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14.30 Introduction to Statistical Methods in Economics
Spring 2009

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Problem Set #1

14.30 - Intro. to Statistical Methods in Economics

Instructor: Konrad Menzel

Due: Tuesday, February 17, 2009

Instructions

You may work together to solve the problems but must each hand in independently-written solutions, so make sure to show all of your work. Each part of each question is worth 1 point, although partial credit may be rewarded for incorrect answers.

Question 1

A delegation of three is to be chosen from the untenured faculty of the MIT Economics Department (numbering ten) to represent the department in an Institute-wide committee. In how many ways

- a) can the delegation be chosen?
- b) can it be chosen, if two people refuse to go together?
- c) can it be chosen, if two particular members insist on either both going or neither going?
- d) can it be chosen, if two people must be chosen from MIT assistant faculty (6 professors) and one person must be chosen from visiting assistant faculty (4 professors)?

Question 2

In the seventeenth century, Italian gamblers used to bet on the total number of spots rolled with three dice. They believed that the chance of rolling a total of 9 ought to equal the chance of rolling a total of 10. They noted that altogether there are six combinations to make 9: (1,2,6), (1,3,5), (1,4,4), (2,3,4), (2,2,5), and (3,3,3). Similarly, there are six combinations for 10: (1,4,5), (1,3,6), (2,2,6), (2,3,5), (2,4,4), (3,3,4). Thus, argued the gamblers, 9 and 10 should have the same chance. Empirically, they found this not to be true, however. Galileo solved the gamblers' problem. How?

- a) How many permutations of three dice are there that sum to 9?
- b) How many permutations of three dice are there that sum to 10?
- c) How many total permutations of three dice are there? What was Galileo's solution? Explain.

Question 3

Venn diagrams or set diagrams are diagrams that show all hypothetically possible logical relations between a finite collection of sets (groups of things). Venn diagrams were invented around 1880 by John Venn. They are used in many fields, including set theory, probability, logic, statistics, and computer science (Wikipedia: http://en.wikipedia.org/wiki/Venn_Diagram).

- a) Draw a Venn diagram for the three events A, B, and C contained in the sample space S and properly label all possible union and intersections of events.
- b) Draw a Venn diagram for the three events A, B, and C contained in the sample space S and properly label all possible combination of events where $A \cap B \cap C = \emptyset$.
- c) Try (but don't spend too much time—it's just for fun) to draw a complete Venn diagram for the four events A, B, C, and D contained in the sample space S where you include all possible unions and intersections of events. How many mutually exclusive regions should such a diagram include?
- d) How many mutually exclusive regions should such a diagram with $k \in \mathbb{N}$ events include?

Question 4

Does a monkey have a better chance of rearranging “ACCLUUS” to spell “CALCULUS,”

or of rearranging “AABEGLR” to spell “ALGEBRA?” (2 points.)

Question 5

In Lecture 1, you learned about event partitions. Give three different examples of partitions of a single draw from a deck of playing cards.

Question 6

The MIT football team plays 12 games in a season. In each game they have $\frac{1}{3}$ probability of winning, $\frac{1}{2}$ probability of losing, and $\frac{1}{6}$ probability of tying. Games are independent. What is the probability that the team has 8-3-1 record? (6 wins, 4 losses, and 2 ties)

Question 7

You and your friends just rented a car from Enterprise for an 8,000 mile cross-country road trip to see all of the sights from from Boston Harbor to the Golden Gate Bridge. Your rental car may be of three different types: brand new (and not a lemon), nearly 1 year old, or a lemon (bound to break down). That many miles can be demanding on a rental car. If the car you receive is brand new

(New), it will break down with probability 0.05. If it is one year old (One), it will break down with probability 0.1. If it is just a lemon (Lemon), it will break down with probability 0.9. The probability that the car Enterprise gives you a car that is New, One, or Lemon is 0.8, 0.1, and 0.1, respectively. Compute the probability that your car is going to break down on your road trip.

Question 8

- a) Bayes' formula is really important. Write down Bayes' formula and describe it in words.

Further, here are a couple of common applications.

- b) Suppose that five percent of men and 0.25 percent of women are color blind. A colorblind person is chosen at random. What is the probability of this person being male? Assume that there are an equal number of males and females. What if there were twice as many males as females?
- c) Suppose that there exists an imperfect test for Tuberculosis (TB). If someone has TB, there is a ninety-five percent chance that the test will come up "red." If someone does not have TB, there is only a two percent chance that the test will come up red. Finally, the chance that anyone has TB is, say, five percent (in the United States; in other countries Tuberculosis is endemic). Once someone takes the test and it comes up red, what is the probability that they have TB?