could commit to I'm only signing a one-year contract, then someone's going to enter, that would be better. But you-- if you know that everybody else is signing the five-year contract, you holding out and not signing the five-year contract is not going to do anything. It's not going to affect entry. So there's a free rider problem. Even if it's bad for all the consumers in aggregate, you could sign it because of the free rider problem.

Aghion and Bolton note that even with a monopoly buyer, we can still tell stories where the long-term contracts get signed even though they're not optimal or not socially optimal. And the reason is that the long-term contracts are in some sense stealing money from firm two, the potential entrant.

And you, in some sense, take that business stealing profit and divide it among the customer signing the contract and the initial monopolist. And by stealing the profits away from the future firm two, you can divide those up among and get a situation where it's bad socially to have these long-term tie-ups. But the monopolist wants to offer them, and the customers want to sign them.

So anyway, I will not go through that argument. It's written there. I had one more.

Anyway. So Wednesday, I'm going to go back to empirical papers on strategic investment. I think these three are the ones that I'm mostly going to talk about. OK. See you then.