

14.30 PROBLEM SET 8

Due: Tuesday, May 2, by 4:30 p.m.

Note: The first three problems are required, and the remaining two are practice problems. If you choose to do the practice problems now, you will receive feedback from the grader. Alternatively, you may use them later in the course to study for exams. Credit may not be awarded for solutions that do not use methods discussed in class.

Problem 1

Suppose that the sample statistics for a random sample of 10 observations from a $N(\mu, \sigma^2)$ population are the following:

$$\begin{aligned}\bar{X} &= 5 \\ S^2 &= 4\end{aligned}$$

- Construct a 95% confidence interval for μ .
- Suppose that you want the length of the confidence interval to be no greater than 3.57. What is the associated confidence level?

Problem 2

Assume that X_1, X_2, \dots, X_n is a random sample from a $N(\mu, \sigma^2)$ population.

- Write down the distribution of

$$\sqrt{n} \frac{\bar{X} - \mu}{S}$$

- Write down the distribution of

$$n \frac{(\bar{X} - \mu)^2}{S^2}$$

- What do your answers imply about the relationship between the t-student and F distributions?

Problem 3

Suppose that a random sample is drawn from a $N(\mu, \sigma^2)$ population, where σ^2 is known.

a. How large of a sample must be drawn so that the confidence interval for μ has a length less than 0.01σ ?

b. Suppose that the known value of σ^2 is 2 and that you draw a sample with $\bar{X} = 5$. However, due to time and budgetary constraints, you were only able to draw a sample half as large as the sample you came up with in part a. If we hold the confidence interval length constant, what is our new confidence level?

c. What is the 95% confidence interval for the sample you used in part b? How does it compare to the desired length from part a?

Problem 4

A Cambridge landlord is trying to decide what rent to charge for her apartments. She decides to poll a group of students about how much rent they are able to pay. She asks nine students what they are able to pay per month, X_t , and finds the following:

$$\bar{X} = \frac{1}{9} \sum_{t=1}^9 x_t = 600$$

$$S^2 = \frac{1}{8} \sum_{t=1}^9 (x_t - \bar{X})^2 = 10000$$

She infers that 95% of the students in the area can pay between \$400 and \$800 per month.

a. What assumptions are underlying this inference? What are the potential problems with her inference? How will the size of her confidence interval change as she polls more students?

b. Now suppose that the assumptions made in part a are true. Construct a 95% confidence interval for the mean of the underlying distribution of X_t . How will the size of this confidence interval depend on the number of students that are polled? How many students would the landlord need to poll to get a confidence interval with a width of \$100 or less?

Problem 5

Suppose that you are going to draw a random sample from a Bernoulli distribution in an effort to estimate the parameter p . How large of a sample will you need to draw in order to ensure that the width of the 95% confidence interval for your estimate no greater than 0.01? (Hint: assume that n will be large and consider bounds on the quantity $p(1-p)$.)