Static Competition

Glenn Ellison

Introduction

Strategic interaction among firms involves many decision variables. They differ in the longevity of their effects.

LR-- SR

product development / entry → capacity → advertising → price / output

We'll spend the next few weeks on the most short-run of these, focusing on the determination of prices and markups holding technologies and market structure fixed.

We focus on markups because deadweight loss is an important welfare concern, but will also highlight other welfare considerations.

I'll start with a quick review of classic models and then spend more time on the differentiated product demand models that are now most commonly used.

Cournot Competition (1838)

- N firms
- Inverse demand $P(X)$ for homogeneous good
- Cost functions $c_i(x_i)$
- Firms simultaneously choose outputs $x_1, x_2, \cdots, x_N$. Price is $P(\sum_i x_i)$

Antoine Cournot
1801-1877

The literal model is most obviously appropriate for old-economy goods like wheat, natural gas, and iron ore. Sometimes it is also thought of as a reduced form for a situation where firms choose capacities of factories that will always run at full capacity.

While Cournot is rarely the recommended model these days, students should know some of the better known implications.

Nash equilibrium FOC: If $(x_1^*,\ldots, x_N^*)$ is a NE then

$$[P(\underbrace{\textstyle\sum_i x_i^*}_{X^*}) - c_i'(x_i^*)] + P'(\textstyle\sum_i x_i^*)\, x_i^* = 0 \text{ for all } i \text{ with } x_i^* > 0$$

Cournot Competition

$$[P(\underbrace{\textstyle\sum_i x_i^*}_{X^*}) - c_i'(x_i^*)] + P'(\textstyle\sum_i x_i^*)\, x_i^* = 0 \text{ for all } i \text{ with } x_i^* > 0$$

Some implications:

1. Price exceeds the marginal cost of all firms with positive sales.
2. Production is inefficient with asymmetric firms: $c_i'(x_i^*) \neq c_j'(x_j^*)$.
3. Firm outputs are *usually* "strategic substitutes": $\frac{dBR_i}{x_{-i}} < 0$. (Constant elasticity demand is one of the exceptions.)
4. $L_i \equiv \frac{P - c_i'(x_i^*)}{P} = -\frac{x_i^*}{X^*}\frac{1}{\varepsilon}$. Markups roughly decline like 1/N.
5. The industry-wide Lerner index is $\frac{P - \sum_i \frac{x_i^*}{X^*} c_i'(x_i^*)}{P} = -\frac{H}{\varepsilon}$, where $H = \sum_i (\frac{x_i^*}{X^*})^2$ is the industry "Herfindahl Index".
6. We should not think of H as a welfare proxy. For example, in a symmetric model reducing one firm's cost raises welfare but also increases H.

Bertrand Competition (1883)

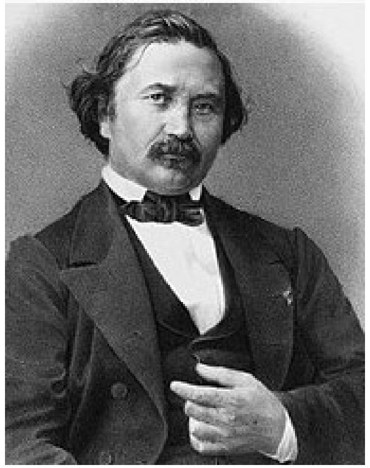
- 2 firms (could be *N*)
- $X(p)$ is market demand function. Assume $X(p)$ is weakly decreasing and $pX(p)$ is bounded.
- c is unit cost
- Firms simultaneously announce prices. All demand goes to lowest price firms.

Unique Nash equilibrium: $p_1^* = p_2^* = c$.

Bertrand is an "exemplifying theory." It illustrates forces using extreme assumptions that we would not see in practice.

- No product differentiation creates infinitely elastic firm-level demand
- Constant returns to scale with no capacity constraints
- One-shot interaction

With asymmetric costs, $c_1 < c_2$, an equilibrium is $p_1^* = p_2^* = c_2$ with all consumers purchasing from firm 1.



Joseph Bertrand
1822-1900

Bertrand's Postulate: For n≥3 there is at least one prime between n and 2n-2.

Schoenfeld's Theorem: For n≥ 2,010,760 there is at least one prime between n and $\left(1+\frac{1}{16597}\right)$n.

Hotelling Competition (1929)

- Continuum of consumers with types $\theta \sim U[0,1]$ have unit demands
- Utility is $v - t\theta - p_1$ if buy from firm 1, $v - t(1-\theta) - p_2$if buy from firm 2, and 0 if they don't buy.
- Constant marginal cost c
- Firms simultaneously announce p_1, p_2.

If v is sufficiently large relative to p_1, p_2, then all consumers will purchase from one firm or the other. The indifferent type has

$$v - t\hat{\theta} - p_1 = v - t\left(1 - \hat{\theta}\right) - p_2 \Longrightarrow \hat{\theta} = \frac{1}{2} + \frac{p_2 - p_1}{2t}$$

Has 1140 advising descendants listed on the Mathematics Geneology Project.

Hotelliing→Arrow→Maskin→Fudenberg→Ellison

STABILITY IN COMPETITION [1]

After the work of the late Professor F. Y. Edgeworth one may doubt that anything further can be said on the theory of competition among a small number of entrepreneurs. However, one important feature of actual business seems until recently to have escaped scrutiny. This is the fact that of all the purchasers of a commodity, some buy from one seller, some from another, in spite of moderate differences of price. If the purveyor of an article gradually increases his price while his rivals keep theirs fixed, the diminution in volume of his sales will in general take place continuously rather than in the abrupt way which has tacitly been assumed.

Hotelling Competition (1929)

Assuming that equilibrium prices are sufficiently low relative to v so that this case applies:

$$BR_i(p_j) = \underset{p}{\operatorname{argmax}}(p - c)\left(\frac{1}{2} + \frac{p_j - p}{2t}\right)$$

$$\text{FOC} \quad \Longrightarrow \frac{1}{2} + \frac{p_j - BR_i(p_j)}{2t} - \frac{1}{2t}\left(BR_i(p_j) - c\right) = 0$$

$$\Longrightarrow BR_i(p_j) = \frac{1}{2}\left(c + t + p_j\right)$$

Solving, we find $p_1^* = p_2^* = c + t$.

Notes:

1. Markups are proportional to the product differentiation parameter t.
2. In an N-firm circular version markups decline like 1/N as in Cournot.
3. Actions are “strategic complements”: firms increase prices when rivals increase prices.

Vertical Differentiation

- Firms L and H produce goods of quality s_L and s_H, respectively, with $s_L < s_H$.
- Consumers with types $\theta \sim U[\underline{\theta}, \overline{\theta}]$ have unit demands with utility $u_i(\theta) = \theta s_i - p_i$ if buy from i and 0 from outside good. For simplicity assume mass $\overline{\theta} - \underline{\theta}$ of consumers.
- Both firms have constant marginal cost c.
- Firms simultaneously choose p_L, p_H.

Given prices p_L, p_H let $\hat{\theta}$ be solution to $u_L(\hat{\theta}) = u_H(\hat{\theta})$,
and let θ' be solution to $u_L(\theta') = 0$.

When $\theta' < \underline{\theta} < \hat{\theta}$ demands are given by

$$D_H(p_L, p_H) = \overline{\theta} - \hat{\theta} = \overline{\theta} - \frac{p_H - p_L}{s_H - s_L}$$

$$D_L(p_L, p_H) = \hat{\theta} - \underline{\theta} = \frac{p_H - p_L}{s_H - s_L} - \underline{\theta}$$

Vertical Differentiation

Again, finding BRs and NE is easy with linear demand curves:

$$BR_H(p_L) = \underset{p}{\operatorname{argmax}}(p-c)\left(\overline{\theta} - \frac{p-p_L}{s_H-s_L}\right)$$
$$= \frac{1}{2}\left(p_L + c + \overline{\theta}(s_H - s_L)\right)$$
$$BR_L(p_H) = \frac{1}{2}\left(p_H + c - \underline{\theta}\,(s_H - s_L)\right)$$

The solution to these is the NE provided $\overline{\theta} \geq 2\underline{\theta}$ and $c + \frac{\overline{\theta}-2\underline{\theta}}{3}(s_H - s_L) < \underline{\theta}s_L$:

$$p_L^* = c + \frac{\overline{\theta}-2\underline{\theta}}{3}(s_H - s_L) \qquad p_H^* = c + \frac{2\overline{\theta}-\underline{\theta}}{3}(s_H - s_L)$$

Notes:

1. Vertical differentiation also creates finite elasticities and positive markups.
2. Firm H sets a higher price and earns higher profits.
3. When $\overline{\theta}$ and $\underline{\theta}$ are too close together firm L is shut out of the market.

Back to Horizontal Differentiation

Variants of Hotelling's model (sometimes with some vertical differentiation as well) have become the dominant approach in empirical IO.

The standard N-firm implementation assumes consumers have an N+1 dimensional type $\varepsilon_{i0}, \varepsilon_{i1}, \cdots, \varepsilon_{iN}$ with joint CDF G and utility is

$$u_{ij} = \begin{cases} v_j - \alpha p_j + \varepsilon_{ij} & \text{if i purchases good j} \\ \varepsilon_{i0} & \text{if i consumes "outside good" 0} \end{cases}$$

Demand in this model in the general case is given by an N+1-dimensional integral:

$$x_j(p_1, \cdots p_N) = \iiint_{\{\varepsilon_{i0}, \varepsilon_{i1}, \cdots, \varepsilon_{iN} | u_{ij} > u_{ik} \forall k \neq j\}} dG(\varepsilon_{i0}, \varepsilon_{i1}, \cdots, \varepsilon_{iN})$$

Empirical papers sometimes approximate this by simulating draws of the ε_{ik}.

A more tractable special case is when there is no outside good, the v_j are all equal, and the ε_{ik} are iid with density f. Demand when others all charge p is a one-dimensional integral:

$$x_j(p_j, p, \cdots, p) = \int [1 - F(\theta + (p_j - p))] \underbrace{(N-1) F(\theta)^{N-2} f(\theta)}_{\substack{\text{density of highest draw} \\ \text{among } N-1 \text{ rivals}}} d\theta$$

Horizontal Differentiation

Perloff and Salop (REStud 1985) analyze this symmetric model and show:

Proposition. *In this model the symmetric NE prices are*

$$p_N^* = c + \frac{1}{M(N)}\frac{1}{\alpha},$$

with $M(N) = N(N-1)\int_{-\infty}^{\infty} F(\varepsilon)^{N-2} f(\varepsilon)^2 d\varepsilon$

Some corollaries of this result are:

1. When *F* is a uniform distribution this behaves just like the Hotelling model. $\frac{1}{\alpha}$ is analogous to the *t* and *M(N)=N* so the formula is saying $p_N^* = c + \frac{1}{N}t$.
2. If ε is bounded above or $\lim_{\varepsilon\to\infty}\frac{f'(\varepsilon)}{f(\varepsilon)} = -\infty$, then $\lim_{N\to\infty} p_N^* = c$.
3. In the "logit" model, $F(\varepsilon) = e^{-e^{-(\varepsilon+\gamma)}}$, $p_N^* = c + k\left(1+\frac{1}{N-1}\right)\frac{1}{\alpha}$ for some constant k.
4. Prices can even increase in N if the distribution of ε has a thick upper tail. Intuitively, the gain from raising prices must be exactly offset by the likelihood that a consumer who likes your product best likes some other product nearly as much.

Horizontal Differentiation

Application to Mergers

Suppose two single-product firms merge.

- Premerger costs are (c_1,c_2), merger cost changes are (Δc_1, Δc_2)

The premerger FOC is $(p_i - c_i)\frac{\partial x_i}{\partial p_i} + x_i = 0$

The postmerger derivative is $\frac{\partial \pi}{\partial p_i} = (p_i - c_i - \Delta c_i)\frac{\partial x_i}{\partial p_i} + x_i + (p_j - c_j - \Delta c_j)\frac{\partial x_j}{\partial p_i}$

The merger creates "upward pricing pressure" for product *i* if

$$(p_j - c_j - \Delta c_j)\underbrace{\left(-\frac{\frac{\partial x_j}{\partial p_i}}{\frac{\partial x_i}{\partial p_i}}\right)}_{\substack{Diversion\\ ratio}} + \Delta c_i > 0$$

The Logit formula, $p_N^* = c + k\left(1 + \frac{1}{N-1}\right)\frac{1}{\alpha}$, provides another know-your-theory cautionary tale. If you predict the effect of a merger from 5 to 4 symmetric firms, you will predict that markups increase by a factor of (4/3) / (5/4) ≈ 1.067 regardless of what's in the data.

Horizontal Differentiation

Application to Bundling

Zhou (*Econometrica* 2017) discusses the effects of pure bundling on welfare.

- Consumer i gets utility $v - p_{jm} + \varepsilon_{ijm}$ from M distinct products.
- Firms $j = 1, 2, \cdots, N$ all sell M goods. Compares good-by-good price competition and with **pure** bundling competition where *j* charges P_j.
- Assume the ε_{ijm} are iid.

Consumers prefer M$v - P_j + \sum_m \varepsilon_{ijm}$ to M$v - P_k + \sum_m \varepsilon_{ikm}$ if and only if they prefer $v - \frac{1}{M}P_j + \frac{1}{M}\sum_m \varepsilon_{ijm}$ to $v - \frac{1}{M}P_k + \frac{1}{M}\sum_m \varepsilon_{ikm}$, so the equilibrium per-good price with bundling is the equilibrium of the Perloff-Salop model with idiosyncratic preferences $\frac{1}{M}\sum_m \varepsilon_{ijm}$.

To think about whether prices are higher with ε_{ijm} or $\frac{1}{M}\sum_m \varepsilon_{ijm}$, think about the price competition FOC: $\left(p_j - c\right)\frac{-\partial x_j}{\partial p_j} = x_j$. In equilibrium $x_j = \frac{1}{N}$ and $\frac{-\partial x_j}{\partial p_j}$ is the density of consumers with $U_{ij} - \max_{k \neq j} U_{ik} = 0$.

With N fixed, the FOC implies that equilibrium prices are lower if this density is higher.

The distribution of $\frac{1}{M}\sum_m \varepsilon_{ijm}$ is more concentrated around its mean and thinner in the tails.

Horizontal Differentiation

Application to Bundling

Zhou (*Econometrica* 2017) discusses the effects of pure bundling on welfare.

- Consumer i gets utility $v - p_{jm} + \varepsilon_{ijm}$ from M distinct products.
- Firms $j = 1, 2, \cdots, N$ sell all M goods. Compares separate good-by-good price competition and competition with **pure** bundling.
- Assume the ε_{ijm} are iid.

Consumer surplus depends on (1) price levels and (2) match quality. The latter is always better if the firms do not bundle.

Results:

1. Bundling reduces per-good prices when $N = 2$. The density of $\frac{1}{M}\sum_m \varepsilon_{ijm}$ is more concentrated. This is what makes demand price-sensitive when $N = 2$ because $Prob\{|u_{i1} - u_{i2}| < \Delta\} \approx 2\Delta \int f_\varepsilon(x)^2 dx$.
2. When $N = 2$ consumers are worse off despite the lower prices.
3. Bundling increases profits when *N* is above some threshold. For large *N* the marginal consumers have upper tail values, and the upper tail of $\frac{1}{M}\sum_m \varepsilon_{ijm}$ is thinner.
4. For fixed *N* $\lim_{M\to\infty} \frac{P^{*,b}}{M} = c$, i.e. bundling drives per-good prices to cost as $M \to \infty$.

On Monday I'll finish up oligopoly price discrimination and discuss a couple empirical papers:

- Bresnahan
- Miller and Weinberg

See you then!

(Much of what Tobias will talk about in his demand lectures can be thought of as techniques for estimating models of oligopoly competition.)

Oligopoly Price Discrimination

Price discrimination is typically taught as a monopoly topic. With perfectly competition p = MC, so there is no price discrimination. But we can easily see price discrimination given any differentiation.

- Borenstein-Rose (*JPE* 1994) study of airline markets gives evidence of price discrimination (i) in pretty competitive city-pair markets, and (ii) greater price discrimination in more competitive markets.

Oligopoly Price Discrimination

3rd Degree

- Firms 1, 2 compete in markets m=1, 2 with no cross-market arbitrage.
- *Example*. Each market has Hotelling preferences. Type θ consumers in market m get utility $v\ - p_{1m} - t_m\theta$ if they buy from 1 and $v\ - p_{2m} - t_m(1-\theta)$ if they buy from 2.

If the firms can discriminate the outcome would be $p^*_{jm} = c + t_m$. If discrimination in banned we get $p^*_j = c + \frac{2t_1t_2}{t_1+t_2}$.

More generally, banning price discrimination has several effects:

- Typically "high types" better off. ("High types" means population with more differentiated preferences.)
- Typically "low types" worse off.
- Misallocation across markets eliminated.
- Welfare can go either way.

Note: The Hotelling example doesn't bring out the the 3rd or 4th because there is no DWL in either market (provided v is large enough). Profit is higher with discrimination in the example by the arithmetic-geometric mean inequality.

Oligopoly Price Discrimination

3rd Degree

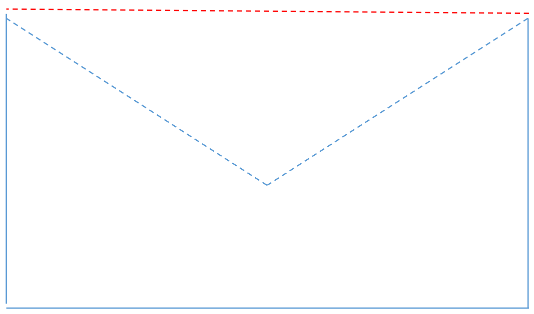
The “typical” price effects don’t need to hold.

Thisse and Vives (*AER* 1988) noted that allowing price discrimination can make prices lower for *all* consumers: allowing firms to aggressively target a rival’s natural consumers can intensify price competition.

- *Example*. Consider again the Hotelling model. Suppose that each θ is a different market. Firms can observe θ and charge θ-dependent prices without arbitrage.

Without discrimination the outcome is $p_j^* = c + t$.

With discrimination we get asymmetric Bertrand competition at each θ. Consumers buy from the closer firm at $p_j^*(\theta) = c + t|\theta - (1-\theta)|$. The distant firm sets $p_k^*(\theta) = c$.



Oligopoly Price Discrimination

3rd Degree

Corts (*Rand* 1998) noted that allowing price discrimination can also make prices *higher* for all consumers: a second effect is that firms can more effectively exploit captive consumers.

- *Example*. Consider a variant of the Hotelling model where consumers only get utility from the better-matched product. Suppose $\theta \sim U[0,2]$ and gross values are:

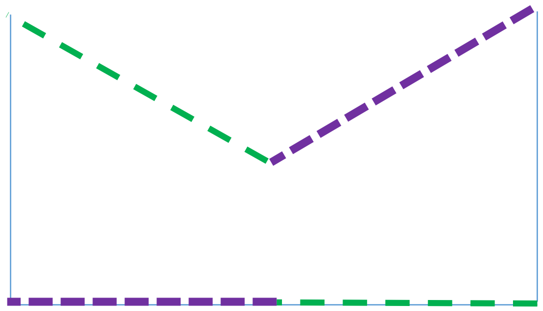
$$v_1(\theta) = \begin{cases} 2-\theta & \text{if } \theta < 1 \\ 0 & \text{if } \theta \geq 1 \end{cases} \qquad v_2(\theta) = \begin{cases} 0 & \text{if } \theta < 1 \\ \theta & \text{if } \theta \geq 1 \end{cases}$$

- Again, suppose that each θ is a different market. Firms can observe θ and charge θ-dependent prices without arbitrage.

With no discrimination this is like monopoly pricing with $D_j(p) = 2 - p$. The outcome is $p_j^* = 1$.

With discrimination asymmetric Bertrand at each θ is essentially θ by θ monopoly pricing. Consumers buy from the closer firm at $p_j^*(\theta) = v_j(\theta)$.

Corts shows that this can happen only if consumer preferences are asymmetric with one firm's high types being the other firm's low types.



Oligopoly Price Discrimination

2rd Degree

Developing tractable models of competitive 2nd degree discrimination is difficult. One wants both vertical and horizontal differentiation. Demand reflects both cross-firm and within-firm substitution.

The theoretical literature has focused on a few different special cases:

- Assuming independent horizonal and vertical preferences produces a tractable model.
- Discrete vertical types can make IC constraints nonbinding.
- Low dimensional models without vertical differentiation.

Oligopoly Price Discrimination

2rd Degree – Independent Vertical and Horizontal Types

Stole's *Handbook of IO* chapter surveys a large literature.

- Firms *j* can produce range of qualities *s* at constant marginal cost *c(s)*.
- Consumer *i's* utility from buying a quality *s* good from firm *j* is

$$u_{ij} = v_i s - p_{js} + \varepsilon_{ij}$$

- Assume v_i and the ε_{ij} are all independent with $v_i \sim F_v$ and $\varepsilon_{ij} \sim F_\varepsilon$.

Write $s^*(v) \equiv \max_s vs - c(s)$ for the efficient quality for a type v buyer.

Example. Consider a Hotelling-like version with two firms: suppose $\varepsilon_{i1} \sim U[0, t]$ and $\varepsilon_{i2} = -\varepsilon_{i1}$. Then, a NE is $p_{js}^* = c(s) + t$ for all *j, s*, with all consumers buying quality $s^*(v_i)$ from the firm with the largest ε_{ij}.

The intuition for the result is straightforward. When all goods are offered at the same markup all consumers compare the prices at which firms sell $s^*(v_i)$. This gives the same FOC as in the Hotelling model.

Notes:

1. Stole surveys a literature that contains similar results for general distributions.
2. Independence is a very strong assumption. For many applications the step from uncorrelated to independent will not be appropriate.

Oligopoly Price Discrimination

2rd Degree – Discrete Vertical Types

- Firms *j* = 1, 2 can produce qualities s_L and s_H at marginal cost *c*. Define $\Delta s \equiv s_H - s_L$.
- Consumers have vertical type $v_i \in \{t_L, t_H\}$ and horizontal types $\epsilon_{i1} \sim U[0,1]$ and $\epsilon_{i2} = -\epsilon_{i1}$. Assume i's utility from buying quality *s* from j is

$$u_{ij} = v_i(s - \varepsilon_{ij}) - p_{js}$$

- Assume v_i and the ε_{i1} are independent with $\text{Prob}\{v_i = t_H\} = \frac{1}{2}$.

Note that types with a higher WTP for quality have stronger horizontal preferences.

Example. In the model above suppose $\frac{t_H}{t_L} \in [3.2, 10]$, $t_L \Delta s < t_H - t_L < t_H \Delta s$, and $\Delta s \leq \overline{\Delta s} \equiv \frac{2(t_H + t_L)}{\sqrt{t_H t_L}} - 4$. Then, there is a NE with $p^*_{js} = c + t_s$ for all *j, s. All consumers buy from the closest firm, with type t_H consumers buying quality s_H and type t_L consumers buying s_L.*

The calculation is straightforward. Assume t_H types buy s_H and t_L types buy s_L we just have two Hotelling games with transportation costs t_H and t_L. The restrictions on Δs imply that only the t_H types are willing to pay the price difference that results. Other constraints rule out non-local deviations.

Notes:

1. The model allows high WTP consumers to have stronger horizontal preferences (in dollars).
2. Again, IC constraints are nonbinding in equilibrium and competition determines markups.
3. For some of the same parameters the model also has a NE with higher welfare where all buy s_H.

Loss Leaders

Lal and Matutes (*J Business* 1994)

- Continuum of consumer have unit demands for products m = 1, 2.
- Firms $j = 1, 2$ sell both products. Costs c_m and qualities v_m common across firms.
- Consumers have horizontal types $\theta \sim U[0, 1]$. Utilities are

  $v_1 + v_2 - p_{11} - p_{12} - t\theta$ if buy both products from firm 1
  $v_1 + v_2 - p_{21} - p_{22} - t(1 - \theta)$ if buy both products from firm 2
  $v_1 + v_2 - p_{11} - p_{22} - t\theta - t(1 - \theta)$ if buy good 1 from firm 1 and good 2 from firm 2
- Consider multistage game where (1) firms choose and advertise p_{j1}, (2) firms choose unadvertised price p_{j2}, (3) consumers visit one firm incurring cost $t\theta$ or $t(1 - \theta)$ and learn its unadvertised price, and (4) consumers purchase or visit the other firm incurring another transportation cost

Proposition. In the model above:

(a) Equilibrium prices satisfy $p_{j1}^* + p_{j2}^* = c_1 + c_2 + t$

(b) Individual prices are $p_{j2}^* = v_2$ and $p_{j1}^* = c_1 + c_2 + t - v_2$.

The argument for p_{j2}^* is that consumers will always pay ε more than they had anticipated when they get to the store unless $p_{j2}^* = v_2$. Competition in p_{j1}^* is then as in Hotelling, but with consumers paying v_2 more than announced price.

Notes:

1. Product 1 is the "loss leader". Its price can be below cost, but need not be.
2. Loss leaders are profit neutral. Profits are unchanged if both prices are advertised or if good 2 does not exist.
3. With per-product advertising costs firms would choose to advertise just one product.

Add On Pricing

In some examples of 2nd degree discrimination the high quality product involves add-ons with less visible prices: hotel restaurant meals and minibar items, rental car insurance and car seats, bank account overdraft fees, printer cables and toner.

Consider a model that combines a loss-leader information structure with discrete 2nd degree discrimination.

- Firms $j = 1, 2$ advertise prices p_{jL} for goods of quality s_L. Quality s_L and s_H both have cost c.
- Consumers who incur a small cost to visit *j* learn the price p_{jU} for an upgrade to $s_H = s_L + \Delta s$.
- Consumers have vertical type $v_i \in \{t_L, t_H\}$ and horizontal types $\epsilon_{i1} \sim U[0, 1]$ and $\epsilon_{i2} = -\epsilon_{i1}$. v_i and the ε_{i1} are independent with $\text{Prob}\{v_i = t_H\} = \frac{1}{2}$. *i*'s utility from buying quality *s* from *j* is

$$u_{ij} = v_i(s - \varepsilon_{ij}) - p_{js}$$

Proposition. In the model above with the same parameter restrictions as two slides ago:

(a) There is a sequential equilibrium with $p^*_{jU} = t_H \Delta s$ and $p^*_{jL} = c + \bar{t}\left(1 - \frac{\Delta s}{2}\right)$ where $\bar{t} = \frac{2t_1 t_2}{t_1 + t_2}$.

(b) Profits in this game are higher than the profits in the game in which both p_L and p_H are visible.

(c) All consumers are worse off than in the game with no low-quality good.

Add On Pricing

Proposition. In the model above with the same parameter restrictions as three slides ago:

(a) There is a sequential equilibrium with $p_{jU}^* = t_H \Delta s$ and $p_{jL}^* = c + \bar{t}(1 - \frac{\Delta s}{2})$.

(b) Profits in this game are higher than the profits in the game in which both p_L and p_H are visible.

(c) All consumers are worse off than in the game with no low-quality good.

Sketch of Proof:

The fact that $p_{jU}^* = t_H \Delta s$ is just like the argument that $p_{j2}^* = v_2$ in Lal-Matutes. The firm has an incentive to make p_{jU} slightly higher unless it is at the monopoly price. The parameter restrictions make it is better to sell upgrades just to the high types.

p_{jL} is then chosen to maximize $(p - c)X(p, p_L^*) + p_{jU}^* X_H(p, p_L^*)$. The FOC for this is

$$(p_L^* - c)X'(p_L^*, p_L^*) + X(p_L^*, p_L^*) + p_{jU}^* X_H'(p_L^*, p_L^*) = 0$$

$$\Longrightarrow \quad (p_L^* - c)(-\frac{1}{2\bar{t}}) + \frac{1}{2} + t_H \Delta s(-\frac{1}{2t_H}) = 0$$

Other parameter restrictions ensure that the FOC solutions are global optima.

Add On Pricing

Proposition. In the model above with the same parameter restrictions as three slides ago:

(a) There is a sequential equilibrium with $p_{jU}^* = t_H \Delta s$ and $p_{jL}^* = c + \bar{t}(1 - \frac{\Delta s}{2})$.

(b) Profits in this game are higher than the profits in the game in which both p_L and p_H are visible.

(c) All consumers are worse off than in the game with no low-quality good.

Intuition:

Think about reducing all prices to $p^* - dp$ in the add-on model and the model with no low-quality good.

In both models the firms lose $\frac{1}{2} dp$ from charging lower prices to their existing customers.

In both models the gain is (# of customers gained) × (per-customer profit on the marginal customers gained).

The FOC implies that per-customer profits on marginal consumers are the same in both models.

Equilibrium profits depend on per-customer profits on the **average** consumer. These are higher than profits on the marginal consumers attracted by a price cut, because marginal consumers are disproportionately (in a ratio of t_H/t_L) cheapskates who only buy the low-quality good.

The ratio of average to marginal profits is larger in the add-on pricing model because the add-on is more expensive. This makes the per-consumer profit ratio between high and low types larger.

Briefly: Selling add-ons creates an adverse selection problem that makes firms hesitant to cut in prices.

Oligopoly Price Discrimination

2^{rd} Degree – Mixed Bundling in a Model without Vertical Types

Armstrong and Vickers (*REStud* 2010) ask why industries like phone-internet-cable offer discounts for bundled purchases even though many consumers presumably prefer a single provider.

- Continuum of consumer have unit demands for products *m* = 1, 2.
- Firms $j = A, B$ sell both products. Costs c_m and qualities $v_m$common. Mixed bundling prices T_1^j, T_2^j, T_{12}^j.
- Consumers have horizontal types $\theta_1, \theta_2 \sim U[0,1] \times [0,1]$. Utilities are

  $v_1 + v_2 - T_{12}^A - t\theta_1 - t\theta_2$ if buy both products from firm A
  $v_1 + v_2 - T_{12}^B - t(1-\theta_1) - t(1-\theta_2)$ if buy both products from firm B
  $v_1 + v_2 - T_1^A - T_2^B - t\theta_1 - t(1-\theta_2) - z$ if buy good 1 from A and good 2 from B.

Proposition. Let $\Phi(d)$ be the fraction splitting purchases when both firms charge T_1, T_2, $T_1 + T_2 - d$ and all consumers purchase. Then, the optimal bundling discount is positive and satisfies $d = -\frac{2\Phi(d)}{\Phi'(d)}$.

The formula again comes out of a simple FOC. Consider simultaneously raising the prices of the unbundled goods by dp and leaving the bundle price unchanged.

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Notes:

1. This result adds to our earlier discussion of mixed bundling, characterizing what is optimal.
2. Intuition for why bundles are discounted comes from price discrimination. When a consumer is marginal for buying just good 1 from me instead of the bundle, it must be that they have a low value for good 2. This makes me want to offer a discount for buying good 2 as well.
3. The paper also discusses a more general model in which consumers also have a vertical type. It is set up like the "independent vertical and horizontal types" models and has a similar outcome – markups are quality-independent and each consumer buys the quality that is optimal for them.