

## 14.273: Advanced Topics in IO

---

Empirical Models of Search

## Motivation — why search?



*“Price dispersion is a manifestation – and, indeed it is the measure – of ignorance in the market”*

(Stigler 1961, p. 214)

*“Price dispersion is a manifestation – and, indeed it is the measure – of ignorance in the market”*  
(Stigler 1961, p. 214)

## Positive

- *Law of one price* empirically almost never holds.
- Search models explain **price dispersion** for **homogeneous goods**.
- In markets with symmetric firms and homogeneous products, prices may differ in equilibrium if consumers incur search costs to obtain price information.

*“Price dispersion is a manifestation – and, indeed it is the measure – of ignorance in the market”*  
(Stigler 1961, p. 214)

## Positive

- *Law of one price* empirically almost never holds.
- Search models explain **price dispersion** for **homogeneous goods**.
- In markets with symmetric firms and homogeneous products, prices may differ in equilibrium if consumers incur search costs to obtain price information.

## Normative

- Search costs are a source of market power!
- Search costs are pure social costs.
- Search costs can lead to mis-allocation.

## Relevance

- Common in many retail markets: automobiles, real estate, mortgages, all kinds of services.
- Knowing the “fair” price is hard for consumers if there is an element of idiosyncratic pricing.
- Distributional implications of pricing.

## Empirical Challenge

- Typically, only transaction prices are observed, which are a selected set of offered prices.

1. Hong and Shum (2006).
2. Allen, Clarke, and Houde (2014): search and negotiations in mortgage markets.
3. Salz (2022): intermediation in a decentralized market.

**Hong and Shum (2006)**

## Estimation + identification:

- Back to Stigler's quote: what is the empirical content of the price distribution?
- Identify and estimate search cost distribution under the assumption of a mixed strategy pricing equilibrium. Burdett and Judd (1983)

## Data:

- Observe only prices, no quantity data.

## Sequential vs. non-sequential:

- Non-sequential: commit to taking  $N$  draws, pick lowest price.
- Sequential: Draw and observe price, decide whether to draw again → Cut-off strategy.



**Consumer objective function:**

$$l^*(c_i) \equiv \operatorname{argmin} c_i \cdot (\ell - 1) + \int_p^{\bar{p}} \ell \cdot p (1 - F_p(p))^{\ell-1} f_p(p) dp$$

Consumers are assumed to draw i.i.d. samples from the equilibrium price distr.  $F_p$ .

**Consumer objective function:**

$$l^*(c_i) \equiv \operatorname{argmin}_p c_i \cdot (l - 1) + \int_p^{\bar{p}} l \cdot p (1 - F_p(p))^{\ell-1} f_p(p) dp$$

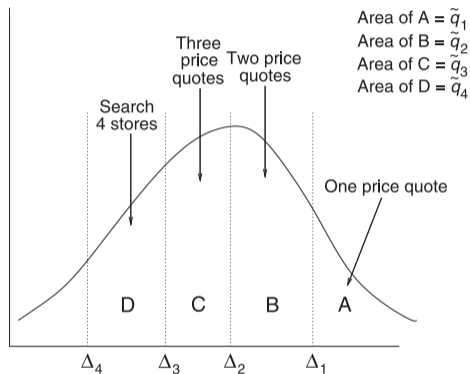
Consumers are assumed to draw i.i.d. samples from the equilibrium price distr.  $F_p$ . The marginal expected savings from searching  $i + 1$  versus  $i$  stores is:

$$\Delta_i \equiv E p_{1:i} - E p_{1:i+1}, \quad i = 1, 2, \dots$$

- The sequence of marginal expected savings  $\Delta_i$ ,  $i = 1, 2, \dots$  is non-increasing in  $i$  for any price distribution  $F_p$ , while the cost per search is constant.
- A consumer will search as long as the marginal expected savings  $\Delta_i$  exceeds his marginal search cost  $c$ .

# Hong and Shum (2006) — non-sequential search

Let  $F_c(\cdot)$  be the distribution of search cost



- $\tilde{q}_1 \equiv 1 - F_c(\Delta_1)$ : proportion of consumers with one price quote;
- $\tilde{q}_2 \equiv F_c(\Delta_1) - F_c(\Delta_2)$ : proportion of consumers with two price quotes;

A firm's profits from following the **mixed pricing** strategy  $F_p(\cdot)$  (See Burdett and Judd (1983)):

$$\Pi(p) = (p - r) \cdot \left[ \sum_{k=1}^K \tilde{q}_k \cdot k \cdot (1 - F_p(p))^{k-1} \right]$$

**Equilibrium:** Firm must be indifferent between charging the monopoly price  $\bar{p}$  (selling only to people who never search but receive an initial free draw) and any other price  $p$  in the equilibrium support  $[\underline{p}, \bar{p}]$ :

$$(\bar{p} - r) \cdot \tilde{q}_1 = (p - r) \cdot \left[ \sum_{k=1}^K \tilde{q}_k \cdot k \cdot (1 - F_p(p))^{k-1} \right]$$

Let  $K \leq n - 1$  denote the maximum number of firms from which a consumer obtains price quotes in this market.

**Indifference condition and estimation:**

$$(\bar{p} - r) \cdot \tilde{q}_1 = (p_i - r) \cdot \left[ \sum_{k=1}^K \tilde{q}_k \cdot k \cdot (1 - \hat{F}_p(p_i))^{k-1} \right]$$

Since  $\tilde{q}_K = 1 - \sum_{k=1}^{K-1} \tilde{q}_k$  the above constitutes  $n - 1$  linear equations from which we can solve for  $\{r, \tilde{q}_1, \dots, \tilde{q}_{K-1}\}$ .

After each search, consumers can choose to purchase at the lowest price observed so far, or make an additional search. At any price, there is an option value associated with searching again.

**Cut-off** rule: A standard result in the sequential-search literature is that the consumers' optimal stationary-search strategy is a reservation price policy, where they search until they obtain a price that is no larger than some reservation price  $z^*$ .

For consumer  $i$ , who has per-price search costs  $c_i$ , let  $z^*(c_i)$  denote the price  $z$  that satisfies the indifference condition, then:

$$c_i = \int_{\underline{p}}^{z^*(c_i)} (z - p) f_p(p) dp$$

### Can't use the same argument for identification

Let the reservation price be  $p_i = \min\{z^*(c_i), \bar{p}\}$  and  $G(p_i)$  be the mass of customers with reservation values less than  $p$ .

### Can't use the same argument for identification

Let the reservation price be  $p_i = \min\{z^*(c_i), \bar{p}\}$  and  $G(p_i)$  be the mass of customers with reservation values less than  $p$ . A firm charging price  $p$  will only sell to consumers with a reservation price higher than  $p$ . Indifference condition:

$$(\bar{p} - r) \cdot (1 - G(\bar{p})) = (p_i - r) \cdot (1 - G(p_i))$$



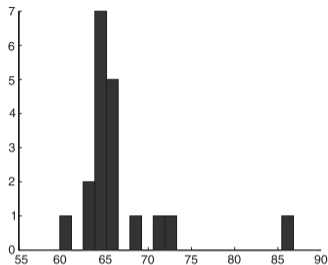
## Can't use the same argument for identification

Let the reservation price be  $p_i = \min\{z^*(c_i), \bar{p}\}$  and  $G(p_i)$  be the mass of customers with reservation values less than  $p$ . A firm charging price  $p$  will only sell to consumers with a reservation price higher than  $p$ . Indifference condition:

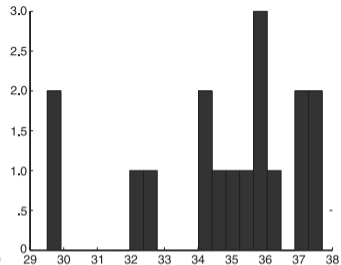
$$(\bar{p} - r) \cdot (1 - G(\bar{p})) = (p_i - r) \cdot (1 - G(p_i))$$

- $n+1$  unknowns ( $G(p_i)$   $i$  and  $r$ ) but only  $n - 1$  indifference conditions
- Intuition: for non-sequential search there are only  $K$  possible cutoff-types, whereas for sequential search there is a continuum of reservation values.
- Instead, they estimate a parametric search cost distribution  $f_c(\cdot|\theta)$ .

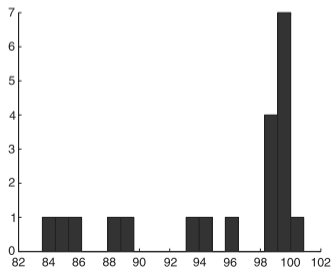
# Hong and Shum (2006) — results



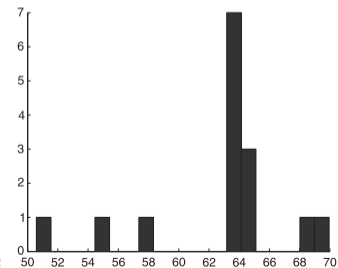
Stokey-Lucas: *Recursive Methods...*



Lazear: *Personnel Economics*



Billingsley: *Probability and Measure*



Duffie: *Dynamic Asset Pricing Theory*

x-axis: prices (in dollars)

Product	$K^a$	$M^b$	$\tilde{q}_1^c$	$\tilde{q}_2$	$\tilde{q}_3$	Selling Cost $r$	MEL Value
<b>Parameter estimates and standard errors: nonsequential-search model</b>							
Stokey-Lucas	3	5	.480 (.170)	.288 (.433)		49.52 (12.45)	102.62
Lazear	4	5	.364 (.926)	.351 (.660)	.135 (.692)	27.76 (8.50)	84.70
Billingsley	3	5	.633 (.944)	.309 (.310)		69.73 (68.12)	199.70
Duffie	3	5	.627 (1.248)	.314 (.195)		35.48 (96.30)	109.13
<b>Search-cost distribution estimates</b>							
	$\Delta_1$	$F_c(\Delta_1)$	$\Delta_2$	$F_c(\Delta_2)$	$\Delta_3$	$F_c(\Delta_3)$	
Stokey-Lucas	2.32	.520	.68	.232			
Lazear	1.31	.636	.83	.285	.57	.150	
Billingsley	2.90	.367	2.00	.058			
Duffie	2.41	.373	1.42	.059			

Makes the most of a very limiting data scenario. One of the first empirical papers on search.

Is the **mixed strategy** pricing equilibrium plausible?

In many settings homogeneous marginal cost not very realistic.

Some issues that many empirical search models have in common:

- Typically find very large search cost.
- All price variation is interpreted as structural. When might this be problematic?

**Allen, Clarke, and Houde (2013)**

**Research Question:** what is the effect of a merger in a negotiated price market?

- Different people get different prices → mergers might have distributional implications.

**Research Question:** what is the effect of a merger in a negotiated price market?

- Different people get different prices → mergers might have distributional implications.

**Setting:** Canadian mortgage market.

- Administrative data on insured mortgage contracts, collected from insurer.
- Extremely homogeneous product, idiosyncratic risk is insured away.

**Research Question:** what is the effect of a merger in a negotiated price market?

- Different people get different prices → mergers might have distributional implications.

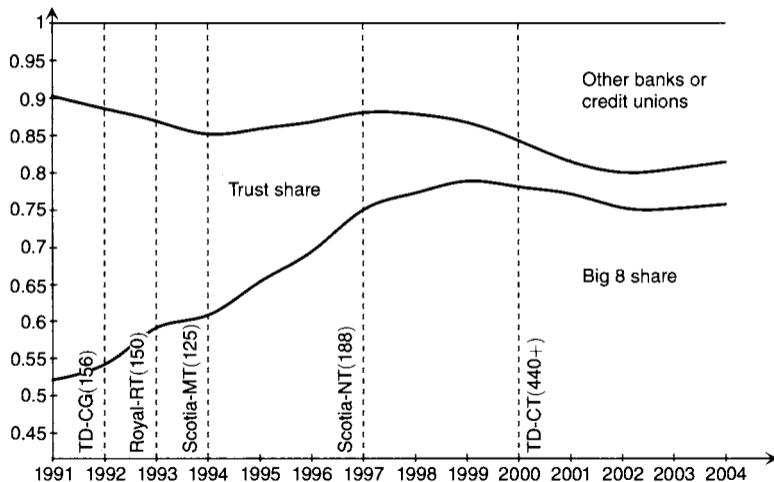
**Setting:** Canadian mortgage market.

- Administrative data on insured mortgage contracts, collected from insurer.
- Extremely homogeneous product, idiosyncratic risk is insured away.

**Research Design:** structural model + treatment effect analysis.

- Use quasi-experimental variation induced by horizontal merger between two large Canadian mortgage lenders. Creates discrete changes in choice sets for consumers.
- Examine estimates in light of a search model.



FIGURE 1. EVOLUTION OF FINANCIAL INSTITUTION MARKET SHARES FOR NEWLY INSURED MORTGAGES (*Smoothed*)

## **Institutional details:**

- Banks post rates and consumers can negotiate discounts with local branch managers.
- Branch managers receive commission that are affected by discounts.

## **Institutional details:**

- Banks post rates and consumers can negotiate discounts with local branch managers.
- Branch managers receive commission that are affected by discounts.

## **Market definition / consumer choice sets:**

- Financial characteristics of the contract, i.e., rate, loan size, house price, debt ratio, risk type.  
Demographics: income, prior relationship with banks, ...
- Market defined as a 5KM radius around consumer's home FSA.

## Institutional details:

- Banks post rates and consumers can negotiate discounts with local branch managers.
- Branch managers receive commission that are affected by discounts.

## Market definition / consumer choice sets:

- Financial characteristics of the contract, i.e., rate, loan size, house price, debt ratio, risk type.  
Demographics: income, prior relationship with banks, ...
- Market defined as a 5KM radius around consumer's home FSA.

## Treatment and control group:

- Focus on short-run effect of a merger. Long run market-conditions (branch openings and closings) might reflect unobserved local market conditions.
- No effects on consumers who had only one or neither type of branch in their neighborhood (Control Group). Number of competing branches reduced in the neighborhoods that have both types of branches. Markets with branches of both merging firms are in the Treatment Group.

## Descriptive statistics / outcome variables:

- Few consumers pay a rate equal to the posted rate.
- Residual margin is the main outcome variable of interest:

$$\text{Margin}_i = \beta' \mathbf{X}_i + m_i$$

where:  $\text{Margin}_i = \text{Rate}_i - \text{Bond}_i$ ,  $\mathbf{X}_i$  is a vector of control variables.

- The residual,  $m_i$ , is called the negotiated margin.

TABLE 2—SUMMARY STATISTICS ON MORTGAGE CONTRACTS AND HOUSEHOLD CHARACTERISTICS

	Control/before				Control/after			
	Mean	SD	P(25)	P(75)	Mean	SD	P(25)	P(75)
Margin	1.07	0.46	0.73	1.42	1.43	0.56	1.02	1.82
N-Margin	1.07	0.43	0.78	1.35	1.67	0.46	1.36	2.02
$I(r_i = \bar{r}_i)$	36.38	48.12			26.92	44.36		
Income	61.95	25.00	43.70	74.77	62.84	24.49	45.37	75.29
House	121.11	55.49	82.18	145.30	118.25	52.16	81.18	143.74
Loan	113.4	49.7	78.7	137.2	110.0	46.9	76.9	133.5
LTV	91.58	4.26	90.00	95.00	91.25	4.34	90.00	95.00
FICO	67.23	46.95			64.27	47.93		
Renter	68.30	46.55			70.05	45.82		
Parents	5.73	23.24			6.64	24.90		
Switch	30.21	45.93			36.84	48.25		
Broker	21.91	41.37			30.05	45.86		

	Treatment/before				Treatment/after			
	Mean	SD	P(25)	P(75)	Mean	SD	P(25)	P(75)
Margin	0.93	0.49	0.65	1.27	1.44	0.56	1.12	1.82
N-Margin	1.06	0.43	0.78	1.35	1.72	0.47	1.44	2.00
$I(r_i = \bar{r}_i)$	23.67	42.51			22.23	41.58		
Income	69.33	26.48	50.71	82.23	70.99	26.49	52.46	84.11
House	162.93	63.38	116.77	201.83	161.07	64.58	114.84	200.72
Loan	152.3	57.6	110.5	188.2	149.9	57.6	108.2	187.0
LTV	91.35	4.25	90.00	95.00	90.99	4.48	89.60	95.00
FICO	62.40	48.44			62.56	48.40		
Renter	68.28	46.54			71.15	45.31		
Parents	8.31	27.61			9.32	29.08		
Switch	26.68	44.23			38.43	48.65		
Broker	15.66	36.34			27.73	44.77		

Notes: The sample size is 18,121 divided between the control and treatment group, pre- and post-merger with 62.8 percent of contracts in the treatment and 42.2 percent observed post-merger. It includes a random sample of homogeneous term and amortization contracts insured by CMHC or Genworth within one year of the merger. Margins and

TABLE 3—AVERAGE EFFECT OF THE MERGER ON MARGINS AND DISCOUNTS

	Margin		Zero discount	
	Baseline	With trend	Baseline	With trend
<b>Linear DiD (OLS)</b>				
Merger ATE	0.0607*** (0.0183)	0.0719*** (0.0242)	0.0646*** (0.0154)	0.0477*** (0.0188)
Observations	18,121	18,121	18,121	18,121
$R^2$	0.408	0.420	0.181	0.189
<b>Matching DiD</b>				
Merger ATE	0.0739*** (0.0249)	0.0692** (0.0273)	0.0670*** (0.0206)	0.0538** (0.0218)
Observations	17,220	17,220	17,220	17,220
<b>Change-in-change</b>				
Merger ATE	0.057*** (0.015)	0.0661*** (0.021)		
Observations	18,103	18,103		

*Notes:* Standard errors clustered at the FSA level are in parentheses. The dependent variable in columns 1 and 2 is the transaction rate minus bond rate, in columns 3 and 4 is an indicator variable for the transaction rate within ten basis points of the posted rate, which we take to imply

- ATE margin corresponds to 10 to 15% of standard deviation in sample.
- Rejects the null that merger did not increase market power.
- Not consistent with models that explain interest rate dispersion purely in terms of cost and risk.
- For an average loan size (\$152000) merger led to \$5.7 increase in monthly loan payments.
- For informed customers, this should be a fairly competitive market.



Let  $\hat{F}_{G,T}(m)$  be the CDF where (G) indexed treatment status and (T) time period. **Want:**  $\hat{F}_{1,1}^c$ .

Let  $\hat{F}_{G,T}(m)$  be the CDF where (G) indexed treatment status and (T) time period. **Want:**  $\hat{F}_{1,1}^c$ .

**Changes-in-Changes Estimator** (Athey and Imbens (2006)):

$$\hat{F}_{1,0}^{-1} \left( \hat{F}_{1,1}^c (m_i) \right) = \hat{F}_{0,0}^{-1} \left( \hat{F}_{0,1} (m_i) \right)$$

Let  $\hat{F}_{G,T}(m)$  be the CDF where (G) indexed treatment status and (T) time period. **Want:**  $\hat{F}_{1,1}^c$ .

**Changes-in-Changes Estimator** (Athey and Imbens (2006)):

$$\hat{F}_{1,0}^{-1} \left( \hat{F}_{1,1}^c (m_i) \right) = \hat{F}_{0,0}^{-1} \left( \hat{F}_{0,1} (m_i) \right) \Leftrightarrow \hat{F}_{1,1}^c (m_i) = \hat{F}_{1,0} \left( \hat{F}_{0,0}^{-1} \left( \hat{F}_{0,1} (m_i) \right) \right)$$

**Intuitively**, obtain counterfactual distribution by transforming the observed negotiated margin distribution in the treatment group ( $F_{1,0}(m)$ ) to mimic the changes in the control group.

**Distributional effect:**

$$\alpha(q_i) = \hat{F}_{1,1}^{-1}(q_i) - \hat{F}_{1,1}^{c-1}(q_i)$$

FIGURE 2. EMPIRICAL DISTRIBUTION OF NEGOTIATED MARGINS WITH AND WITHOUT THE MERGER

TABLE 4—DISTRIBUTIONAL EFFECT OF THE MERGER ON NEGOTIATED MARGINS

	Baseline			With trend		
	Est.	95% CI		Est.	95% CI	
<b>Quantile effects</b>						
$q_5$	0.0766	0.0123	0.157	0.0782	0.0080	0.18
$q_{10}$	0.0914	0.0505	0.13	0.0844	0.0379	0.138
$q_{25}$	0.0762	0.0454	0.0994	0.0767	0.0397	0.119
$q_{50}$	0.0842	0.0433	0.114	0.0874	0.0446	0.128
$q_{75}$	-0.0008	-0.0621	0.0686	0.0185	-0.0473	0.0885
$q_{90}$	-0.0042	-0.0485	0.0383	0.0069	-0.0393	0.0591
$q_{95}$	0.056	0.0053	0.0949	0.0513	-0.0125	0.0973
<b>Dispersion effects</b>						
$\Delta SD$	-0.0287	-0.0587	-0.0066	-0.0256	-0.056	-0.0034
$\Delta CV$	-0.0462	-0.0766	-0.0211	-0.0455	-0.0792	-0.0184
$\Delta q_{75} - q_{25}$	-0.077	-0.131	-0.0099	-0.0582	-0.114	-0.0044
$\Delta q_{90} - q_{10}$	-0.0957	-0.153	-0.0457	-0.0776	-0.134	-0.0261
$H_0 : F_{1,1}^c = F_{1,1}$						
KS	4.7***			4.84***		

Notes: The dependent variable is negotiated margins.  $\Delta SD$  and  $\Delta CV$  measure the changes in the standard deviations and coefficient of variation respectively. Confidence intervals were cal-

Interpret results through a **search model**:  $n + 1$  lenders, negotiation takes place over three stages.

1. Consumers get an initial quote  $m_i^0$  from some lender.
2. Can reject offer and engage in search effort  $e_i$  to get additional quotes. With  $s(e_i)$  matched with all  $n$  banks and with  $(1 - s(e_i))$  they receive two quotes.
3. Competition takes place, consumers choose lowest price offer.

Working backwards starting with competition (Stage 3):

- English auction.
- Lenders have a common cost component  $c$  and an idiosyncratic cost component  $\epsilon_j$ .
- Unique dominant strategy price eq: banks offer up to their private cost  $c + \epsilon_j$ . The most efficient bank wins the contract at  $c_{(2)} = c + \epsilon_{(2)}$
- Let  $E(m^* | \tilde{n}) = c + E(\epsilon_{(2)} | \tilde{n})$  denote the expected second stage transaction price under  $\tilde{n}$  quotes.
- Gain from searching:  $\Delta(n) = E(\epsilon_{(2)} | 2) - E(\epsilon_{(2)} | n) > 0$

## Search effort decision (Stage 2):

- Effort cost has variable and fixed component:  $\kappa_i = u_i(1 + e_i) + \eta_i$ . Assume that marginal cost of effort ( $u_i$ ) is publicly observed and fixed cost component ( $\eta_i$ ) privately observed.

## Search effort decision (Stage 2):

- Effort cost has variable and fixed component:  $\kappa_i = u_i(1 + e_i) + \eta_i$ . Assume that marginal cost of effort ( $u_i$ ) is publicly observed and fixed cost component ( $\eta_i$ ) privately observed.

- Matching probability assumed to be pareto:  $s(e) = 1 - (1 + e)^{-\lambda}$

- Consumers optimize:

$$r(u_i, n) \equiv \min_{e \geq 0} u_i \cdot (1 + e) + c + E(\epsilon_{(2)}|2) - s(e)\Delta(n) = c + \pi(u_i, n)$$

- Optimal effort level has a threshold property: zero effort with marginal cost larger than  $\bar{u}(n)$ .



## Allen, Clarke, and Houde (2013) — model: initial offer stage



Initial offer stage (Stage 1), **search probability**:  $H_\eta \left( m_i^0 | u_i, n \right) = \Pr \left( \eta_i < m_i^0 - r(u_i, n) \right)$ .

## Allen, Clarke, and Houde (2013) — model: initial offer stage



Initial offer stage (Stage 1), **search probability**:  $H_\eta \left( m_i^0 | u_i, n \right) = \Pr \left( \eta_i < m_i^0 - r(u_i, n) \right)$ .

Bank that makes **initial offer** maximizes expected profits:

$$\max_{m_i^0 \geq r(u_i, n)} \left( m_i^0 - c \right) \left[ 1 - H_\eta \left( m_i^0 | u_i, n \right) \right]$$

Initial offer stage (Stage 1), **search probability**:  $H_\eta \left( m_i^0 | u_i, n \right) = \Pr \left( \eta_i < m_i^0 - r(u_i, n) \right)$ .

Bank that makes **initial offer** maximizes expected profits:

$$\max_{m_i^0 \geq r(u_i, n)} \left( m_i^0 - c \right) \left[ 1 - H_\eta \left( m_i^0 | u_i, n \right) \right]$$

This leads to a monopolist pricing rule in which consumers earn some information rent due to  $\eta_i$ :

$$m_i^0 - c = \frac{1 - H_\eta \left( m_i^0 | u_i, n \right)}{h_\eta \left( m_i^0 | u_i, n \right)}.$$

However, in the empirical implementation  $\eta_i = 0$ , consumers **earn no information rent** and are offered the reservation value implied by their search cost type  $m_i = m_i^0 = r(u_i, n)$ .

Map to observed distribution of **negotiated margins**:

$$F_{1,1}(m) = \Pr(c + \pi(u_i, n) < m) = H_u(\pi^{-1}(m - c, n))$$

→ quantiles of marginal search cost map to quantiles of negotiated margins.

Map to observed distribution of **negotiated margins**:

$$F_{1,1}(m) = \Pr(c + \pi(u_i, n) < m) = H_u(\pi^{-1}(m - c, n))$$

→ quantiles of marginal search cost map to quantiles of negotiated margins.

**Heterogeneous treatment effects:**

$$\begin{aligned}\alpha(u) &= \pi(u, n - 1) - \pi(u, n) \\ &\geq \pi(u', n - 1) - \pi(u', n) = \alpha(u'), \quad \text{for all } u' \geq u\end{aligned}$$

→ pricing function under symmetric information has the following properties: (i) monotonically increasing in  $u_i$ , (ii) weakly decreasing in  $n$ , (iii) increasing marginal effects of  $u_i$  with respect to  $n$ .

Construct moment estimator that targets those changes:

$$J = \min_{\theta = \{c, \lambda, \sigma_\epsilon\}} (\mathbf{M}(\theta) - \hat{\mathbf{M}})^T \Omega^{-1} (\mathbf{M}(\theta) - \hat{\mathbf{M}})$$
$$\text{s.t. } u_{q_i} = \pi^{-1} \left( m_i^l - c, n - 1 | \theta \right), \quad i = 1, \dots, S$$

**Moments** describe distributional effect of the merger. **Parameters:**  $\lambda$ : search effort parameter,  $c$ : bank's common cost parameter,  $\sigma_\epsilon$ : variance of banks idiosyncratic cost component.

Construct moment estimator that targets those changes:

$$J = \min_{\theta = \{c, \lambda, \sigma_\epsilon\}} (\mathbf{M}(\theta) - \hat{\mathbf{M}})^T \Omega^{-1} (\mathbf{M}(\theta) - \hat{\mathbf{M}})$$

$$\text{s.t. } u_{q_i} = \pi^{-1} \left( m_i^l - c, n - 1 | \theta \right), \quad i = 1, \dots, S$$

**Moments** describe distributional effect of the merger. **Parameters:**  $\lambda$ : search effort parameter,  $c$ : bank's common cost parameter,  $\sigma_\epsilon$ : variance of banks idiosyncratic cost component.

**Results** of the structural model:

- Bank markup 5% to 10%, large differences across consumers (perfect price discrimination).
- Average search cost correspond to \$1760 for a \$152000 (25 year) loan.
- Market power effect much more consequential for consumers with low search cost.

Nice paper that...

- ...highlights a new angle of the effects of mergers
- ...tight link between treatment effect analysis and the structural model.

Some shortcomings of the model:

- No information asymmetries imply that there is no search in equilibrium. → Sunk cost of search are never incurred in equilibrium.
- No information rents for consumers is a strong assumption. Implies no search externalities.



## How do intermediaries affect buyers and sellers?

- Delegating buyers benefit directly from better “search technology” (*direct effect*).
- Selection creates search-externality, Salop and Stiglitz (1977), (*indirect effect*).

## What is the welfare effect? No quantity distortion, but:

- Demand side: Reduction in search cost.
- Supply side: More efficient production (reallocation).

## Modeling contribution:

- Tractable model for a decentralized market with two-sided heterogeneity

## Data contribution:

- Identify setting where decentralized market prices can be observed.

## Intermediaries

- **Large fraction of economic activity**, Spulber (1996).
- Focus here: search for **prices**.

## The Market

- Non-residential waste disposal market, fully decentralized.
- **Sellers:** waste carters, **buyers:** all private entities in the city.
- $\approx 100,000$  buyers and  $\approx 100$  sellers.

## Why looking at this market?

- Rare insight in segmented, decentralized market.
- Many contract characteristics are observed.
- Institutional features aid identification.

## **Business Integrity Commission (BIC)**

- Established by city in 1995
- Response to property rights and racketeering system.

## **Data**

- Ranging from 2009 to 2014.
- Price, quantity, composition, transfer station, business type, zip-code, ...

## The buyers

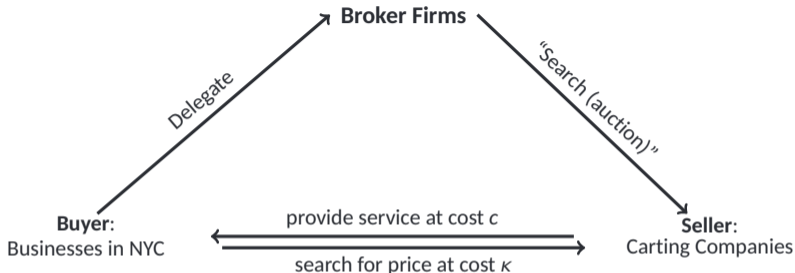
- Delegate or search (non-sequential).

## The sellers

- Price competition for customers.

## The brokers

- Procure through competitive bidding (exogenous market share/technology).



- **Most previous search models** avoid two-sided unobserved heterogeneity. Hortacsu and Syverson (2003), Hong and Shum (2006)
- Idiosyncratic service  $\rightarrow$  customer-specific cost, prices.
- **Identification:** observe two different markets + price formation in brokered market known.

*“When I recently walked down a four-block stretch of Broadway on the Upper West Side of Manhattan, I identified about forty businesses — restaurants, clothing shops, bodegas, banks. Licenses in windows listed the commercial-waste haulers they use — at least fourteen in all, by my count, for a stretch that covers only a fifth of a mile. If there was a pattern, I couldn’t grasp it: the Starbucks at Ninety-third and Broadway uses a different commercial-waste company from the Starbucks at Ninety-fifth and Broadway.” — **New Yorker (2009)***

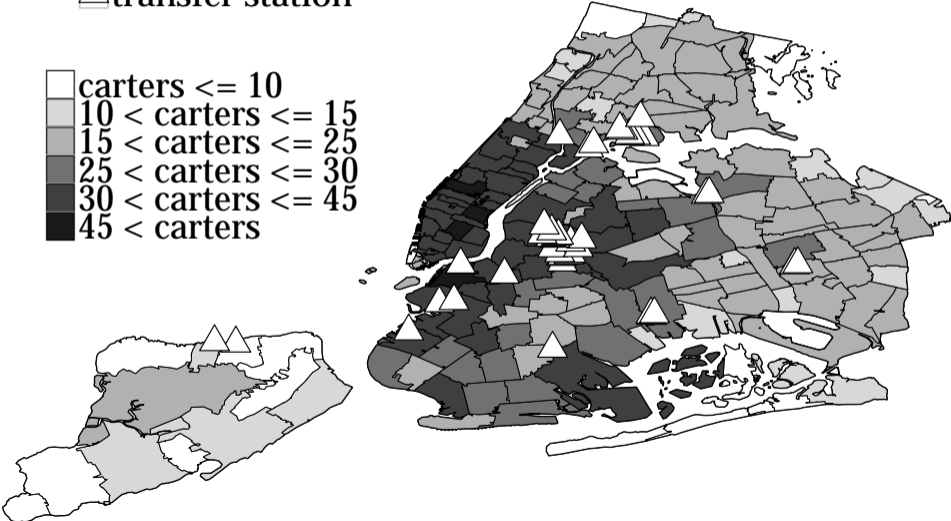
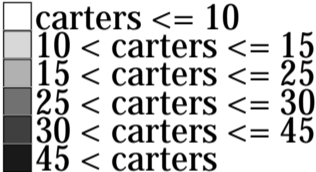
© *The New Yorker* and City of New York. All rights reserved. This content is excluded from our Creative Commons license. For more information, see <https://ocw.mit.edu/help/faq-fair-use/>

*“Both the data and customer outreach reveal a lack of consistency in how rates are established. Without posted rate formulas, customer-initiated service requests require a direct phone call to the carter or a quote request submitted on the carter’s web site, making comparison shopping fairly difficult.” — **DSNY, Private Carting Study, 2016***

# Salz (2022) — fragmented supply



△ transfer station





## Specification

- $p_{ijt} = \mathbf{X}_{ijt} \cdot \beta + \tilde{p}_{ijt}$ .
- Separate for broker and search-market.

## Included Controls

$\mathbf{X} = \{ \text{business type FE, recyclables FE, time FE,}$   
transfer station FE, zip code FE,  $q, \dots, q^5,$   
Number of Pickup FE}  $\times$  {Carter FE, No Carter FE}

TABLE 2  
DOCUMENTING PRICE DISPERSION

	FIRST SPECIFICATION (No Carter Fixed Effects)		SECOND SPECIFICATION (Carter Fixed Effects)	
	Not Brokered	Brokered	Not Brokered	Brokered
$1 - R^2$	.71	.35	.51	.24
SD ( $p_{ijt}$ )	2.9	4.4	2.9	4.4
SD ( $\tilde{p}_{ijt}$ )	2.5	2.2	2.13	1.97
Mean ( $p_{ijt}$ )	12.4	10.9	12.4	10.9

- Coefficient of variation:  $\frac{SD(\tilde{p}_{ijt})}{\text{mean}(p)} > 0.24$  versus 0.22 in Sorensen (2000).
- $+SD(\tilde{p}_{ijt})$  implies extra \$1280 additional chargers over two years for average customer

## Primitives:

- Search expenses:  $\kappa \sim \mathcal{H}(\cdot)$
- Carter cost:  $\mathcal{C}(\mathbf{z}, c)$  with  $c \sim \mathcal{G}(\cdot)$

## Timing:

- Customer draws  $\kappa$ , carters  $c$  (both *private*, *iid*)
- Customer chooses:

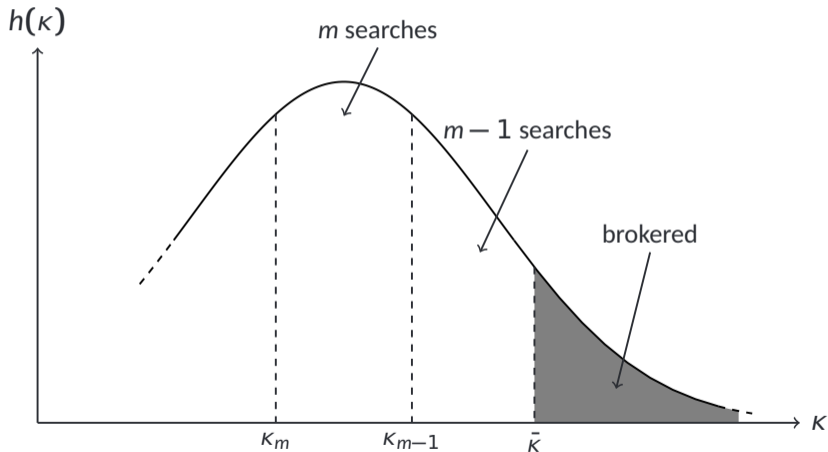
**Delegate:** Broker *RFP* amongst  $N_b$  carters, fee  $\Phi$

**Not Delegate:**  $\min_{m \in \{2, \dots, M\}} \left( q \cdot \mathbb{E}[p|m] + m \cdot \kappa \right)$

- Carters bid:

If delegated: Knowing their  $N_b$  competitors.

If not delegated: Under stochastic  $m \in \{2, \dots, M\}$



## Fraction of customers that make $m$ searches

- $w_m, m \in \{1, \dots, M\}$ .
- $w_m = \mathcal{H}(\kappa_{m-1}) - \mathcal{H}(\kappa_m)$ .

## Optimal strategies of carters:

- Broker market:  $\beta_b(\cdot)$  for broker  $b$  with  $N_b$  bidders.
- Search market:  $\beta_s(\cdot)$ .

**Carter objective function in the search market:**

$$\max_p (p - c) \cdot \left[ \sum_{m=1}^M w_m \cdot (1 - \mathcal{G}(\beta_s^{-1}(p)))^k \right]$$

**Price offer function:**

$$\beta_s(c) = \sum_{m=1}^{M-1} \left[ \frac{w_m \cdot (1 - \mathcal{G}(c))^{(m-1)}}{\sum_{k=1}^{M-1} w_k \cdot (1 - \mathcal{G}(c))^{(k-1)}} \cdot \left( c + \frac{1}{(1 - \mathcal{G}(c))^{(m-1)}} \int_c^{\bar{c}} (1 - \mathcal{G}(u))^{(m-1)} du \right) \right].$$

→ weighted average of symmetric first price procurement IPV auction bidding functions

## Observed objects:

- Prices + contract covariates in both market, number of bidders for brokers, broker fees.

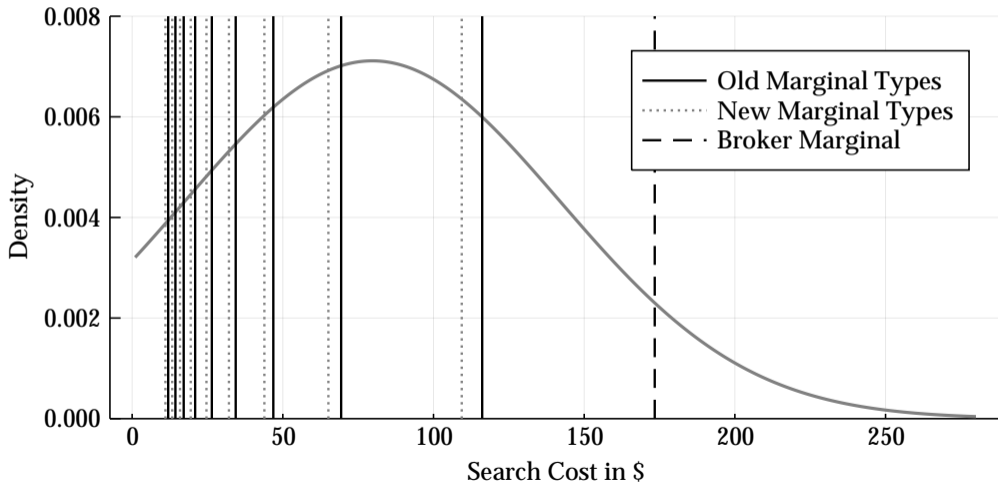
## Identification:

1. Carter cost in the broker market are identified by GPV (we will learn this later in semester).
2. Once  $\mathcal{G}(c)$  is known the search-weights are uniquely identified from prices in the broker market.
3. Can fit a (flexible) parametric function to the weights.

## Estimation:

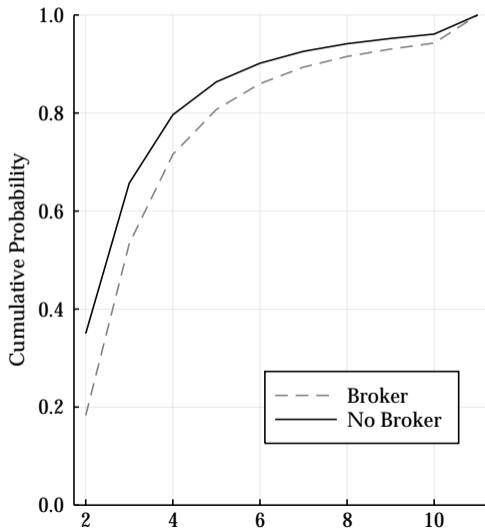
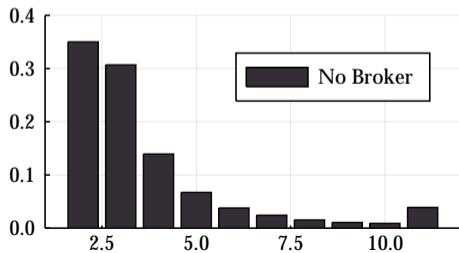
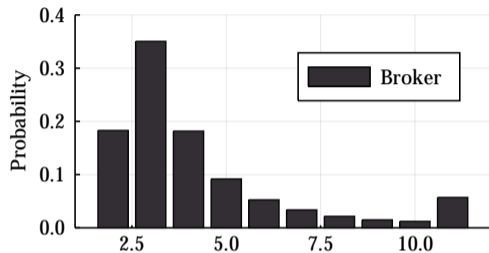
1. Guess any  $w_m$  and iteratively update bidding function/search strategy. Contraction mapping.
2. Once converged, simulate prices for broker market and search market.
3. Match moments of the price distribution.

Counterfactual: No Brokers

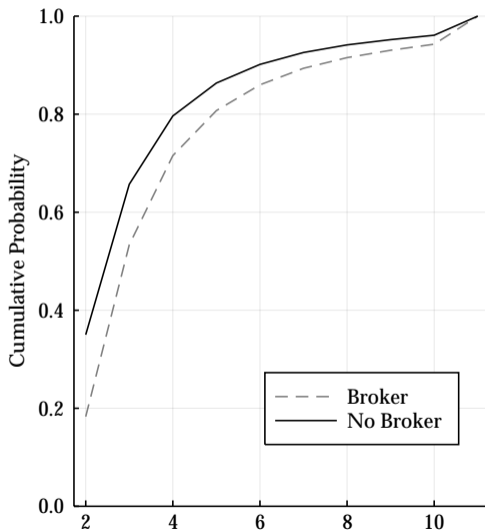
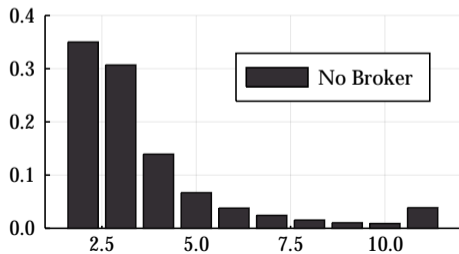
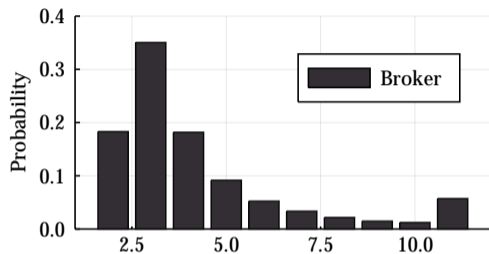




## No Broker Counterfactual, Example



## Consumer Welfare and Profits



**Weitzman (1979)**

General sequential search model that can handle ex-ante product heterogeneity.

**Dynamic program:**

$$W(\tilde{u}_i, \bar{S}_i) = \max \left\{ \tilde{u}_i, \max_{j \in \bar{S}_i} \left\{ -c_{ij} + F_j(\tilde{u}_i) W(\tilde{u}_i, \bar{S}_i - \{j\}) + \int_{\tilde{u}_i}^{\infty} W(u, \bar{S}_i - \{j\}) f_j(u) du \right\} \right\}$$

**Notation:**

- $\tilde{u}_i$ : consumer  $i$ 's highest utility sampled so far.
- $c_{ij}$ : consumer  $i$ 's cost of searching product  $j$ .
- $\bar{S}_i$ : set of firms consumer  $i$  has not searched yet.

Weitzman (1979) shows that the solution to this problem can be stated in terms of  $J$  static optimization problems. Specifically, for each product  $j$ , consumer  $i$  derives a reservation utility  $z_{ij}$ . This reservation utility  $z_{ij}$  equates the benefit and cost of searching product  $j$ , i.e.,

$$c_{ij} = \int_{z_{ij}}^{\infty} (u_{ij} - z_{ij}) f_j(u) du$$

Given this, the optimal strategy is:

1. Search companies in a decreasing order of their reservation utilities  $z_{ij}$  (“selection rule”).
2. Stop searching when the maximum utility among the searched firms is higher than the largest reservation utility among the not-yet-searched firms (“stopping rule”).
3. Purchase from the firm with the highest utility among those searched (“choice rule”).

Given this, the optimal strategy is:

1. Search companies in a decreasing order of their reservation utilities  $z_{ij}$  (“selection rule”).
2. Stop searching when the maximum utility among the searched firms is higher than the largest reservation utility among the not-yet-searched firms (“stopping rule”).
3. Purchase from the firm with the highest utility among those searched (“choice rule”).

**Example:**

Option	Reservation Utilities ( $z_{ij}$ )	Utilities ( $u_{ij}$ )
A	14	11
B	12	7
C	10	9

**Choi, Dai, and Kim (2018)**

**Setup:**

- Suppose we have  $u_{ij} = \delta_{ij} - p_j + \varepsilon_{ij}$ , where  $\delta_{ij}$  and  $p_j$  is known to the consumer but  $\varepsilon_{ij}$  is not.
- Consumers order products and search sequentially and their reservation utility for uncovering  $j$  is  $\varepsilon_{ij}^*$ .
- Consumers search for a good match.

**Problem:** many different search path that may have to be integrated out. With three firms there  $3! = 6$  paths. Especially for empirical purposes this is cumbersome. How to characterize demand for product  $j$ ?



In a **duopoly** product 1 gets purchased if:

1.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2^* - p_2$  (stop at 1)
2.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first),  $\delta_1 + \epsilon_1 - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 as well) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)
3.  $\delta_1 + \epsilon_1^* - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 first),  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2 - p_2$  (visit 1 as well), and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)

In a **duopoly** product 1 gets purchased if:

1.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2^* - p_2$  (stop at 1)
2.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first),  $\delta_1 + \epsilon_1 - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 as well) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)
3.  $\delta_1 + \epsilon_1^* - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 first),  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2 - p_2$  (visit 1 as well), and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)

**Note that:**

1. First condition can be simplified to:  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 > \delta_2 + \epsilon_2^* - p_2$
2. While the second and the third conditions together can be reduced to  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 \leq \delta_j + \epsilon_2^* - p_2$  and  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 > \delta_2 + \epsilon_2 - p_2$ .

In a **duopoly** product 1 gets purchased if:

1.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2^* - p_2$  (stop at 1)
2.  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2^* - p_2$  (visit 1 first),  $\delta_1 + \epsilon_1 - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 as well) and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)
3.  $\delta_1 + \epsilon_1^* - p_1 < \delta_2 + \epsilon_2^* - p_2$  (visit 2 first),  $\delta_1 + \epsilon_1^* - p_1 > \delta_2 + \epsilon_2 - p_2$  (visit 1 as well), and  $\delta_1 + \epsilon_1 - p_1 > \delta_2 + \epsilon_2 - p_2$  (prefer 1 to 2)

**Note that:**

1. First condition can be simplified to:  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 > \delta_2 + \epsilon_2^* - p_2$
2. While the second and the third conditions together can be reduced to  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 \leq \delta_j + \epsilon_2^* - p_2$  and  $\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 > \delta_2 + \epsilon_2 - p_2$ .

**Combining:**

$$\delta_1 + \min \{ \epsilon_1, \epsilon_1^* \} - p_1 > \delta_2 + \min \{ \epsilon_2, \epsilon_2^* \} - p_2$$

## Eventual Purchase Theorem

Let  $w_i \equiv \delta_i + \min \{ \epsilon_i, \epsilon_i^* \}$  for each  $i$ . Given  $G(\mathbf{p}, \delta, \epsilon)$ , the consumer purchases product  $i$  if and only if  $w_i - p_i > u_0$  and  $w_i - p_i > w_j - p_j$  for all  $j \neq i$ .

- Reframes Weitzman's optimal search solution as a simple discrete choice problem.
- Can be extended to unobservable prices as long as beliefs are, on average, correct.
- Markup rule for firms can be derived in terms of simple consumer demand functions.
- Applications to new car purchases: Moraga-Gonzalez et al. (2018): "*Consumer Search and Prices in the Automobile Market*".

**Distribution Function** of new random variable:

$$H_i(w_i) \equiv \int_{\underline{\epsilon}_i}^{\epsilon_i^*} F_i(w_i - \epsilon_i) dG_i(\epsilon_i) + \int_{\epsilon_i^*}^{\bar{\epsilon}_i} F_i(w_i - \epsilon_i^*) dG_i(\epsilon_i)$$

Then consumer's best alternative to product  $i$  is  $X_i \equiv \max\{u_0, \max_{j \neq i} W_j - p_j\}$  and the corresponding distribution function  $\tilde{H}_i(x_i) \equiv \Pr\{X_i \leq x_i\}$ .

**Aggregate demand:**

$$D_i(\mathbf{p}) = \int (1 - H_i(x_i + p_i)) d\tilde{H}_i(x_i)$$

**Optimal pricing:**

$$\frac{1}{p_i - c_i} = - \frac{dD_i(\mathbf{p})/dp_i}{D_i(\mathbf{p})}$$