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Problem Set #2

Due September 28, 2022

1. Consider a two-type model of price discrimination without unit demands. A monopolist produces a divisible good at a constant marginal cost of zero. There is a unit mass of consumers. They may buy any nonnegative real number of units of the good. Half of the consumers are type $\theta = 1$. Each has inverse demand function $P_1(q) = 1 - q$. The other half of consumers are of type $\theta = 2$. Each has inverse demand function $P_2(q) = A - bq^2$ with $A > 0$ and $b > 0$.

(Note that there is no free disposal in this question. A consumer's utility is reduced if he or she is given more units than he or she wants.)

(a) Consider first the best situation from the monopolist's perspective: suppose that θ is observable and the monopolist can charge any tariff $T(q, \theta)$, i.e. the firm can use nonlinear prices and can set separate prices in the two populations with no worries about monitoring, arbitrage, etc.
Find an optimal pricing policy for the firm. What is the firm's profit?

(b) Are there values for (A, b) for which the monopolist would be able to receive the same profit as in part (a) even if the monopolist could not observe θ, i.e. if the monopolist were restricted to using a tariff of the form $T(q)$?
Prove that this is never possible or provide a set of parameter values for which you can show that a tariff of the form $T(q)$ suffices.

(c) Suppose now that $A = 1$ and $b = \frac{1}{2}$. The type 2 consumers can now be thought of as "high types" who have at least a weakly higher valuation for each unit.
Suppose again that θ is unobservable. Suppose also that the monopolist can monitor which consumers are using the good, but cannot prevent resale among the consumers. Hence, the only feasible tariffs will be two part tariffs of the form $T(q) = A + pq$.
Show that the optimal policy for the monopolist will have $p > 0$. What about this situation is different from the textbook example of two-part tariffs where the monopolist sets $p = c$ and extracts all the surplus using a fixed fee?

2. Suppose you are the manufacturer of surfboards which are sold in two separate markets: California and Hawaii. You have factories in both locations, and each can produce an unlimited number of surfboards at a constant marginal cost of \$10 per surboard. Over the last fourteen weeks you've conducted an experiment by varying your prices each week. Your sales at various prices were:

Price	Q in Calif.	Q in Hawaii
10	130	31
11	106	27
12	105	31
14	100	24
15	60	24
16	70	25
17	65	18
18	60	23
20	48	21
22	28	14
24	12	18
25	2	14
26	1	10
30	0	9

(a) Use an OLS regression to estimate linear demand curves for each market.

(b) Given these estimated demand curves what prices would you set in each market? How would you change these prices if antitrust laws required that you set a common price across both markets?

(c) How would profits and consumer surplus be affected by the shift to uniform pricing?

(d) Suppose retailers can ship surfboards between California and Hawaii for \$4 per board. Would this disturb your discriminatory pricing strategy, and if so what would your response be? How is this problem similar to and different from a standard 2^{nd} degree price discrimination model?

3. Consider a model with consumers uniformly distributed on the interval $[0, 1]$. Two suppliers selling the same good are located at points a and $1 - b$ with $0 \leq a, b \leq \frac{1}{2}$. Their production costs per unit are c_1 and c_2, respectively. Consumers buy zero or one unit of the good. They receive zero utility if they don't buy the good and utility $v - p - tx^2$ if they buy the good from a firm at a distance of x from their location. Assume that the firms choose prices simultaneously, and that their objective is to maximize profits.

(a) Find the Nash equilibrium prices and profits in this model assuming that v is sufficiently large so that the equilibrium involves all consumers purchasing the good. How large can firm 1's cost disadvantage be if it does make positive profits in equilibrium?

(b) Suppose that before choosing prices the firms play a first period game where they simultaneously choose where to locate. Assume that the firms costs are equal, $c_1 = c_2 = c$. Show that in equilibrium the firms are maximally differentiated.

4. Consider the model of vertical differentiation discussed in class (and in section 7.5.1 of Tirole). Suppose that the firms' costs are higher than I assumed so that the equilibrium prices end up being such that some consumers do not buy the product. Write down the equations for demand when prices are such that the highest value consumers buy from firm H, some buy from firm L and some do not buy at all. Assuming that the best responses are always given by the first order conditions obtained by maximizing relative to these demands find the best response functions and solve for the Nash equilibrium. For what values of c do the equations you've written really give the Nash

equilibrium of the game?

5. Shiller's 2020 *IER* paper, "Approximating Purchase Propensities and Reservation Prices from Broad Consumer Tracking," models every consumer as having a different willingness to pay for Netflix. In class I noted that in a model of 3rd degree discrimination with linear demand curves allowing price discrimination always (weakly) increases profits and (weakly) reduces social welfare. Discuss how you might explore whether a similar result applies to demand specifications like that in Shiller, and how you might assess the extent to which his welfare conclusions are really estimated rather than assumed.

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