# Problem Set \#5 Solutions 14.41 Public Economics 

DUE: Dec 3, 2010

## 1 Tax Distortions

This question establishes some basic mathematical ways for thinking about taxation and its relationship to the marginal rate of substitution between goods. Consider individuals that have preferences

$$
u\left(c_{1}, c_{2}, L\right)
$$

over two goods, $c_{1}, c_{2}$, and leisure, $L$. Let $p_{1}, p_{2}$ be the (before-tax) prices of goods $c_{1}$ and $c_{2}$. Let $w$ be the (before-tax) wage. The agent has an endowment of non-labor wealth of $m$. As usual, we assume that $u$ is continuously differentiable in $\left(c_{1}, c_{2}, L\right)$. Let $u_{1}\left(c_{1}, c_{2}, L\right)$ denote the marginal utility of $c_{1}, u_{2}\left(c_{1}, c_{2}, L\right)$ denote the marginal utility of $c_{2}$, and $u_{L}\left(c_{1}, c_{2}, L\right)$ denote the marginal utility of leisure.

1. Write the agent's budget constraint assuming no taxes and that the individual is endowed with 1 unit of leisure so that $l+L=1$, where $l$ is the amount of time spent working for a wage.

The budget constraint is given by any of the following expressions:

$$
\begin{aligned}
p_{1} c_{1}+p_{2} c_{2} & =m+w l \\
p_{1} c_{1}+p_{2} c_{2} & =m+w(1-L) \\
p_{1} c_{1}+p_{2} c_{2}+w L & =m+w
\end{aligned}
$$

where $m+w$ is the agent's "full income".
2. Derive the first-order conditions for the agents maximization problem by placing a lagrange multiplier, $\lambda$, on the budget constraint. Derive two equations that combined with the budget constraint characterize the solution to the maximization problem, $\left(c_{1}^{*}, c_{2}^{*}, L\right)$. One condition should be for the marginal rate of substitution between $c_{1}$ and $c_{2}$ and another for the MRS between $c_{1}$ and $L$. Explain the intuition of both equations. [Note: you do not (and cannot in
general) solve for the solution explicitly; however, you should notice that you have 3 equations and 3 unknowns, so that given any well-specified function $u$ one could solve the system of equations. Also, note that one could also write a condition for the MRS between $c_{2}$ and $L$; however this would be redundant given the other two equations].

The lagrangian is given by

$$
u\left(c_{1}, c_{2}, L\right)+\lambda\left[m+w-p_{1} c_{1}-p_{2} c_{2}-w L\right]
$$

so that the FOCs are

$$
\begin{aligned}
& {\left[c_{1}\right] }: \\
& {\left[u_{1}\left(c_{1}, c_{2}, L\right)=p_{1} \lambda\right.} \\
& {[L] } u_{2}\left(c_{1}, c_{2}, L\right)=p_{2} \lambda \\
& {\left[u_{L}\left(c_{1}, c_{2}, L\right)=w \lambda\right.}
\end{aligned}
$$

so that the MRSs are given by

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{2}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{p_{2}}
$$

and

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{L}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{w}
$$

The first expression shows that the MRS between $c_{1}$ and $c_{2}$ is equated to the price ratio between $p_{1}$ and $p_{2}$. Intuitively, the agent must be indifferent between trading $c_{2}$ for $c_{1}$ at price $\frac{p_{1}}{p_{2}}$. The second equation shows that the MRS between $c_{1}$ and $L$ is equated to the price ratio between $p_{1}$ and the wage $w$. Intuitively, the agent must be indifferent between consuming more leisure for one more unit of $c_{1}$ at price $\frac{p_{1}}{w}$.
Now, suppose that the government levies a lump-sum tax of $\tau$ on all individuals, so that their net non-labor wealth is now $m-\tau$ (Assume for simplicity that this tax is used to finance things for which individuals have no utility).
3. Set up the new budget constraint (assuming the agents never receive the money paid to the government and the money is used to finance things for which individuals have no utility). Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for $c_{1}$ vs $c_{2}$ and for $c_{1}$ vs $L$ ). Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would be in the absence of taxation?) Why or why not?

The new budget constraint is now simply

$$
p_{1} c_{1}+p_{2} c_{2}+w L=m+w-\tau
$$

and the FOCs are the same as in part 2. The MRS is not distorted (always equals the price
ratio). Lump sum taxes provide no marginal incentive to consume one particular good over another particular good - it is simply a wealth effect.

Now, suppose that instead of the lump-sum tax, the government institutes a tax on $c_{1}$ of $\tau_{1}$, so that individuals must now pay $p_{1}+\tau_{1}$ per unit of $c_{1}$.
4. Set up the new budget constraint (assuming the agents never receive the money paid to the government and the money is used to finance things for which individuals have no utility). Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for $c_{1}$ vs $c_{2}$ and for $c_{1}$ vs $L$ ). Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would be in the absence of taxation?) Why or why not?

The new budget constraint is given by

$$
\left(p_{1}+\tau_{1}\right) c_{1}+p_{2} c_{2}+w L=m+w
$$

and the FOCs are

$$
\begin{aligned}
{\left[c_{1}\right] } & : u_{1}\left(c_{1}, c_{2}, L\right)=\left(p_{1}+\tau_{1}\right) \lambda \\
{\left[c_{2}\right] } & : u_{2}\left(c_{1}, c_{2}, L\right)=p_{2} \lambda \\
{[L] } & : u_{L}\left(c_{1}, c_{2}, L\right)=w \lambda
\end{aligned}
$$

so that the MRSs are

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{2}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}+\tau_{1}}{p_{2}}
$$

and

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{L}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}+\tau_{1}}{w}
$$

The tax on $c_{1}$ causes the MRS between $c_{1}$ and $c_{2}$ and between $c_{1}$ and $w$ to be higher than it would otherwise be. Now, the agent needs to be willing to trade $c_{2}$ for $c_{1}$ at relative price $\frac{p_{1}+\tau_{1}}{p_{2}}$, instead of just $\frac{p_{1}}{p_{2}}$. Similarly, the agent needs to be willing to trade leisure $L$ for $c_{1}$ at relative price $\frac{p_{1}+\tau_{1}}{w}$, instead of just $\frac{p_{1}}{w}$. Thus the MRS is distorted - the agent adjusts her allocation so that her willingness to pay for $c_{1}$ (MRS in terms of other goods) rises to equate to the after-tax price, $p_{1}+\tau_{1}$.

Now, suppose that, in addition to the tax on $c_{1}$, the government institutes a tax on $c_{2}$ of $\tau_{2}$ and on labor earnings, $\tau_{w}$ (so that the after-tax wage is $w-\tau_{w}$ ).
5. Set up the new budget constraint assuming the agents never receive the money paid to the government. Derive the first order conditions and provide the analogous two conditions for the marginal rates of substitutions (for $c_{1}$ vs $c_{2}$ and for $c_{1}$ vs $L$ ). Explain the intuition of both equations. Is the MRS distorted? Why or why not?

The new budget constraint is given by

$$
\begin{aligned}
\left(p_{1}+\tau\right) c_{1}+\left(p_{2}+\tau_{2}\right) c_{2} & =m+\left(w-\tau_{w}\right) l \\
\left(p_{1}+\tau\right) c_{1}+\left(p_{2}+\tau_{2}\right) c_{2}+\left(w-\tau_{w}\right) L & =m+w-\tau_{w}
\end{aligned}
$$

Note that a tax on labor is equivalent to a subsidy on leisure. The FOCs are given by

$$
\begin{aligned}
& {\left[c_{1}\right]: u_{1}\left(c_{1}, c_{2}, L\right)=\left(p_{1}+\tau_{1}\right) \lambda} \\
& {\left[c_{2}\right]: u_{2}\left(c_{1}, c_{2}, L\right)=\left(p_{2}+\tau_{2}\right) \lambda} \\
& {[L]: u_{L}\left(c_{1}, c_{2}, L\right)=\left(w-\tau_{w}\right) \lambda}
\end{aligned}
$$

so that the MRSs are given by

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{2}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}+\tau_{1}}{p_{2}+\tau_{2}}
$$

and

$$
\frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{L}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}+\tau_{1}}{w-\tau_{w}}
$$

Now, the tax on $c_{1}, c_{2}$, and $l$ causes the MRS to be equated to the after-tax price ratios. The MRS between $c_{1}$ and $c_{2}$ is not distorted if and only if $\tau_{1} / p_{1}=\tau_{2} / p_{2}$. Likewise, the MRS between $c_{1}$ and $L$ is not distorted if and only if $\tau_{1} / p_{1}=-\tau_{\omega} / w$. Intuitively, if the tax rates across the goods are not the same, the taxes will induce a distortion in the MRS (i.e. the willingness to trade one good for the other).

Now, consider a conceptually different, but mathematically similar economy. Suppose there are two time periods, 1 and 2. In the first time period, agents can consume and work. In the second time period, agents can only consume. Agents are endowed with non-labor wealth of $m$ in the first period. Agents can save and/or borrow at a gross interest rate of $R$ so that savings of $s$ in period 1 yield $R s$ in period 2. Assume that the price of consumption in both periods is 1 (in terms of money within the period). Also, assume the wage is equal to $1, w=1$. Agents utility functions are, as before, given by $u\left(c_{1}, c_{2}, L\right)$.
6. Denote the agents net savings/borrowing position in the first period by $s$. Write the agents' two budget constraints, one for each time period. Then, combine these budget constraints into a single budget constraint over $c_{1}, c_{2}$, and $L$. Show how this economy relates to the economy described in parts 1 and 2 by providing prices for $p_{1}, p_{2}$, and $w$ for the general economy (parts $1 \& 2$ ) that make this economy mathematically equivalent (you should normalize $p_{1}=1$ ).

The budget constraint in period 1 is given by

$$
\begin{aligned}
c_{1}+s & =m+l \\
c_{1}+s & =m+(1-L) \\
c_{1}+s+L & =m+1
\end{aligned}
$$

and in period 2 is given by

$$
c_{2}=R s
$$

so that the combined budget constraint is given by

$$
c_{1}+\frac{c_{2}}{R}+L=m+1
$$

so that if we had $p_{1}=1, p_{2}=\frac{1}{R}$, and $w=1$, we are equivalent to the general economy described earlier.
7. Suppose the government institutes a tax on savings of $\tau$ (and no other taxes). What is the agent's marginal rates of substitution between $c_{1}$ and $c_{2}$ and between $L$ and $c_{1}$ ? Explain the intuition of both equations. Is the MRS distorted? (i.e. are they different than they would be in the absence of taxation?) Why or why not? [Note: no derivations should be necessary - just apply the results from parts 1-4]

With the tax on savings, we have $p_{2}$ translated to $1 /(1-\tau)$ instead of 1 so that the MRSs are given by

$$
\begin{aligned}
& \frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{2}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{p_{2}}=R(1-\tau) \\
& \frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{L}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{w}=1
\end{aligned}
$$

So that the MRS is distorted between consumption in period 1 and consumption in period 2 . The tax on savings makes individuals less likely to save: their willingness to trade between period 1 and 2 is now $R(1-\tau)$ instead of $R$. But, the MRS is not distorted between $c_{1}$ and $L$.
8. Suppose the government institutes a tax on labor earnings of $\tau$ (and no other taxes). Solve for the agent's marginal rate of substitution between $c_{1}$ and $c_{2}$, and between $L$ and $c_{1}$. Explain the intuition of both equations. Is the MRS distorted? Why or why not? [Note: no derivations should be necessary - just apply the results from parts 1-4]

With the tax on labor earnings of $\tau$, the MRSs are given by

$$
\begin{aligned}
& \frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{2}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{p_{2}}=R \\
& \frac{u_{1}\left(c_{1}, c_{2}, L\right)}{u_{L}\left(c_{1}, c_{2}, L\right)}=\frac{p_{1}}{w}=\frac{1}{1-\tau}
\end{aligned}
$$

so that there is no distortion between the consumption of $c_{1}$ and $c_{2}$, but there is a distortion between $c_{1}$ and $L$, since leisure $L$ is being "subsidized" (because its perfect complement, labor supply, is being taxed).

## 2 Firm Taxation

Consider an economy populated by a set of individuals who each supply labor, $l$, and capital, $k$, to a competitive market of firms at before-tax prices $w$ and $r$. Each firm has an identical production function $F(k, l)=k^{\alpha} l^{1-\alpha}$. For the first part of the problem, assume that the capital stock is supplied inelastically, so that $k=\bar{K}$ for any set of prices. Also, assume labor is supplied according to a supply function $l(w)=b w$ where $b>0$.

1. Write the firms maximization problem and solve for the demand for $l$ as a function of $w$ and $\bar{K}, l(w, \bar{K})$. Solve for the equilibrium labor quantity, $l^{*}$, the equilibrium wage $w^{*}$, and the competitive price of capital, $r^{*}$, that equates demand with the inelastic supply at $\bar{K}$.

Firms maximize profits

$$
\pi=k^{\alpha} l^{1-\alpha}-w l-r k
$$

where $r$ is the gross cost of capital and $w$ is the wage. The maximization problem yields

$$
\begin{aligned}
& {[l]:(1-\alpha)\left(\frac{k}{l}\right)^{\alpha}=w} \\
& {[k]: \alpha\left(\frac{l}{k}\right)^{1-\alpha}=r}
\end{aligned}
$$

since capital is supplied inelastically, we have $k=\bar{K}$, so that

$$
l(w, \bar{K})=\frac{(1-\alpha)^{\frac{1}{\alpha}} \bar{K}}{(w)^{\frac{1}{\alpha}}}
$$

Now, in equilibrium we will have labor demand equal to labor supply so that

$$
l\left(w^{*}, \bar{K}\right)=b w^{*}
$$

or

$$
\begin{aligned}
\frac{\bar{K}(1-\alpha)^{\frac{1}{\alpha}}}{\left(w^{*}\right)^{\frac{1}{\alpha}}} & =b w^{*} \\
\bar{K}(1-\alpha)^{\frac{1}{\alpha}} & =b w^{*}\left(w^{*}\right)^{\frac{1}{\alpha}} \\
\bar{K}(1-\alpha)^{\frac{1}{\alpha}} & =b\left(w^{*}\right)^{1+\frac{1}{\alpha}} \\
\left(\frac{\bar{K}(1-\alpha)^{\frac{1}{\alpha}}}{b}\right)^{\frac{1}{1+\frac{1}{\alpha}}} & =w^{*} \\
\left(\frac{\bar{K}(1-\alpha)^{\frac{1}{\alpha}}}{b}\right)^{\frac{\alpha}{1+\alpha}} & =w^{*} \\
(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b}\right)^{\frac{\alpha}{1+\alpha}} & =w^{*}
\end{aligned}
$$

so now we can solve for the equilibrium quantity of labor

$$
\begin{aligned}
& l^{*}=b\left((1-\alpha)^{\frac{1}{1+\alpha}} \frac{\bar{K}}{b}\right)^{\frac{\alpha}{1+\alpha}} \\
& l^{*}=(1-\alpha)^{\alpha} b^{\frac{1}{1+\alpha}} \bar{K}^{\frac{\alpha}{1+\alpha}}
\end{aligned}
$$

and we can solve for the interest rate, $r^{*}$, using the marginal product of capital equation,

$$
\begin{aligned}
\alpha\left(\frac{l^{*}}{\bar{K}}\right)^{1-\alpha} & =r^{*} \\
\left(\frac{(1-\alpha)^{\alpha} b^{\frac{1}{1+\alpha}} \bar{K}^{\frac{\alpha}{1+\alpha}}}{\bar{K}}\right)^{1-\alpha} & =r^{*} \\
(1-\alpha)^{\alpha(1-\alpha)}\left(b^{\frac{1}{1+\alpha}} \bar{K}^{\frac{\alpha}{1+\alpha}-1}\right)^{1-\alpha} & =r^{*} \\
(1-\alpha)^{\alpha(1-\alpha)}\left(\frac{b}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}} & =r^{*}
\end{aligned}
$$

so that we have

$$
\begin{aligned}
l^{*} & =(1-\alpha)^{\alpha} b^{\frac{1}{1+\alpha}} \bar{K}^{\frac{\alpha}{1+\alpha}} \\
w^{*} & =(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b}\right)^{\frac{\alpha}{1+\alpha}} \\
r^{*} & =(1-\alpha)^{\alpha(1-\alpha)}\left(\frac{b}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}}
\end{aligned}
$$

2. Suppose now the government institutes a labor tax of $\tau$ which requires agents to pay a tax to the government so that their after-tax wage is $\tilde{w}=w(1-\tau)$. Discuss graphically and
mathematically what happens to the agent's labor supply function (as a function of the pretax wage) as a result of the tax.

Agents now only receive $w(1-\tau)$ instead of $w$. The agent will therefore supply $l=b w(1-\tau)<$ $b w$ for any pre-tax wage of $w$. Graphically, we have

3. Solve for the new equilibrium allocation of labor, $l^{*}$, the equilibrium before-tax wage $w^{*}$, the equilibrium after-tax wage $\tilde{w}^{*}$, and the competitive price of capital $r^{*}$. How do these relate to your solution in part 1? Why?

In equilibrium, labor supply equals labor demand, so that

$$
\begin{aligned}
l\left(w^{*}, \bar{K}\right) & =b w^{*}(1-\tau) \\
\frac{(1-\alpha)^{\frac{1}{\alpha}} \bar{K}}{\left(w^{*}\right)^{\frac{1}{\alpha}}} & =b w^{*}(1-\tau) \\
\frac{(1-\alpha)^{\frac{1}{\alpha}} \bar{K}}{b(1-\tau)} & =\left(w^{*}\right)^{1+\frac{1}{a}} \\
(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b(1-\tau)}\right)^{\frac{\alpha}{1+\alpha}} & =w^{*}
\end{aligned}
$$

is the before-tax wage, and

$$
\begin{aligned}
\tilde{w}^{*} & =(1-\tau) w^{*} \\
& =(1-\tau)(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b(1-\tau)}\right)^{\frac{\alpha}{1+\alpha}} \\
& =(1-\tau)^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b}\right)^{\frac{\alpha}{1+\alpha}}
\end{aligned}
$$

is the after-tax wage. Notice that the before tax wage is higher than part 1, while the after tax wage is lower than in part 1. The tax makes labor more expensive, so firms have to be more willing to pay for labor (thus the before-tax wage goes up). But, the rise in productivity is less than one-for-one; on net, the after tax wage drops as a result of the tax.

The new allocation of labor is given by

$$
\begin{aligned}
l^{*} & =b \tilde{w}^{*} \\
& =b(1-\tau)^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}\left(\frac{\bar{K}}{b}\right)^{\frac{\alpha}{1+\alpha}} \\
& =(b(1-\tau))^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}}
\end{aligned}
$$

which is less than before (since labor is taxed). Finally, the competitive price of capital solves

$$
\begin{aligned}
& r^{*}=\alpha\left(\frac{l^{*}}{\bar{K}}\right)^{1-\alpha} \\
& r^{*}=\alpha\left((b(1-\tau))^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}} \frac{(\bar{K})^{\frac{\alpha}{1+\alpha}}}{\bar{K}}\right)^{1-\alpha} \\
& r^{*}=\alpha\left(\frac{b(1-\tau)(1-\alpha)}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}}
\end{aligned}
$$

so that $r^{*}$ is lower than in part 1. The tax on labor reduces the amount of labor supplied in equilibrium. This reduces the marginal product of capital (since capital and labor are complements in our production function), which thereby lowers the return on capital.
4. What is the net change in gross labor earnings (i.e. pre-tax wage x labor)? What is the net change in total capital earnings (interest rate * capital)? If capital earnings changed, explain why.

The net change in gross labor earnings is

$$
\begin{aligned}
\Delta E & =l_{\text {tax }}^{*} w_{\text {tax }}^{*}-l^{*} w^{*} \\
& =(b(1-\tau))^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}}(b(1-\tau))^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}}-b^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}} b^{\frac{1}{1-}} \\
& =b^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}} b^{\frac{1}{1+\alpha}}(1-\alpha)^{\frac{1}{1+\alpha}}(\bar{K})^{\frac{\alpha}{1+\alpha}}\left[(1-\tau)^{\frac{1}{1+\alpha}+\frac{1}{1+\alpha}}-1\right]<0
\end{aligned}
$$

so that gross labor earnings fall.

The net change in total capital earnings is

$$
\begin{aligned}
\Delta C & =\alpha\left(\frac{b(1-\tau)(1-\alpha)}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}} \bar{K}-\alpha\left(\frac{b(1-\alpha)}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}} \bar{K} \\
& =\alpha\left(\frac{b(1-\alpha)}{\bar{K}}\right)^{\frac{1-\alpha}{1+\alpha}} \bar{K}\left((1-\tau)^{\frac{1-\alpha}{1+\alpha}}-1\right)<0
\end{aligned}
$$

Capital earnings fall because the tax on labor reduces the marginal product of capital. Since capital supply is inelastic, this reduction in the marginal product of capital in these firms leads
to a lower payment to owners of capital. Intuitively, since capital is supplied inelastically, it bears some of the burden of the tax on labor.

Now, suppose that instead of a tax on labor, the government institutes a tax on capital, so that individuals return on capital is given by $\tilde{r}=r(1-\tau)$; for every $r$ that owners of capital receive from the firm, they must pay $\tau r$ to the government.
5. Solve for the equilibrium $l^{*}$, the equilibrium wage $w^{*}$, the equilibrium after-tax return on capital $r$. Without solving for the deadweight loss, is the tax on capital more or less efficient than the tax on labor? [Note: you are not required to make any unnecessary/duplicate calculations if you don't need to].

The equilibrium is the same as in part $1 \& 2$. The only difference is that now the gross interest rate is $r^{*}$, while the after-tax interest rate is $\tilde{r}^{*}=r^{*}(1+\tau)$. Since the tax introduces no distortions, it is more efficient than the labor tax.

Now, suppose that capital is no longer supplied inelastically. Rather, let's make the polar opposite assumption. Let's assume that firms have access to an infinite amount of capital at a world price of $\hat{r}$.
6. Without doing any math, discuss the impact of imposing a capital tax. How much of the tax would be paid by capital owners? How much by labor owners?

Since supply of capital is perfectly elastic, the owners of capital will always receive an after-tax return of $\hat{r}$. Therefore, their earnings will not change in equilibrium (although they will invest less in the firms, they will invest more in their outside option which provides a return of $\hat{r}$ ). Labor owners will bear the full cost of the tax.
7. Again without doing any math, discuss the impact of imposing a labor tax. How much of the tax would be paid by capital owners? How much by labor owners? Why?

Again, since the supply of capital is inelastic, the owners of capital will not pay any of the tax. The labor owners will pay all of the cost of the labor tax.

## 3 Empirical Evaluation

Barack Obama is back from his trip overseas and is considering a change in the tax code to help reduce the deficit. He liked your advice from problem set 3 and decided to give you a call back. He asks you a couple of questions about what would happen under various changes to the tax code. For each question, he asks you to do two things:

- Discuss briefly what economic theory predicts and what existing studies may have shown on this question. How confident are we in these predictions/results?
- Discuss a potential empirical method that would allow you to answer my question if you had access to any reasonable amount of data that could potentially be required. Discuss the potential limitations of your approach ${ }^{1}$.

He asks you the following questions:

1. "I'm considering increasing the tax on savings (interest income), but am worried that this might decrease the amount of savings. How much would this tax increase reduce savings?"

A tax on interest income theoretically reduces savings, however theory does not predict the precise magnitude of the reduction in savings.

The ideal empirical approach would randomly subject individuals to differential interest taxation and see if they have a reduction in savings. However, the downside of this approach is that it requires randomly varying tax rates (which violates horizontal equity!). An alternative approach would be if some states had different tax rates on income taxation interest rate taxes and one could do a difference-in-difference around the times at which the policy changed. The downside here is that the states would need to have parallel trends in savings. Also, a downside to many approaches is that it's not clear whether we'll pick up the short-term or long-term response to taxation. In the short run, individuals may find it difficult to adjust their savings holdings which are subject to taxation; but over the long run they might adjust their savings more than we see initially.
2. "I'm considering reducing EITC credits to the poor, but am worried this might decrease their labor supply. To what extent does changing the level of EITC benefits affect labor supply?"

Existing studies suggest that the EITC has very large effects on labor supply, and the results are quite robust. Also, theory predicts that providing a subsidy to labor will increase labor supply. Therefore, theory and existing studies suggest that reducing these credits would reduce their labor supply.

A potential empirical approach to estimate the magnitude of this effect would be to do an analysis of the labor supply before and after of individuals subjected to the EITC as compared to individuals not subjected ot the EITC (i.e. do a difference-in-difference around the time of introduction of EITC between those who are poor enough to qualify relative to those who are just above the qualification threshold). A potential downside of this approach is that those who are just above the qualification threshold may not be on parallel trends relative to those who qualify.
3. "I'm considering raising the highest tax bracket by from $35 \%$ to $40 \%$ to help balance the budget. But, I'm worried that if I raise the tax rate this will decrease the amount of income

[^0]that people report (either through illegal evasion, or because the rich choose to work less in the face of higher taxation). How much will this increase lead to a decline in taxable income amongst the rich?"

There has been substantial work on the so-called "taxable income elasticity", and in general the evidence suggests that there is some reduction in reported taxable income, but there is no real consensus (FYI: Martin Feldstein argues that this magnitude is huge, while Austan Goolsbee has argued that it's large initially but not in the long-run, since the rich individuals just change the timing of when they accrue their capital gains earnings - which biases the estimates of Feldstein).

An ideal empirical approach would randomly assign tax rates to individuals and analyze how their reported taxable income varies as a function of the tax rate. However, this is not feasible for a variety of ethical and legal reasons. A potentially more realistic empirical approach would be to analyze a difference in difference in reported taxable income around the time when the tax brackets are changed. A time series estimator could be used (this is what feldstein uses), but this is subjected to the significant problem that individuals may know when their tax rate will change. If someone knows that they have a $35 \%$ tax today and a $40 \%$ tax tomorrow, they might shift some of the profitable activity from tomorrow to today. therefore, the estimated quantity may be more of a short-term magnitude than the true long-run response. To counter this, one could look at longer time differences from the time of the tax change (e.g. compare 2 years before vs 2 years after). Or, one could analyze unexpected tax changes (which is hypothetically a possibility, but may not be actually feasible).
4. "I'm considering raising the benefits provided to poor single mothers with children. But, how much will this increase the number of single mothers?"

As Prof. Gruber mentioned in class, current work on this is somewhat inconclusive, but largely suggests that there's not a huge effect of welfare benefits on the number of single mothers. Theory predicts that raising benefits to single mothers would at least somewhat increase the number of single mothers, but does not predict a magnitude for this effect.

One potential empirical approach, among many, is to analyze past increases in benefits provided to poor single mothers around some income threshold. If some states increased benefits, while others did not, we could do a difference-in-difference estimation with the states that don't change their benefits as control groups. A potential downside to this approach is that single mothers may move to states with greater benefits, which would make it look like there was a big effect of the policy (since single mothers would move from the control to treatment group). Augmenting the study with data on migration patterns of poor single women could help rule out this possibility.

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Fall 2010

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[^0]:    ${ }^{1}$ President Obama explicitly mentions that some of his advice he received last time failed to mention the potential limitations/qualifications of the results. So he reminds you to include a brief discussion of the potential limitations of your proposed approach.

