Monopoly

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Single-Product Monopoly Pricing

Consider a monopoly seller of a single product.

- Monopolist faces inverse demand function: $P(x)$
- Total cost function: $C(x)$

Monopolist solves: $max_x \; P(x)x - C(x)$

- FOC:

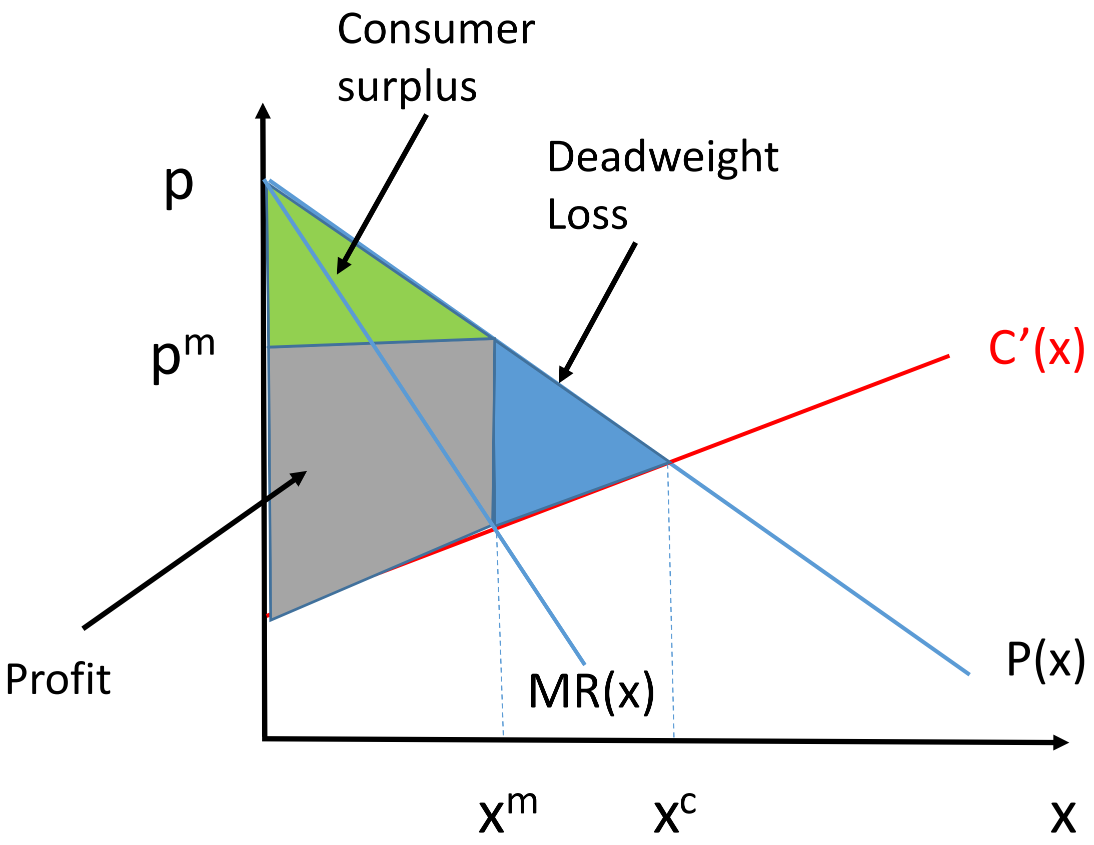
$$\underbrace{P(x) + P'(x)x}_{MR} - \underbrace{C'(x)}_{MC} = 0$$

- Can rewrite as: $\frac{P(x) - C'(x)}{P(x)} = -\frac{P'(x)x}{P(x)} = -\frac{1}{\epsilon}$

Lerner Index

Elasticity of demand at price *P(x)*

Single-Product Monopoly Pricing



Other possible costs of monopoly:

- Productive inefficiency
- Quality distortions
- Rent seeking
- Equity

Multiproduct Monopoly Pricing

Suppose now that the monopolist sells two goods. For simplicity assume constant marginal costs.

- Demands: $X_1(p_1, p_2)$ and $X_2(p_1, p_2)$
- Cost: $C(x_1, x_2) = c_1 x_1 + c_2 x_2$

Monopolist solves: $max_{p_1 p_2}\ (p_1 - c_1) X_1(p_1, p_2) + (p_2 - c_2) X_2(p_1, p_2)$

- FOC:

$$(p_i - c_i)\frac{\partial X_i}{\partial p_i} + X_i(p_1, p_2) + (p_{-i} - c_{-i})\frac{\partial X_{-i}}{\partial p_i} = 0;\ i = 1,2$$

or

$$(p_i - c_i) + \frac{X_i(p_1,p_2)}{\partial X_i/\partial p_i} + (p_{-i} - c_{-i}) \underbrace{\left[\frac{\partial X_{-i}/\partial p_i}{\partial X_i/\partial p_i}\right]}_{\text{Diversion ratio}} = 0;\ i = 1,2$$

This gives the markup formula $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} - \frac{p_{-i} - c_{-i}}{p_i\, dX_i/dp_i}\left[\frac{\partial X_{-i}(p_1,p_2)}{\partial p_i}\right]$

Multiproduct Monopoly Pricing

Markup: $\frac{p_i - c_i}{p_i} = -\frac{1}{\epsilon_{ii}} - \frac{p_{-i} - c_{-i}}{p_i \, dX_i/dp_i} \left[\frac{\partial X_{-i}(p_1, p_2)}{\partial p_i}\right]$

- **Substitutes:** $\left[\frac{\partial X_{-i}(p_1, p_2)}{\partial p_i}\right] > 0.$
  - Margins are both positive (Pf: Can only get negative margin if other margin is negative. Should stop selling both products.)
  - Margins exceed single product Lerner formula level.
    - Makes sense: If monopolist is selling two perfect substitutes it will not set prices close to marginal cost just because each individually has very high elasticity.

- **Complements:** $\left[\frac{\partial X_{-i}(p_1, p_2)}{\partial p_i}\right] < 0.$
  - A product's margin is below the single product Lerner formula level when other product has positive margin, e.g. newspapers, ink-jet printers, Android, Windows.
  - Optimal margins can easily be negative.

Monopoly and Product Quality

Suppose the monopolist also chooses the quality $s \in \mathbb{R}$ of its good.

Suppose it has a constant marginal cost $c(s)$ with $c'(s) > 0$, i.e. $C(q, s) = qc(s)$.

Suppose a unit mass of consumers with types $\theta \sim U[0,1]$ have unit demands:

Utility from one unit is $v(s;\theta) - p$ where $v_s(s;\theta) > 0$ and $v_\theta(s;\theta) > 0$.

If the firm sells *q* units, it will sell to consumers with $\theta \in [1-q, 1]$. Hence, the price at which it can sell *q* units of quality *s* is $v(s; 1\text{-}q)$.

The **monopolist** solves:

$$max_{q,s}\ q[v(s;1-q) - c(s)]$$

- FOC:

$$\frac{\partial v}{\partial s}(s^m; 1-q^m) = \frac{\partial c}{\partial s}(s^m)$$

The effect on the unit cost of increasing s is the benefit to the marginal consumer.

Monopoly and Product Quality

A **social planner** would solve

$$max_{q,s} \int_{1-q}^{1} \big(v(s;\theta) - c(s)\big)\, d\theta$$

- FOC for s:

$$\frac{1}{q}\int_{1-q}^{1} \frac{\partial v}{\partial s}(s;\theta)d\theta = \frac{\partial c}{\partial s}(s)$$

The effect on the per-unit cost of increasing s is the average marginal benefit across served consumers. Write s^{FB} for the solution to this problem.

1. s^m and s^{FB} differ for two reasons:
   - The monopolist focuses the marginal rather than the average consumer.
   - The monopolist serves fewer consumers.

   Often the two go in opposite directions: quality is lower than is optimal holding q fixed, but only high types are served so quality is relatively high.

2. In a homogeneous population model, monopolists choose the optimal quality.

Durable Goods Monopoly

In many settings, firms sell durable goods that provide services over many periods.

This is a potential advantage: you could move down the demand curve over time.

Consumers, however, can choose to buy/use used goods instead of buying from the monopolist – so, in a sense, the monopolist "competes with itself."

A simple model with commitment power:

- Two periods; consumers can buy in either period
- Good is durable
- Consumer per-period consumption value $\theta \in [0,1] \sim F(\theta)$
    - Utility is $2\theta - p_1$ if buy at t=1, $\theta - p_2$ if buy at t=2, and 0 if don't buy.
- Production cost $c \geq 0$ per unit
- Monopolist can *commit* to prices in the two periods: (p_1, p_2)

Observation: In this model the monopolist usually sells at the monopoly price at t=1 and commits not to sell at t=2.

Durable Goods Monopoly

We can think of the monopolist as choosing cutoffs $\hat{\theta}_1, \hat{\theta}_2$ such that

$\theta \in [0, \hat{\theta}_2]$ don't buy $\theta \in [\hat{\theta}_2, \hat{\theta}_1]$ buy at t=2 $\theta \in [\hat{\theta}_1, 1]$ buy at t=1

For $\hat{\theta}_2$ to be the 2nd period cutoff we'll need $p_2 = \hat{\theta}_2$. In the first period, a type $\hat{\theta}_1$ consumer wouldn't buy at $2\hat{\theta}_1$, better to wait until t=2 and get surplus.

$$2\hat{\theta}_1 - p_1 = \hat{\theta}_1 - p_2 \implies p_1 = \hat{\theta}_1 + p_2 = \hat{\theta}_1 + \hat{\theta}_2.$$

The monopolist's problem is then

$$\max_{\hat{\theta}_1 \geq \hat{\theta}_2} \Pi(\hat{\theta}_1, \hat{\theta}_2) = (\hat{\theta}_1 + \hat{\theta}_2 - c)[1 - F(\hat{\theta}_1)] + (\hat{\theta}_2 - c)[F(\hat{\theta}_1) - F(\hat{\theta}_2)]$$

$$= \hat{\theta}_1[1 - F(\hat{\theta}_1)] + (\hat{\theta}_2 - c)[1 - F(\hat{\theta}_2)]$$

Durable Goods Monopoly

The monopolist's problem is then

$$\max_{\hat{\theta}_2 \leq \hat{\theta}_1} \Pi(\hat{\theta}_1, \hat{\theta}_2) = (\hat{\theta}_1 + \hat{\theta}_2 - c)[1 - F(\hat{\theta}_1)] + (\hat{\theta}_2 - c)[F(\hat{\theta}_1) - F(\hat{\theta}_2)]$$

$$= \hat{\theta}_1[1 - F(\hat{\theta}_1)] + (\hat{\theta}_2 - c)[1 - F(\hat{\theta}_2)]$$

If the constraint is not binding, then this is just two separate monopoly pricing problems. The first is monopoly pricing with cost 0. The second is monopoly pricing with cost c. Hence we would have $\hat{\theta}_1 \leq \hat{\theta}_2$. This implies that $\hat{\theta}_2 = \hat{\theta}_1$ and the monopolist only sells at t=1.

(Pf. Note that $(\theta - c)[1 - F(\theta)]$ has increasing differences in θ and c. Recall from 14.121 that this implies that the maximizing θ is increasing in c.)

An alternate intuition is just to imagine you produce goods and rent them at t=1, then take them back and think about producing more to rent at t=2.

Durable Goods Monopoly

Example

- Suppose $\theta \sim U(0,1)$ so that $F(\theta) = \theta$, and $c = 0$. This is monopoly pricing with D(p) = 1 – p.
- Monopolist solves $max_{\hat{\theta}} \;\; 2\hat{\theta}(1 - \hat{\theta}) \;\; \rightarrow \hat{\theta} = \frac{1}{2}$
- Implemented by setting $p_1 = 1$ and $p_2 \geq 1/2$.

Where could price drops come from?

- Different preferences: some people want to be early adopters
- Cost reduction/capacity increase from learning-by-doing in production
- Commitment problem

Durable Goods Monopoly: No Commitment

Example

- Suppose $\theta \sim U(0,1)$ so that $F(\theta) = \theta$, and $c = 0$. This is monopoly pricing with D(p) = 1 – p.
- Assume now that the monopolist chooses p_1 at t=1 and p_2 at t=2 and **cannot commit**. Price p_2 must be optimal at t=2 given the consumers in the market.

If the firm has already sold to consumers with $\theta \in [\hat{\theta}_1, 1]$ at t=1, then the optimal p_2 will satisfy

$$p_2 = Argmax_p\, p\left(\hat{\theta}_1 - p\right) \Rightarrow p_2 = \hat{\theta}_1/2.$$

Consumers at t=1 will anticipate this price cut, so the type $\hat{\theta}_1$ consumer can only be indifferent if $p_1 = \frac{\hat{\theta}_1}{2} + \hat{\theta}_1$. Hence,

$$\pi(\theta_1) = \frac{3}{2}\theta_1(1 - \theta_1) + \theta_1^2/4.$$

This is maximized for $\theta_1 = \frac{3}{5}$, which gives $p_1 = \frac{9}{10}, q_1 = \frac{2}{5}, p_2 = \frac{3}{10}, q_2 = \frac{3}{10}$

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Note:

1. We have a decreasing price sequence $p_2 < \frac{1}{2} p_1$.
2. Profits are lower than with commitment: $\frac{9}{10} \cdot \frac{2}{5} + \frac{3}{10} \cdot \frac{3}{10} = \frac{45}{100} < \frac{1}{2}$.

These are both very general properties.

Coase Conjecture

Consider a more extreme commitment problem. Suppose the monopolist has no commitment power, can set prices at t = 0, $\Delta T, 2\Delta T, 3\Delta T$, …, and consumers are fully rational and get gross utility θe^{-rt} if they purchase at t.

Coase Conjecture: *Under some conditions the monopolist's profits go to zero as* $\Delta T \rightarrow 0$.

Intuition:

- Suppose that $\theta \sim U(0,1)$ and $c \in (0,1)$
- Suppose that in the limiting time path of prices is not an immediate drop to zero.
- Hence, prices must be dropping linearly in T at some point and quantities must also be proportional to ΔT.
- Suppose the monopolist jumps ahead in its price sequence and charges $p_{t+\Delta T}$ instead of p_t.
- The **gain** from having all sales occur ΔT earlier is first order in ΔT.
- The **loss** from earning less on the sales at time t is second order in ΔT the price difference and quantity on which you get the lower price are both of order ΔT. Hence, it is better to jump ahead and cut prices faster.

The proof of the Coase conjecture is somewhat involved and the extreme case is probably not so practically relevant.

Whether the result is even true depends on whether the lower-bound of the value distribution is above or below c.

Coase Conjecture

- Durable goods producers don’t seem to earn zero. *Why?*

  - Delays/costs of changing prices
  - Reputation
  - Inflows of new high willingness-to-pay customers (Fuchs-Skrzypacz *AER* 2010)
  - Outside options (Board-Pycia *AER* 2014)
  - Consumers not all rational and forward-looking
  - Strategic actions:
    - Rent rather than sell (but can run into moral hazard or antitrust problems)
    - Most favored customer contracts
    - Destroy/limit ability to produce

Monopoly Information Design

We see a lot of variation in information firms provide to consumers:

- Laptops, mattresses, wholesale used cars

Many consumers now purchase through intermediaries which provide information. Efforts to help consumers find products they will like could be offset by firms raising prices.

- Amazon, Yelp

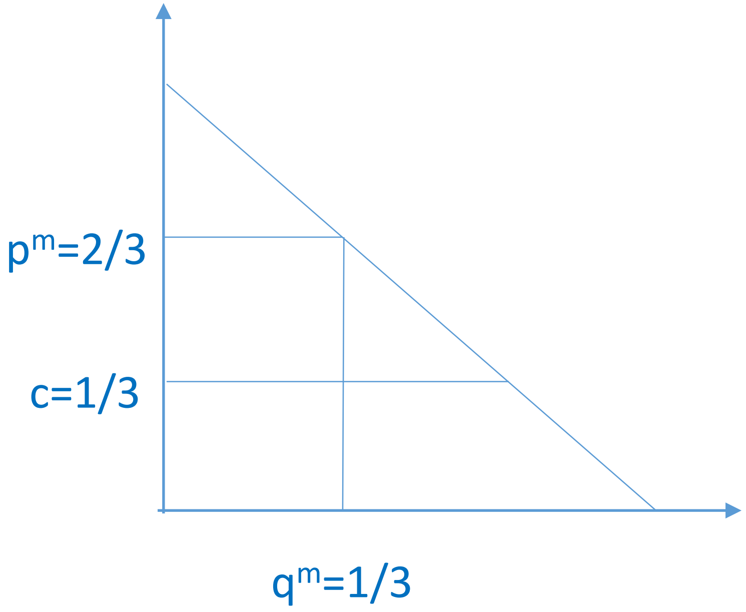
Both the platform application and developments in theory have spurred the growth of interest in "information design."

Baseline Example for Illustrations:

- Fully informed consumers would have values $v \sim U[0, 1]$
- Monopolist has constant marginal cost of 1/3.

Monopoly price is $p^m = \underset{p}{\text{argmax}}(p - {}^1/_3)(1 - p) = {}^2/_3$

- Profit $\pi = \frac{1}{3} \cdot \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{9}$. CS $= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{18}$. DWL $= \frac{1}{18}$.



Monopoly Information Design

Limiting Information Can Increase Profits

- Fully informed consumers would have values $v \sim U[0,1]$
- Monopolist has constant marginal cost of 1/3.

Full information monopoly had $\pi = \frac{1}{3} \cdot \left(\frac{2}{3} - \frac{1}{3}\right) = \frac{1}{9}$. CS $= \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{3} = \frac{1}{18}$. DWL = $\frac{1}{18}$.

Suppose consumers learn their valuations v by observing signals about the product's attributes. If none are available, rational consumers would all have $E(v|s) = \frac{1}{2}$.

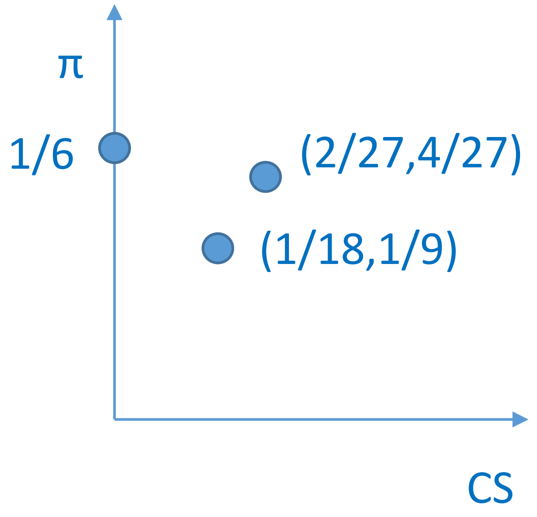
Hence, the monopolist can sell to all consumers at $p^m = \frac{1}{2}$.

- Profit $\pi = 1 \cdot \left(\frac{1}{2} - \frac{1}{3}\right) = \frac{1}{6}$. CS =0. Loss from inefficient purchases = $\frac{1}{18}$.

Suppressing information has transferred surplus from consumers to the monopolist.

Monopoly Information Design

Partial Information Can Increase Consumer Surplus



Example:

- Fully informed would have consumers have values $v \sim U[0,1]$
- Monopolist has constant marginal cost of 1/3.

Suppose a platform provided consumers with partial information.

$$s = \begin{cases} \text{L if v} \in \left[0, {}^{1}/_{3}\right] & E(v|L) = {}^{1}/_{6} \\ \text{M if v} \in \left[{}^{1}/_{3}, {}^{7}/_{9}\right] & E(v|M) = {}^{5}/_{9} \\ \text{H if v} \in \left[{}^{7}/_{9}, 1\right] & E(v|H) = {}^{8}/_{9} \end{cases}$$

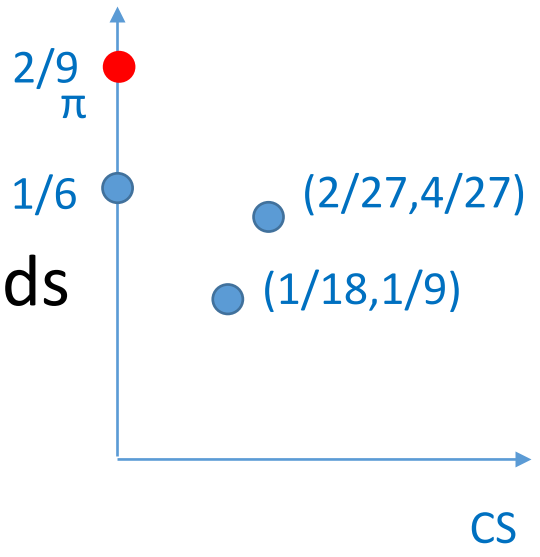
$\pi\left(\frac{5}{9}\right) = \frac{6}{9} \cdot \left(\frac{5}{9} - \frac{1}{3}\right) = \frac{12}{81}$ and $\pi\left(\frac{8}{9}\right) = \frac{2}{9} \cdot \left(\frac{8}{9} - \frac{1}{3}\right) = \frac{10}{81}$, so the monopolist chooses $p^m = \frac{5}{9}$.

- Profit $\pi = \frac{4}{27}$. CS $= \frac{2}{9} \cdot \frac{3}{9} = \frac{2}{27}$. DWL= 0.

The signal structure achieves full efficiency and limits the monopolist's rents.

Monopoly Information Design

Seller Optimal Information Structures



The **seller optimal** information structure with unit demands is obvious:

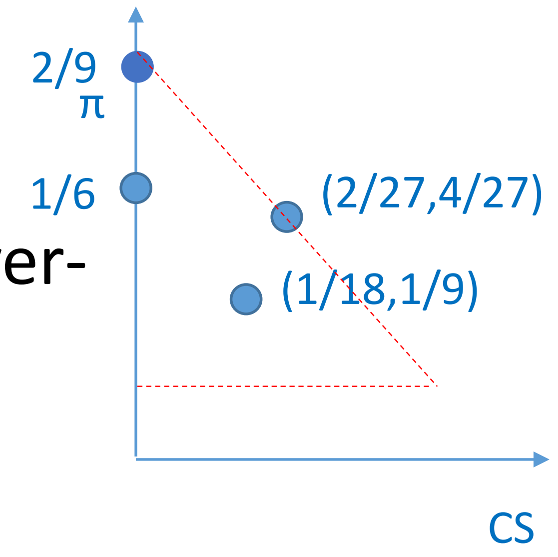
$$s = \begin{cases} \text{L if } v < c \\ \text{H if } v > c \end{cases}$$

The monopolist maximizes surplus and extracts all of it by setting $p^m = E(v|v > c)$.

In the example we would have $p^m = \frac{2}{3}$. $\pi = \frac{2}{9}$. CS = 0. DWL= 0.

Monopoly Information Design

Buyer Optimal Information Structures



Roesler and Szentes (*AER* 2017) characterize both the buyer-optimal information policy and the full set of possible surplus divisions for the problem with unit demands with $v \sim F$ on [0, 1] and c=0. Write $\mu = E_F(v)$.

Suppose the platform can choose any joint distribution on (*v, s*).

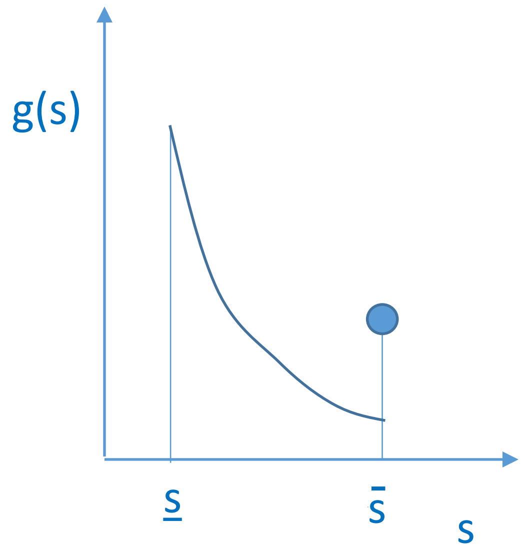
WLOG we can assume $s = E(v|s)$. Consumer decisions will depend only on $E(v|s)$, so we might as well give them this number.

Write G for the CDF of s. Note that $v = s + \varepsilon$, with $E(\varepsilon|s) = 0$, so F is a mean-preserving spread of G. This implies that $E_G(s) = \mu$.

The monopolist's price depends on the joint distribution of (*v, s*) only through G. So profits and consumer surplus only depend on G. We focus on choosing G.

Monopoly Information Design

Roesler-Szentes (2017)



Step 1: Choose best G from a limited class.

For parameters $\underline{s}, \bar{s}$ define $G_{\underline{s},\bar{s}}(s) = \begin{cases} 0 & if\ s < \underline{s} \\ 1 - \frac{\underline{s}}{s} & if\ s \in [\underline{s}, \bar{s}) \\ 1 & if\ s \geq \underline{s} \end{cases}$

If the platform chooses $G_{\underline{s},\bar{s}}(s)$, then $\pi(p) = p\left(1 - G_{\underline{s},\bar{s}}(p - dp)\right) = p\left(\underline{s}/p\right) = \underline{s}$ is constant for all $p \in [\underline{s}, \bar{s}]$, so $\underline{s}$ is a profit-maximizing price.

Trade always occurs at this price, so

Consumer Surplus = Maximized Social Surplus – Profit = Max Social Surplus – $\underline{s}$.

Observation 1: Within this class of G's the consumer optimal solution is clear: we want to choose the smallest possible $\underline{s}$ subject to the constraint that F must be a mean preserving spread of $G_{\underline{s},\bar{s}}$.

For any $\underline{s} < \mu$ we can find an $\bar{s}$ for which $E_{G_{\underline{s},\bar{s}}}(s) = \mu$. But the required $\bar{s}$ might be bigger than 1. So there is a strictly positive lower bound on $\underline{s}$.

Monopoly Information Design

Roesler-Szentes (2017)

Step 2: Show that no other distribution G can give greater consumer surplus than is possible with some $G_{\underline{s},\overline{s}}(s)$.

An outline of this argument is:

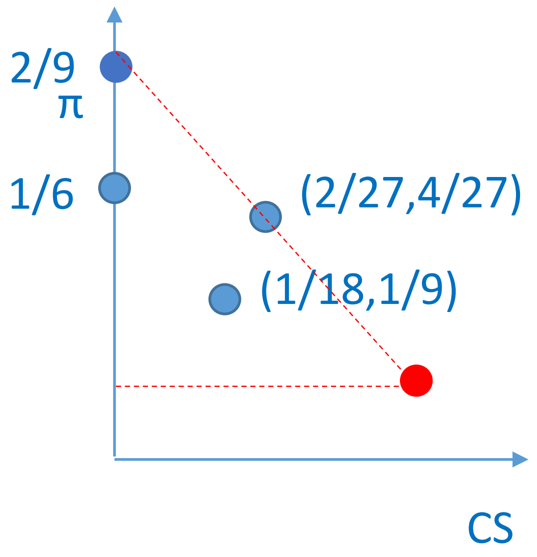
1. Suppose G is a valid choice. Then F is a mean preserving spread of G. Write π for the profit given this distribution.
2. Consider the distribution $G_{\pi,\overline{s}}$ that has $E_{G_{\pi,\overline{s}}}(s) = \mu$. It gives the same profit as G. It also maximizes social surplus. So it gives at least as much consumer surplus as G.
3. To complete the proof it remains only to show that $G_{\pi.\overline{s}}$ is a valid choice. To show this, it suffices to show that F is a mean-preserving spread of $G_{\pi.\overline{s}}$. By transitivity of the mean-preserving-spread property it suffices for this to show that G is a mean-preserving spread of $G_{\pi.\overline{s}}$.

   To see this, recall that (given the identical means) G is a mean-preserving spread of $G_{\pi.\overline{s}}$ if the CDF of $G_{\pi.\overline{s}}$ crosses the CDF of G once from below as s goes from 0 to 1.

   $G_{\pi.\overline{s}}$ was chosen so that $G_{\pi.\overline{s}}(\pi) = 0$. Obviously, this makes the CDF as small as possible for s < π. For all p ∈ $[\pi, \overline{s})$ we have $p\big(1 - G(p)\big) \leq \pi = p(1 - G_{\pi.\overline{s}}(p))$, so $G(s) \geq G_{\pi.\overline{s}}(s)$ also holds for $s \in [\pi, \overline{s})$. Above $\overline{s}$ the CDFs reverse, $G(s) \leq G_{\pi.\overline{s}}(s)$ because $G_{\pi.\overline{s}}(s)$=1. So they do cross once as desired. (The crossing is at $\overline{s}$.)

Monopoly Information Design

Roesler-Szentes (2017)



Recap of ideas:

- To maximize consumer surplus we want to maximize social surplus and minimize profits.
- A good way to think about information design is to think about choosing the distribution of the consumer's posterior.
- To minimize profit we choose a distribution with just a few high-value consumers and a steep peak of moderate-value consumers that keeps the monopolist just indifferent to raising its price.

Full characterization:

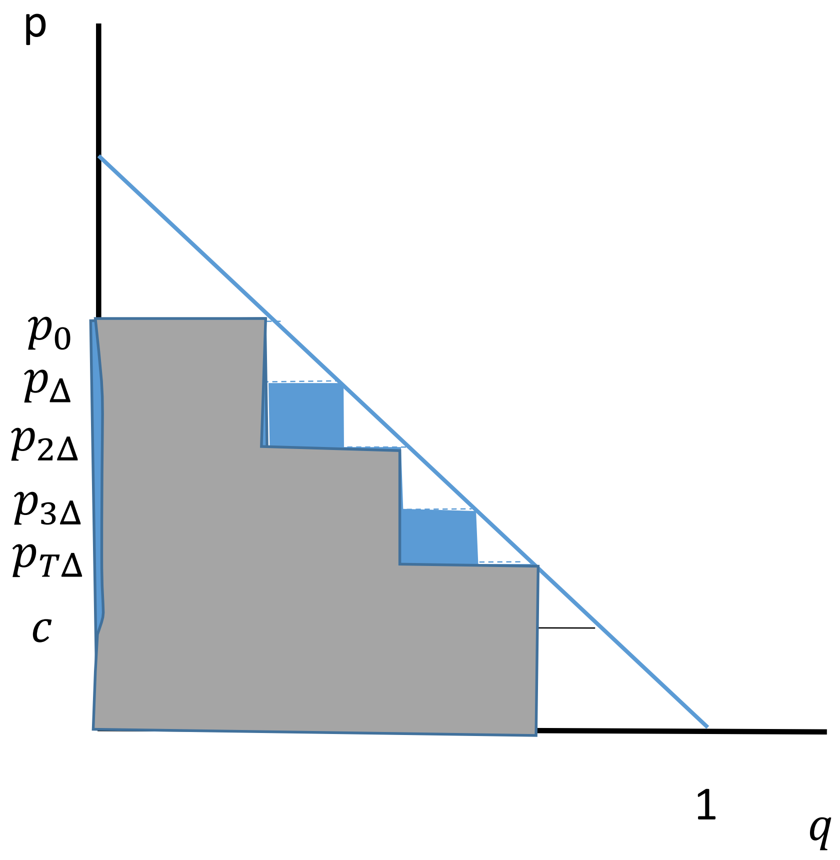
The full set of possible profit/consumer surplus divisions is the right triangle below and to the left of the profit-maximizing and consumer-surplus maximizing points.

On Monday I’ll discuss two empirical papers on monopoly pricing:

- Chevalier-Goolsbee
- Gentzkow-Shapiro

I hope you enjoy the course!

Coase Conjecture



Consider an equilibrium in which consumers adopt reservation price purchase rules: buy in period t if price is below some level (which depends on the consumer's type).

Suppose in equilibrium monopolist charges price p_0 at time 0 and when $\Delta \approx 0$ price falls to $\bar{p}$ at time T. If consumers are willing to buy at time 0, we must have T > 0.

If Δ is small, better to skip steps and do $p_0, p_{2\Delta}, p_{4\Delta}, \ldots$
Lose a little bit of revenue but cut time in half to get it.