14.271: PROBLEM SET 3

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1 Assume that consumer utilities are $u_i = V_j + \epsilon_{ij}$, where ϵ_{ij} is distributed T1EV. We can write the probability that the consumer chooses product j as

$$P_j = \int_{s=-\infty}^{\infty} \left(\prod_{k \neq j} e^{-e^{-(s+V_j-V_k)}} \right) e^{-s} e^{-e^{-s}} ds$$

where s is ϵ_{ij}. Derive the closed form logit choice probabilities discussed in class, starting from the above expression.

Solution:

$$P_j = \int_{-\infty}^{\infty} \left(\prod_{k \neq j} e^{-e^{-(s+V_j-V_k)}} \right) e^{-s} e^{-e^{-s}} \, \mathrm{d}s$$

$$= \int_{-\infty}^{\infty} \left(\prod_{k} e^{-e^{-(s+V_j-V_k)}} \right) e^{-s} \, \mathrm{d}s$$

$$= \int_{-\infty}^{\infty} e^{-\sum_k e^{-(s+V_j-V_k)}} e^{-s} \, \mathrm{d}s$$

$$= \int_{-\infty}^{\infty} e^{-e^{-s} \sum_k e^{-(V_j-V_k)}} e^{-s} \, \mathrm{d}s$$

$$= \int_{0}^{\infty} \left(e^{-\sum_k e^{-(V_j-V_k)}} \right)^u \, \mathrm{d}u$$

$$= \left. \frac{\left(e^{-\sum_k e^{-(V_j-V_k)}} \right)^u}{-\sum_k e^{-(V_j-V_k)}} \right|_0^\infty$$

$$= \frac{-1}{-\sum_k e^{-(V_j-V_k)}}$$

$$= \frac{e^{V_j}}{\sum_k e^{V_k}}$$

2 **a** OLS: Each code is different, but you should get the same results for OLS and minimizing least squares as this is the definition of OLS. The coefficients are given by

[1]based on solutions by Adam Harris, Anton Popov, Vivek Bhattacharya, and Sam Grondahl

β_0	-1.267
β_1	-0.003
β_2	1.017
β_3	0.844
β_4	-0.075
β_5	1.013
β_6	0.509
β_7	-0.873
β_8	-0.697

b MLE:

Again all answers are different, but a reminder as to what the correct maximum is:

$$\hat{\beta} \in \arg\max \sum_i y_i log(p(y_i = 1|X_i, \beta)) + (1 - y_i) log(p(y_i = 0)|X_i, \beta))$$

The results are

β_0	-2.613
β_1	-0.000
β_2	0.129

3 Open ended