14.271: Industrial Organization I

Introduction to Empirical Models of Auctions

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Auctions

Roadmap

1. Empirical techniques for Auctions.
   - Laffont, Ossard, and Vuong (1995)
   - Non-parametric identification.
   - Guerre, Perrigne and Vuong (2001)
2. Athey, Levin, and Seira (2011)
3. Kong (2019)

Auctions

Relevance

- Auctions are a type of monopoly market: auctioneer has monopoly power.
- $\approx$ 10% of GDP contracted through auctions.
- Often direct policy implications. Government organizes purchases and allocation through auctions.
- The study of collusion in auctions has been quite fruitful.

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Fertile ground for structural work

- Well defined rules and strategy sets. Players' strategies are often observed.
- In many cases participants are sophisticated - a setting where game theory is likely to deliver good predictions.
- Want to recover the distribution of bidder valuations/costs.

Single Unit Auctions

The symmetric IPV model — recap

Model Elements:

1. n bidders, indexed by i
2. Bidder i's value is denoted V_i
3. Privately observed signal S_i summarizes bidder i's information
4. Bid is denoted B_i
5. Convention: Upper-case letters refer to random variables, and lower-case letters refer to specific values

Statistical Dependence:

– Independent signals: S is distributed $F_S = \prod_{i=1}^{n} F_{S_i}$

Private Values:

– Private values: $E\left(V_i | S_i, S_{-i}\right) = E\left(V_i | S_i\right)$

The symmetric IPV model — FPA

Under symmetric strategies $\beta(v)$, bidder maximizes:

$$\max_{b_i}(v_i - b_i) \cdot F(\beta^{-1}(b_i))^{n-1}$$

First order condition:

$$(v_i - \beta(v_i)) \cdot (n-1) \cdot F(v_i)^{n-2} \cdot f(v_i)\frac{1}{\beta'(v_i)} - F(v_i)^{n-1} = 0,$$

with boundary condition $\beta(\underline{v}) = \underline{v}$.

Differential equation, $b_i = \beta(v_i)$, with solution (Riley and Samuelson 1981):

$$\beta(v_i, n, F) = v_i - \frac{\int_{\underline{v}}^{v_i} F(x)^{n-1}dx}{F(v_i)^{n-1}}$$

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

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with boundary condition $\beta(\underline{v}) = \underline{v}$.

Reserve price solution:

$$b^i = \beta\left(v^i, n, p^0, F\right) = \begin{cases} v^i - \frac{\int_{p^0}^{v^i} F(x)^{n-1} dx}{F(v^i)^{n-1}} & \text{if } v^i > p^0 \\ 0 & \text{otherwise} \end{cases}$$

Laffont, Ossard, and Vuong (1995)

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

Setup:

- n symmetric bidders with IPV valuations.
- Descending (Dutch) auction with reserve price p_0.

Estimation:

- Valuations log-normal: $v_i \sim F(\cdot|\theta)$.
- Goal is to estimate θ based on observed outcomes.
- Data from 81 auctions, 11 bidders.

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The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

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We would want to evaluate:

$$L(\mathbf{b};\theta) = \prod f\left(\beta^{-1}(b_i;\theta) \mid \theta\right)$$

with

$$b^i = \mathbf{1}\{v^i > p^0\} \cdot \left(v^i - \frac{\int_{p^0}^{v^i} F(x \mid \theta)^{n-1} dx}{F\left(v^i \mid \theta\right)^{n-1}}\right) + \mathbf{1}\{v^i \leqslant p^0\} \cdot 0$$

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Issues with this approach:

- Computationally costly.
- Only the winning bid is observed because of Dutch auction format.
- Support of the winning bid is $[p^o, E\left(\max\left(X, p^o\right)\right)]$, where X is the largest order statistic in $n-1$ draws from F. This violates regularity conditions of ML.

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

Another idea: match winning bid to simulated expectation. (McFadden (1989) and Pakes and Pollard (1989)).

$$
\begin{aligned}
E_{v_{(n)} > p^0}\left(b^w\right) &= \int_{p^0}^{\infty} \beta\left(v_{(n)}, n, p^0, F\right) n \cdot F(v \mid \theta)^{n-1} f(v \mid \theta) dv \\
&= n \int_{p^0}^{\infty} \left(v \cdot F(v \mid \theta)^{n-1} - \int_{p^0}^{\infty} F(x \mid \theta)^{n-1} dx\right) f(v \mid \theta) dv.
\end{aligned}
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Still cumbersome:

- Get $v_s = F^{-1}\left(u_s \mid \theta\right)$ where $u_1, \ldots, u_S$ i.i.d. from the $U[0,1]$
- Compute $\tilde{V}_s = v_s \cdot F\left(v_s \mid \theta\right)^{n-1} - \int_{p^0}^{v_s} F(x \mid \theta)^{n-1} dx$ then average: $\frac{1}{S} \sum_s \tilde{V}_s$

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

One more clever idea: use the Revenue Equivalence Theorem.

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Assume: risk-neutral IPV setting with atom-less signal support.

Revenue Equivalence Theorem (Vickrey 1961): Any auction mechanism which is (i) efficient in awarding the object to the bidder with the highest signal; and (ii) leaves any bidder with the lowest signal with zero surplus yields the same expected revenue for the seller, and results in a bidder with signal s making the same expected payment.

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Using RE of first and second-price auction, **we know that:**

$$\mathbb{E}_{n:n}[b^*\left(v, p^0\right)] = \mathbb{E}_{n-1:n}\left[\max\left(v, p^0\right)\right]$$

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

For each parameter guess θ and each auction L.

- Draw $v_1^s, ..., v_I^s$, simulated valuations from $F(\cdot|\theta)$
- Sort draws in ascending order.
- Set $b_l^{w,s}$ as maximum of second-highest valuation and p_0.
- Approximate revenue, $\mathbb{E}(b_l^w \mid \theta) = \frac{1}{S}\sum_s b_l^{w,s}$
- Estimate θ by NLLS:

$$\min_\theta \frac{1}{L}\sum_{l=1}^{L}\left(b_l^w - \mathbb{E}(b_l^w \mid \theta)\right)^2.$$

The symmetric IPV model — Laffont, Ossard, and Vuong (1995)

Comments:

- Not an approach that is widely used.
- RET does not apply to many cases of interest, for example auctions with entry or asymmetric bidders.
- Reliance on functional forms for the distribution of valuations.

$\rightarrow$ Much of the modern auction literature does not restrict distribution of valuations to one parametric family (i.e. normal, exponential, etc). Literature places strong emphasis on formal identification results.

Digression: Nonparametric Identification

"A model is identified if, given the implications of equilibrium behavior in a particular auction game, the joint distribution of bidders' utilities and signals is uniquely determined by the joint distribution of observables." (Athey and Haile, 2002)

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- Let $m^\star$ be the true vector of functions and distributions.
- Let $P(m)$ denote the joint distribution of observable variables under the assumption that the data is generated under m.

Definition of Identification

$m^\star$ is identified in M if and only if for all $m \in M$, $m \neq m^\star$, $P(m) \neq P(m^\star)$

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- If M is a subset of a finite dimensional space, we say that the model M is **parametric**.
- If M is not a subset of a finite dimensional space, we say that M is:
  - **Semiparametric**, if some of the functions, distributions lie inside a finite dimensional space.
  - **Nonparametric**, if none of the functions, distributions lie inside a finite dimensional space.
- Notice, that we are not worried about finite sample variation. Identification asks: if we had infinite data, can we recover the objects of interest?

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Primitive of interest: distribution of values F. We observe the distribution of bids. Any ideas how to approach this?

- $b_i = \beta_i(v_i) = v_i$ is a (weakly) dominant strategy $\rightarrow$ bid your valuation
- If all bids are observed:
  - we have draws from $F \rightarrow$ **done**

What if only transaction prices are observed?

The symmetric IPV model — second price auction and order statistics

Transaction price is the second-highest valuation in a second-price sealed-bid or English auction

- Let G_W be the distribution of the transaction price (data):

$$G_W(v) = F_{n-1:n}(v)$$

  where the number of bidders n is known.

Relations of order statistics:

- Distribution of i-th order statistic from n draws,

$$F_{i:n}(v) = \frac{n!}{(n-i)!(i-1)!} \int_0^{F(v)} t^{i-1}(1-t)^{n-i} \mathrm{d}t,$$

  is increasing in $F(v)$, hence invertible. See Arnold, Balakrishnan, and Nagaraja (1992).

Guerre, Perrigne and Vuong (2001)

The IPV model — Guerre, Perrigne and Vuong (2001)

Main idea: re-arrange necessary first-order conditions as a functions of objects that are directly recoverable in the data.

- Transform FOC as a function of distribution of bids (G), instead of valuations (F).
- Distribution can be recovered non-parametrically.
- In practice, works best if all bids are observed, but still identified if only winning bid is observed (Athey and Haile, 2002).

The IPV model — Guerre, Perrigne and Vuong (2001)

Steps:

- Remember that before integrating, FOC yields:

$$\beta'(v_i) = (v_i - \beta(v_i)) \cdot (n-1) \cdot \frac{f(v_i)}{F(v_i)}$$

The IPV model — Guerre, Perrigne and Vuong (2001)

Steps:

- Remember that before integrating, FOC yields:

$$\beta'(v_i) = (v_i - \beta(v_i)) \cdot (n-1) \cdot \frac{f(v_i)}{F(v_i)}$$

- **Key:** due to monotonicity $G(\beta(v_i)) = G(b_i) = F(v_i)$, and hence:

$$g(b_i) = f(v_i) \cdot \beta'(v_i)^{-1}$$

- Use expression to substitute equilibrium strategy:

$$v_i = b_i + \frac{G(b_i)}{(n-1) \cdot g(b_i)}$$

Guerre, Perrigne and Vuong (2001)

Alternative Derivation: Consider bidder i's response given that competitors play the eq. strategy β:

- Strategy β results in opponent bid distribution $G(b) = P(\beta(v) \leqslant b)$
- Best response of a v_i-type bidder:

$$\max_b (v_i - b)[G(b)]^{n-1}$$

- First order condition:

$$-[G(b_i^*)]^{n-1} + (v_i - b_i^*)(n-1)[G(b_i^*)]^{I-2} g(b_i^*) = 0$$

Re-arranging, we get bidder i's value as a function of her bid

$$v_i = b_i^* + \frac{G(b_i^*)}{(n-1) \cdot g(b_i^*)}$$

where $G(b_i)$ and $g(b_i)$ are observed.

Guerre, Perrigne and Vuong (2001) — estimation steps

Approximate $\hat{G}(b)$ and $\hat{g}(b)$ from bidding data, e.g.:

$$\hat{G}(b) = \frac{1}{T \cdot n} \sum_t \sum_i 1\{b_{ti} \leqslant b\}.$$

$$\hat{g}(b) = \frac{1}{T \cdot n} \sum_t \sum_i \frac{1}{h} \mathcal{K}\left(\frac{b - b_{it}}{h}\right),$$

Use estimated of density and CDF to recover valuations:

$$\hat{v}_i = b_i + \frac{\hat{G}(b_i)}{(n-1) \cdot \hat{g}(b)}$$

Finally, use kernel again to estimate $f(v)$. Standard errors on outcomes are in practice Bootstrapped.

Guerre, Perrigne and Vuong (2001) — discussion

Comments:

- Computational/estimation simplicity.
- Alternative to solving for equilibria and matching the data.
- Does not rely on functional form restrictions.
- GPV has been very influential in the way auction data is analyzed. Most of the auction literature is non-parametric. Some examples:
  - Multi-unit auctions (Hortacsu, 2002; Wolak, 2003).
  - Dynamics (Jofre-Bonet and Pesendorfer, 2003).
  - Test RET (Athey, Levin and Seira, 2008).
  - Estimating damages of bidding rings (Asker 2010).
- Probably led to too many IPV applications. Testing for common values (Haile, Hong, Shum, 2003).

Athey, Levin, and Seira (2011)

Athey, Levin, and Seira (2011)

Question: should we use a sealed bid or open auction format?

- Timber auctions, $100 billion industry.
- 30% of land publicly owned
- Government auctions from Idaho-Montana border and California.
- U.S. Forest Service uses both open and sealed bidding (sometimes randomly).
- Open auctions are believed to foster collusion.

Image by the Bureau of Land Management Oregon and Washington. CC BY.

Athey, Levin, and Seira (2011) — model setup and predictions

Setup:

- They consider open and sealed bid auctions $\tau \in \{o, s\}$.
- Two types of bidders: Loggers (L) and Mills (M).
- Loggers take iid draws from $F_L(\cdot)$, mills from $F_M(\cdot)$.
- Mills are assumed to be strong bidders, their valuations stochastically dominate those of loggers'.
- Bidders learn their valuation after paying the entry cost.
- Search for equilibrium in type symmetric entry and bidding, $(p_t, \beta_t(\cdot; n))$ strategies.
- Assume that for all $n_L, n_M, \pi^s_M(n_L, n_M + 1) > \pi^s_L(n_L, n_M)$.
- In a collusive eq. only loggers collude.

Image is in the public domain.

Athey, Levin, and Seira (2011) — model setup and predictions

Predictions:

- Unique type symmetric entry eq. with either $p_L = 0$ or $p_M = 1$.
- In the open auction (relative to sealed) (i) loggers are less likely to enter; (ii) mills are more likely to enter; (iii) it is less likely a logger will win.
- Maskin Riley (2000), sealed bidding favors weaker bidders.

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- Maskin Riley (2000), sealed bidding favors weaker bidders.
- For the open auction for any non-collusive type-symmetric equilibrium there is a collusive equilibrium (only mills collude) where (i) loggers are less likely to enter, (ii) mills are more likely to enter, (iii) it is less likely a logger will win.

Athey, Levin, and Seira (2011) — effect of auction format

$Y = \alpha \cdot SEALED + \cdot X\beta + N\gamma + \varepsilon$ and matching estimate

EFFECT OF AUCTION METHOD ON SALE OUTCOMES

Dependent variable:	(1) Ln(logger entry)	(2) Ln(mill entry)	(3) Loggers/entrants	(4) Logger wins	(5) Ln(price)	(6) Ln(price)[a]
A: Northern sales (N = 1071 sales)						
Regression with no interactions between sealed and covariates						
Sealed bid effect	0.089	–0.014	0.056	0.039	0.094	0.055
	(0.036)	(0.030)	(0.016)	(0.026)	(0.038)	(0.032)
Regression with interactions between sealed and covariates						
Sealed bid effect on sample	0.097	–0.010	0.058	0.038	0.099	0.060
	(0.036)	(0.031)	(0.016)	(0.027)	(0.039)	(0.033)
Matching estimate[b]						
Sealed bid effect on sample	0.100	0.018	0.052	0.034	0.118	0.091
	(0.048)	(0.053)	(0.029)	(0.039)	(0.064)	(0.055)
B: California sales (N = 707 sales)						
Regression with no interactions between sealed and covariates						
Sealed bid effect	0.101	–0.026	0.058	0.036	0.027	–0.026
	(0.045)	(0.038)	(0.020)	(0.036)	(0.051)	(0.040)
Regression with interactions between sealed and covariates						
Sealed bid effect on sample	0.099	–0.022	0.056	0.035	0.026	–0.037
	(0.044)	(0.038)	(0.020)	(0.035)	(0.050)	(0.039)
Matching estimate[b]						
Sealed bid effect on sample	0.106	–0.123	0.097	0.107	–0.038	0.005
	(0.062)	(0.067)	(0.034)	(0.051)	(0.127)	(0.087)

Athey, Levin, and Seira (2011) — structural estimation

- Estimation is based on only sealed bid auctions via GPV.
- Parametrize the bid distribution as Weibull, test with Andrews (1997).
- Account for unobserved auction-specific heterogeneity, (Krasnokutskaya (2011), Li and Vuong (1998)).
- Estimate entry cost from optimal entry behavior.
- Generate predictions for both open and sealed bid auctions.

Athey, Levin, and Seira (2011) — effect of auction format

- Model performs very well at predicting sealed auction entry and bidding behavior.
- Logger entry in open auction also well predicted.
- For California the competitive model also predicts prices and revenues in the open auction well.
- For open Northern auctions both the competitive and the collusive model are rejected.

ACTUAL OUTCOMES VERSUS OUTCOMES PREDICTED BY MODEL

	N	(1) Actual	(2) Predicted (bidding only)		(3) Predicted (bidding + entry)	
A: Northern sales						
Sealed bid sales						
Avg. bid	1,492	59.6	58.2	(1.4)	57.4	(1.3)
Avg. logger bid	1,096	50.8	48.7	(1.4)	47.4	(1.4)
Avg. mill bid	396	83.8	84.7	(2.7)	85.2	(2.7)
Avg. sale price ($/mbf)	339	69.4	69.9	(1.4)	70.4	(1.6)
Avg. revenue ($000s)	339	111.4	108.1	(4)	109.9	(4.2)
% sales won by loggers	339	68.1	68.0	(0.90)	65.0	(0.01)
Avg. logger entry	339	3.23			3.23	(0.09)
						(0.1)
Open auction sales						
Avg. sale price (competition)	732	63.3	67.9	(1.8)	67.8	(2.1)
Avg. sale price (collusion)	732	63.3	44.2	(1.3)	44.1	(2.2)
Avg. revenue (competition)	732	144.7	152.7	(6.8)	154.8	(7.9)
Avg. revenue (collusion)	732	144.7	61.0	(2)	64.7	(5.0)
% sales won by loggers	732	59.0	56.0	(0.01)	54.4	(0.02)
Avg. logger entry	732	2.75			2.67	(0.17)

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B: California sales						
Sealed bid sales						
Avg. bid	1,630	73.6	74.7	(2.3)	74.2	(2.3)
Avg. logger bid	1,150	64.0	63.6	(2.1)	62.3	(2.4)
Avg. mill bid	480	96.5	101.2	(3.5)	102.8	(3.8)
Avg. sale price ($/mbf)	382	80.4	83.8	(2.1)	84.4	(2.4)
Avg. revenue ($000s)	382	103.1	110.7	(3.8)	111.9	(4.0)
% sales won by loggers	382	66.8	66.4	(1.2)	62.6	(1.3)
Avg. logger entry	382	3.01			3.01	(0.07)
Open auction sales						
Avg. sale price (competition)	325	85.1	87.2	(2.7)	86.7	(3.1)
Avg. sale price (collusion)	325	85.1	46.1	(1.2)	51.0	(1.6)
Avg. revenue (competition)	325	227.0	244.7	(9.7)	242.4	(10.9)
Avg. revenue (collusion)	325	227.0	93.2	(2.6)	112.9	(5.6)
% sales won by loggers	325	50.5	48.2	(1.1)	43.6	(1.8)
Avg. logger entry	325	1.95			1.90	(0.13)

Comments

- Very complete and well done paper.
- Is this a private value setting?
- Can we attribute part of the effect to risk aversion?

Kong (2019) — risk aversion in open and sealed bid auctions

- New Mexico, Permian Basin oil extraction.
- In 2018 the Permian Basin was the second most productive oil field in the world.
- Lessees pay royalties, a rental rate, and in the auction compete on a lump sump bonus.
- New Mexico State Land Office (NMSLO) uses both open (English) and sealed bid auctions.

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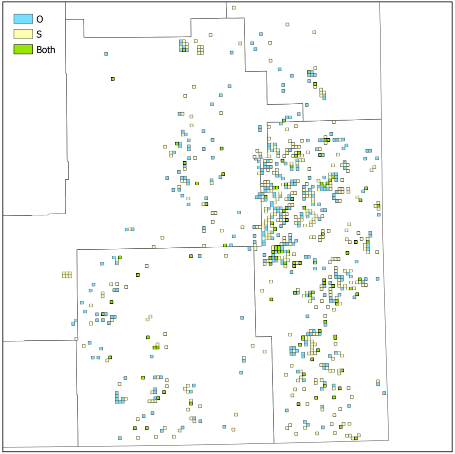
Figure 2: Map of sections by auction format, estimation sample



Courtesy of Yunmi Kong. Used with permission.

Kong (2019) — risk aversion in open and sealed bid auctions

- Sealed-bid FPA generate higher revenue than English auctions.

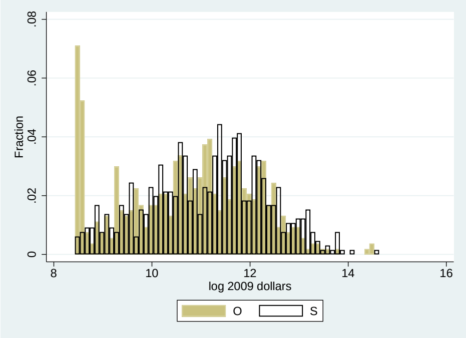
Table 4: Auction format and auction revenue, estimation sample

	(1) auction revenue	(2) auction revenue
auction format S	0.321***	0.305***
	(0.073)	(0.068)
lease prefix VB	0.404***	0.174**
	(0.077)	(0.072)
ln(production) 1970-auction date	0.003	0.002
	(0.011)	(0.010)
ln(production) auction date-2014	0.069***	0.025**
	(0.010)	(0.011)
section drilled before	0.093	0.095
	(0.074)	(0.069)
ln(gas futures)	-0.112	-0.305
	(0.226)	(0.205)
ln(WTI oil price)	0.497**	0.721***
	(0.208)	(0.196)
same quarter BLM price/acre	0.185**	0.242***
	(0.074)	(0.067)
last month price/acre	0.029	0.055
	(0.093)	(0.085)

Kong (2019) — risk aversion in open and sealed bid auctions

- Sealed-bid FPA generate higher revenue than English auctions.
- Many auctions with only one bidder.
- If bidders in sealed-bid auctions knew n they would have bid the reserve price when $n = 1$.

Figure 3: Histogram of ln(price), estimation sample



Prices are in 2009 dollars, deflated by the GDP implicit price deflator.

Kong (2019) — risk aversion in open and sealed bid auctions

- Sealed-bid FPA generate higher revenue than English auctions.
- Many auctions with only one bidder.
- If bidders in sealed-bid auctions knew n they would have bid the reserve price when $n = 1$.
- Under risk neutrality, First-price and English auctions with or without uncertainty about n are all be revenue-equivalent (Harstad, Kagel and Levin (1990)).
- Bidders in first price auctions insure themselves by bidding higher.

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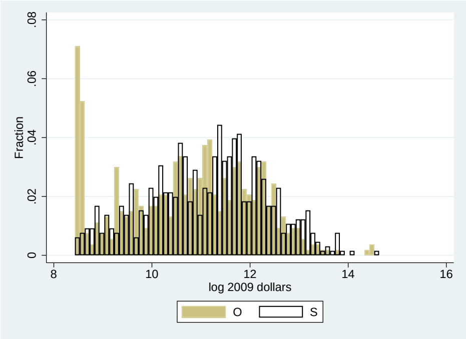

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- Kong shows how open and sealed bid auctions together identify the distribution valuations and the level of risk aversion.

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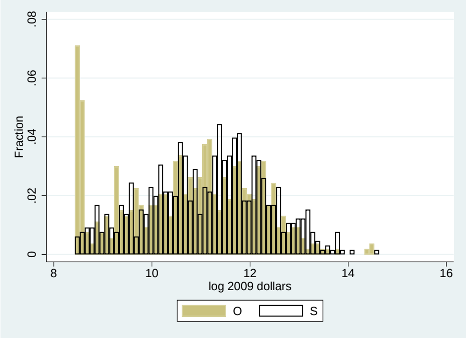

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Thank you and see you (hopefully) in 14.273 next semester!