14.30 PROBLEM SET 2

Due: Tuesday, February 28, 2006 , by 4:30 p.m.

Note: The first four problems are required, and the remaining two are practice problems. If you choose to do the practice problems now, you will receive feedback from the grader. Alternatively, you may use them later in the course to study for exams. Credit may not be awarded for solutions that do not use the methods discussed in this class.

Problem 1

Suppose that the MIT hockey team plays twelve games in a season. In each game, they have a $\frac{1}{3}$ probability of winning, a $\frac{1}{2}$ probability of losing, and a $\frac{1}{6}$ probability of tying. The outcome of each game is independent of all others.

a. What is the probability that the team will end the season with a record of 7-3-2 (7 wins, 3 losses, 2 ties)?

b. Suppose that for each win, the team recieves three points, and for each tie they receive one point. Let X be a random variable that denotes the number of points the team has earned after four games. Calculate the pmf of X and graph it.

c. Calculate the cmf of X and graph it. What is the probability that the team has earned at least five points after four games?

Problem 2

Suppose that you have just purchased a new battery for your smoke detector, and the life of the battery is a random variable X, with pdf $f_x(x) = ke^{\frac{-x}{\beta}}$ where $x \in (0, \infty)$. Assume that t and s are real non-negative numbers.

a. Use the properties of a pdf to find the value of k.

b. Find an expression for $\Pr(X \geq t)$.

c. Find an expression for $\Pr(X \geq t + s \mid X \geq s)$.

d. Suppose that your batteries have lasted s weeks without dying. Based on your above answers, are you more concerned that the battery is about to die than you were when you first put it in?

Problem 3

Suppose that the joint pdf of X and Y is given by

$$f_{X,Y} = \{ \begin{matrix} kx^3y & 0 < x < y < 1 \\ 0 & \text{elsewhere} \end{matrix}$$

a. What is the value of k?

b. What is the marginal pdf of x?

c. What is the value of the marginal cdf of x at $x = \frac{1}{2}$?

d. What is the conditional pdf of y? Are X and Y independent?

e. What is the probability that $X + Y < 1$?

Problem 4

a. Suppose that a random variable has a pdf that is proportional to x on the interval $[0, 1]$. Write down a formula for this pdf. What is the corresponding cdf?

b. Now suppose that the random variable has a cdf that is proportional to x on the interval $[0, 1]$. Write down a formula for this cdf. What is the corresponding pdf?

Problem 5

On a warm spring day, you are sitting outside on your lawn and watching people pass by. Some people are riding bikes (probability p) and some are walking (probability $1 - p$). The probabilities for any two students are independent of each other.

a. Let Y be the number of students riding a bike out of the first n that pass by you. Write down the pmf of Y.

b. Let Z be the number of students riding a bike that pass before the first student that is walking. Write down the joint pmf $f_{Y,Z}(y, z) = \Pr(Y = y, Z = z)$ and verify that it satisfies the properties of a pmf.

c. Calculate the marginal pmf $f_Y(y)$.

d. Calculate the conditional pmf $f_{Z|Y}(y \mid z)$

e. Are Y and Z independent?

Problem 6

Each week, a graduate student named Eliza gives a copy of her job market paper to each of three professors. Each professor then randomly assigns a score to the paper, where the score is an integer ranging from one to four. If the sum of the three scores is 11 or higher, Eliza is allowed to present her paper at that week's seminar. If the combined score is less than 11, she must wait one week and try again. Scores assigned by different professors are independent, and scores assigned by a given professor are independent across weeks. Define X as the number of weeks Eliza must wait before presenting her paper. Derive the pdf of X.