14.30 PROBLEM SET 6

Due: Thursday, April 13, by 4:30 p.m.

Note: The first three problems are required, and the fourth is a practice problem. If you choose to do the practice problems now, you will receive feedback from the grader. Alternatively, you may use them later in the course to study for exams. Credit may not be awarded for solutions that do not use methods discussed in class.

Problem 1

Since the early thirteenth century, coins struck by the Royal Mint of Great Britain have been weighed on a sample basis in the *Trial of the Pyx*. The procedure involves choosing a sample of coins, weighing them, and inflicting punishment on the Master of the Mint if the total weight of the coins does not exceed the prescribed weight minus a 'remedy,'which is set according to manufacturing tolerances. Under the assumption that the Master is honest (and has not pocketed some of the gold from each coin), each coin has a weight in grains with a $N(128, 1)$ distribution.

a. If 100 coins are sampled, what should be the remedy if the British government wants to limit the probability that an honest Master is punished to 1%?

b. What is the probability that the procedure you derived in part a. will detect a dishonest Master who has been pocketing 0.1 grains of gold per coin?

Problem 2

Let $X_1, ..., X_n$ be a random sample (i.i.d.) of size n from a population with distribution $f(x)$ with mean μ and variance σ^2 (both finite). Prove the following:

a. $E\left(S^2\right) = \sigma^2$, where S^2 is the sample variance.

b. $\overline{X} \xrightarrow{p} \mu$ (the sample mean converges in probability to the population mean)

Problem 3

Let S_0 denote the price of a certain stock today. Suppose the price of the stock evolves over time as follows:

$$S_t = S_{t-1} + X_t$$

where

$$X_t = \begin{Bmatrix} 1 \text{ with probability } 0.39 \\ 0 \text{ with probabilty } 0.20 \\ -1 \text{ with probability } 0.41 \end{Bmatrix}$$

a. Express the change in the price of the stock over the first 700 periods, $\Delta S = S_{700} - S_0$, as a function of the X_ts.

b. What is the approximate distribution of the average daily change in the stock's price?

c. What is the probability that the stock is up at least 10 over the first 700 periods?

Problem 4

A manufacturer of booklets packages them in boxes of 100. It is known that on average, each booklet weighs 1 oz., with a standard deviation of 0.05 oz. The manufacturer is interested in calculating the probability that 100 booklets weigh more than 100.4 oz. Calculate the probability for her, citing any theorems that you use.