Problem Set #6 Solutions

Roi Orzach[1]

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Problem 1 (a) Assume the length of the circumference is 1.

If N firms enter, then it is easy to check that the profit of each firm is mt/N^2, as the quantity sold is m/N and the price is $c + t/N$. (This is a minor modification of the standard Hotelling problem.) Then, if the fixed cost of entry is E, the number of entrants with sequential entry will be $\left\lfloor \sqrt{\frac{mt}{E}} \right\rfloor$.

Now consider a mixed strategy in which all firms enter with probability p^*. In this case, a firm who enters will earn a profit of $\mathbb{E}_n \left[mt/(1+n)^2 \right] - E$, where the random variable n is the number of *other* firms who enter. This is distributed binomially with parameters $(N-1, p^*)$. This profit must equal zero for the firm in question to be indifferent between entering and not (so that it would want to mix). Then, we have

$$E = \mathbb{E}\left[\frac{mt}{(1+n)^2}\right] \geq \frac{mt}{(1+\mathbb{E}n)^2} = \frac{mt}{[1+(N-1)p^*]^2},$$

where the middle inequality is by Jensen's inequality. Then,

$$Np^* \geq \sqrt{\frac{mt}{E}} - 1 + p^* \geq \left\lfloor \sqrt{\frac{mt}{E}} \right\rfloor - 1 + p^* = K^* - 1 + p^* > K^* - 1,$$

which is what we wanted to show.

The intuition is that due to convexity of the profit function in the number of entrants, firms get a relatively large profit if they happen to be the only entrant (or one of the few entrants) compared to when there are too many entrants. This causes firms to be more eager to enter if the number of entrants is uncertain (and there is some chance they are one of only a few entrants). Note, however, that the mixed strategy sets the profit *conditional on entry* equal to the fixed cost, whereas Np^* is the *unconditional* number of firms who enter. For certain parameter values, Np^* can be less than K^*.

For example, consider $m = 1, t = 1, E = \frac{3}{4}$. Then $K^* = 1$. Let the number of firms be $N = 2$. For mixed strategy NE we need

$$\begin{aligned}
E &= \mathbb{E}[\pi|\text{firm 1 enters}] = \\
&= 1 \cdot P[\text{firm 2 does not enter}|\text{firm 1 enters}] + \frac{1}{4} \cdot P[\text{firm 2 enters}|\text{firm 1 enters}] = \\
&= 1 \cdot (1-p^*) + \frac{1}{4} \cdot p^* = 1 - \frac{3}{4}p^*
\end{aligned}$$

from which we find $p^* = \frac{1}{3}$.

[1]Based in part on solutions by Adam Harris, Anton Popov, and Vivek Bhattacharya.

The expected number of firms entering is

$$Np^* = 2 \cdot \frac{1}{3} = \frac{2}{3} < K^* = 1$$

(b) Consider a situation with three firms and normalize marginal cost c to 0. Firm 1 enters and positions itself on some point of the circle. Firm 2 enters and positions itself diametrically opposite from Firm 1. Say Firm 3 enters and positions itself a distance θ away from Firm 1 (and thus $1/2 - \theta$ away from Firm 2). Note that the demand from an arc of the circle of length d when you are setting a price p and your opponent is setting a price p' is $d/2 + (p' - p)/2t$. Using this formula, we can write down the pricing problems of all three firms. We have

$$\begin{aligned}
p_1^* &= \arg\max_{p_1} mp_1 \cdot \left[\frac{\theta}{2} + \frac{p_3^* - p_1}{2t} + \frac{1}{4} + \frac{p_2^* - p_1}{2t}\right] \\
p_2^* &= \arg\max_{p_2} mp_2 \cdot \left[\frac{1/2 - \theta}{2} + \frac{p_3^* - p_2}{2t} + \frac{1}{4} + \frac{p_1^* - p_2}{2t}\right] \\
p_3^* &= \arg\max_{p_3} mp_3 \cdot \left[\frac{\theta}{2} + \frac{p_1^* - p_3}{2t} + \frac{1/2 - \theta}{2} + \frac{p_2^* - p_3}{2t}\right].
\end{aligned}$$

Setting $m = 9$ and $t = 1$, we can solve the FOCs simultaneously and get $p_1 = (3 + 2\theta)/10$, $p_2 = (2 - \theta)/5$, and $p_3 = 3/10$, which gives a profit of 0.81 for Firm 3. Setting the entry cost to 0.9 will ensure that this third firm does not enter in this game. But the number of firms in a pure strategy equilibrium of the simultaneous game will be $K^* = 3$.

We can also check that the choices we posited for Firms 1 and 2 are indeed optimal.

Problem 2 (a) Given prices p_1, p_2, the indifferent θ is defined by

$$\widehat{\theta} = \frac{1}{2} + \frac{1}{2t}\left(v_1 - p_1 - (v_2 - p_2)\right)$$

Then, each firm's profits are given by

$$\begin{aligned}
\Pi_1 &= p_1 \left[\frac{1}{2} + \frac{1}{2t}\left(v_1 - p_1 - (v_2 - p_2)\right)\right] \\
\Pi_2 &= p_2 \left[\frac{1}{2} + \frac{1}{2t}\left(v_2 - p_2 - (v_1 - p_1)\right)\right]
\end{aligned}$$

Taking first-order conditions to find best response functions, then solving that set of simultaneous equations, we have that

$$\begin{aligned}
p_1 &= t + \frac{1}{2}(v_1 - v_2) \\
p_2 &= t + \frac{1}{2}(v_2 - v_1)
\end{aligned}$$

(b) Given the equilibrium prices found, we have that

$$\Pi_2 = \frac{1}{18t}\left(3t - v_1 + v_2\right)^2$$

Firm 2 will choose to enter if

$$\frac{1}{18t}\left(3t - v_1 + v_2\right)^2 \geq E$$

(c) Using the valuations above we can calculate that firm 2 enters whenever $E \leq \frac{25}{6}$.
It is socially efficient to enter if

$$\int_{\widehat{\theta}}^{1} (v_2 - t(1-\theta)) - (v_1 - t\theta)\,\mathrm{d}\theta - E \geq 0$$

given the parameters above, this is equivalent to $E \leq \frac{5}{12}$. This means that for some values of E, firm 2 enters even though it is better for consumers for there to be no entry (given the higher quality of good 1). This is the Mankiw Whinston intuition.

(d) Note here that investment lowers firm 1's profit, however, it commits firm 1 to pricing more aggressively if there is entry. Hence we know that when entry costs are very large or very small, this decision does not effect the entry decision and hence firm 1 will not invest. We now just have to check if there is a region in which the decision alters firm 2's entry decision.

If the firm chooses to adopt the technology, we have an indifferent consumer located at $\widehat{\theta} = \frac{5}{12} + \frac{1}{t}(v_1 - v_2 - p_1 + p_2)$. Taking FOCs of profits yields best response curves, from which we derive prices that give per-firm profits of $(\Pi_1, \Pi_2) = (4.898, 1.565)$.

Further note that the firm would rather make the investment and be a monopolist than not make the investment and be a duopolist. Hence, the incumbent invests only when it changes the entrant's decision. Hence they invest if and only if $E \in (1.565, \frac{49}{6})$.