

Problem Set #4
14.41 Public Economics
(100 points)

DUE: November 12, 2010

Question 1 (20 points)

Suppose that doctors must choose a level of care, x , for each of their patients. The cost of such care is $c(x) = 5x^2 + 5$. The benefit of care (translated into dollars) is $f(x) = 90x - 10x^2$. Suppose doctors have utility functions given by

$$U(x) = (1 - \lambda)P(x) + \lambda f(x) - c(x)$$

where $P(x)$ is the monetary payment the doctor receives from treating a patient with x units of care. Doctors care about a weighted average of their own monetary revenue from care $P(x)$ and the patient's benefit $f(x)$, with the weight given by the parameter $\lambda \in [0, 1]$.

1. What is the optimal level of care, x^* ?
2. Suppose that the Medicare system begins with retrospective payments. Under this scheme, doctors choose whichever level of x they feel is appropriate, and then the Medicare program gives the doctors \$40 for every unit of x that they prescribe.
 - (a) Under these incentives, what amount of care will each doctor prescribe? How does your answer depend on λ ? To what does $\lambda = 0$ correspond? To what does $\lambda = 1$ correspond?
 - (b) In general, is the level of care too high or too low (the answer may depend on λ)? Why?

For the remainder of this problem, set $\lambda = 0$.

3. Now suppose that the Medicare system switches to a prospective payment system. For our purposes, let's say that the prospective payment system compensates doctors by a fixed payment π provided that $x \geq \hat{x}$, where \hat{x} is some level set by the Medicare administrators. That is, the prospective payment system gives doctors a fixed dollar amount per patient, but requires that doctor's provide at least a certain amount of care.
 - (a) If the Medicare prospective payment system wants doctors to provide precisely the optimal level of care, what will they set as \hat{x} ?
 - (b) Provided that Medicare wants doctors to care for all patients (and not send patients away), what should they set as π ?
 - (c) Demonstrate mathematically how doctors have limited incentive to provide care beyond \hat{x} .
 - (d) Show that the total health benefits under this system are actually slightly smaller than those for under retrospective payment.
4. In reality, health economists found that PPS lowered costs and had little effect on actual health outcomes. Explain why this happened, citing the results above.

5. Now suppose that there are two types of patients divided into two diagnosis related groups (DRGs). Some patients are “serious” cases (S), and the other patients are “not so serious” (N). The cost of care is as above. Suppose further that the Medicare system compensates doctors with $\pi_N = 10$ for $x_N \geq 1$; and $\pi_S = 60$ for $x_S \geq 3$.
- (a) Calculate a doctor’s profits for each type of patient.
 - (b) If doctors can re-label patients describe what will happen.
 - (c) How did “DRG Creep” undo some of the gains of PPS, both in this model and in reality?

Question 2 (20 points)

You have been hired by the Commonwealth of Massachusetts to evaluate a welfare reform put in place by the state, which made it very unpleasant to be on welfare by imposing harsh training requirements on those in the program. Specifically, Massachusetts wonders if the harsher requirements induced women to supply additional labor supply, and hence earn more in labor income. Welfare is only available to single mothers in the state. Welfare was reformed for residents of Boston in 2005, but not for residents of Springfield (a town in western Massachusetts) until 2007. For your evaluation, you are provided the following data on the average monthly earnings of different groups of women in the two cities for two years, 2004 and 2006.

City	Year	Marital Status	Labor Income
Boston	2004	Single	175
Boston	2004	Married	300
Boston	2006	Single	210
Boston	2006	Married	290
Springfield	2004	Single	120
Springfield	2004	Married	200
Springfield	2006	Single	125
Springfield	2006	Married	210

1. Propose two difference-in-difference estimators of the impact of the 1995 welfare reform in Boston. For each, give
 - (a) The assumptions required for the DD estimator to be valid.
 - (b) A scenario under which the assumptions would be violated
2. Which of these two estimators do you prefer, and why?
3. You are worried that the state will not believe the assumptions underlying your estimators above.
 - (a) Propose another estimator that holds under more general assumptions, and construct this estimator.
 - (b) Explain the assumptions required for your estimator in 3(a) to be valid.
 - (c) Give a scenario under which the assumptions in 3(b) would be violated.
 - (d) Based on your DD estimates and your estimate in 3(a), was the welfare reform successful in increasing the labor earnings of single Boston women?

Question 3 (40 points)

In public economics, we are often interested in transfer programs that maximize some social welfare function. For the current exercise, we will be interested in maximizing a utilitarian social welfare function, which is equivalent to maximizing the utility of a randomly chosen individual.

Suppose a random process assigns individuals different characteristics that affect their ability to earn income. If these random characteristics were observable, the government could use these characteristics as a basis for lump sum transfers and achieve the first best outcome (see the discussion in the course textbook at the beginning of section 17.4). In practice, however, we can only observe rough indicators of these characteristics, such as income. The challenge is to design the best transfer program based only on these indicators, yielding what we call the “second best” solution.

In this problem, we will consider a world where people choose to spend some of their time in leisure l and the remainder of their time $(1 - l)$ to earn an income Y (the maximum available leisure has been normalized to be 1). We will assume that all individuals in our society have a common utility function $U(C, l)$ with consumption and leisure as arguments. If a person receives a wage w , income and leisure are related by $Y = (1 - l) \cdot w$. Consumption is equal to earned income Y plus any government transfers T , so that $C = Y + T$. Thus, the utility function can be written simply

$$U(C, l) = U(Y + T, 1 - Y/w)$$

The particular form of the utility function has intentionally been left unspecified, but in the following problems you should draw your plots for a well-behaved utility function that features a believable trade-off between consumption and leisure. It is sufficient to solve everything drawing graphs only—no need for a mathematical representation of the utility function.

1. Plotting the utility function for a single individual.
 - (a) Suppose there are no government transfers ($T = 0$). Plot utility vs. income for a given individual (i.e. U on the y-axis, and Y on the x-axis). Discuss how the shape of the utility curve relates to decreasing returns and to the resulting trade-off between consumption and leisure. Mark the individual’s optimal choice of earned income.
 - (b) Now suppose the government gives the individual in part (a) a transfer of size T . On the same graph as above, draw the individual’s new utility curve under the transfer. Label the individual’s new optimal choice of earned income, and explain why this does or does not differ from the optimal choice of earned income in (a).
 - (c) Finally, suppose the government levies a tax T on the individual in part (a). On the same graph as above, draw the individual’s new utility curve under the tax.

2. Suppose we have two individuals A and B. Nature has given individuals A and B wages $w_A > w_B$, respectively. Plot the utility curves for A and B on the same graph (with taxes and transfers set to zero). Who chooses to earn more income? Why?
3. The government wants to transfer income from high earners to low earners. To do so, it pays a transfer of T to individuals earning less than a specified limit \bar{Y} , and taxes individuals a lump sum amount T if they earn more than \bar{Y} .

Suppose the two individuals A and B from part 2 above comprise our society. The government uses the transfer scheme just outlined, in the hope that A pays the tax while B receives the transfer. Of course, there is the concern that A may masquerade as a low ability individual by intentionally reducing his pre-tax income to qualify for the transfer. The government realizes that for a given cutoff \bar{Y} , the transfer T must be limited so that A is not an impostor (i.e. does not choose to earn less than \bar{Y}).

- (a) Classify this program as Categorical vs. Means-tested, and Cash vs. In-kind.
 - (b) For a given tax/transfer T and cutoff \bar{Y} , write out the condition that must be satisfied in order for A not to be an impostor.
 - (c) Since the government's goal is to transfer income to B, their intuition is that B's earnings should not be restricted. That is, if Y_B^* is the level of earned income chosen by B if he were given a transfer T , the income cutoff is set at $\bar{Y} = Y_B^*$. Plot B's utility function when $\bar{Y} = Y_B^*$, and mark B's choice of earned income.
 - (d) Continuing from 3(c), suppose the government has chosen the largest possible transfer T so that A is not willing to be an impostor, but that the constraint in part 3(b) binds. On the same graph from 3(c), plot two utility curves for A: the first utility curve in the case that A decides to earn more than \bar{Y} , and the second utility curve in the case that A decides to be an impostor. Make sure that the two utility curves for A correspond to the fact that the constraint in 3(b) binds.
 - (e) Using the graph from 3(d), decide whether it would be welfare-improving to restrict B's income. That is, can social welfare be increased by lowering \bar{Y} slightly to \tilde{Y} ? Describe your reasoning carefully. Does your conclusion match the government's intuition expressed in 3(c)?
4. In one or two sentences, explain the policy lesson learned from this exercise.

Question 4 (20 points)

Suppose Montana establishes an income tax schedule that has a tax rate of 5% on the first \$20,000 of income, 10% on the next \$10,000, and then 20% on all taxable income above \$30,000. Montana provides a \$2,000 exemption (deduction) per family member. (Let's call this tax system A.)

1. The Abrams family has three members. Thomas is the sole earner, and he has two kids. The family's annual income is \$35,000. Calculate their:
 - (a) Statutory marginal tax rate,
 - (b) Effective marginal tax rate,
 - (c) Taxable income,
 - (d) Total taxes, and
 - (e) Effective average tax rate.
2. The Brigham family has four members. Rachel is the sole earner, and she has three daughters. The family's annual income is \$100,000. For the Brigham family, calculate the same 5 items as you did for the Abrams family in 1(a)-1(e).
3. Suppose that under Montana's tax system, taxes are levied at the household level (i.e. the marginal tax rates apply to combined spousal income). Does this tax system provide Thomas and Rachel a financial incentive to marry each other, or does it provide a disincentive?
4. Suppose that Montana decides to move to a flat tax rate of 15%, keeping the \$2,000 per family member exemption. (Let's call this tax system B.)
 - (a) Specify whether each family benefits or suffers from the new tax system.
 - (b) How has the marriage incentive or disincentive changed from that in system A?
5. Rather than implementing the flat tax system, Montana keeps the income tax schedule described first (A), but instead of providing a \$2,000 exemption per family member, introduces a \$2,000 tax credit per family member. (Let's call this tax system C.)
 - (a) Which tax regime will each family prefer? Why?
 - (b) Why does the distinction between a credit versus a deduction matter so much in terms of total tax revenue?
6. Compare and contrast the vertical and horizontal equity implications of each of the three tax regimes A-C.
 - (a) Which tax system is the most vertically equitable, and which is the least?
 - (b) Which regime is the most horizontally equitable, and which is the least?

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