MIT 14.41 – Midterm

October 24, 2022

Instructions

- This is a closed-book exam
- You have 85 minutes to take the exam and there are 85 points on the exam; each point should correspond to approximately one minute
- Use a different blue book for responses to each section (Question 1, Question 2, Question 3)
- Please write neatly, we cannot give credit for illegible work
- You may use a calculator; if you do not have a calculator you may use your phone as a calculator

QUESTION 1: True, False, Uncertain (20 points)

For each question state whether the claim is **true, false, or uncertain** and explain why. You must give reasons or no points will be awarded.

1. (5 points) This is Maisie. She lives in Cambridge with Hannah, and, like many dogs, she **loves** to go to the dog park. But, there aren't many dog parks in Cambridge. Hannah should reassure Maisie that even though there aren't many dog parks for them to visit, they are surely being provided at the efficient level, because (a) a free market always provides goods at the socially efficient level and (b) if for some reason it didn't, the government would provide the optimal level because Cambridge's local government is democratic. (You can assume that Hannah, like all other dog owners, internalizes Maisie's preferences perfectly).

   Please respond separately for whether reasons (a) and (b) are true, false, or uncertain, and give a rationale for each.

Solution: (a) False. Like human parks, dog parks are non-excludable, and assuming that they are not too crowded and dogs are well-behaved, they are non-rival. Therefore, they are a public good, and dog owners are likely to free ride on the contributions of others towards a privately-provided dog park. (b) False/Uncertain. Because of (a), it is unsurprising the dog parks are provided publicly. But even in a democracy and with very strong preferences for dog parks, if dog owners are a minority, they won't be able to win funding for dog parks and no candidate will run for local office on a dog park platform (if assumptions for Median Voter Theorem hold).

Grading notes: 0.5 points for (a) is False, 0.5 points for (b) is False or Uncertain. (a) 1 point for dog parks are a public good (non-rival, non-excludable) and 1 point for this means there will be free-rider problems in a private market for dog parks. **Or,** 1.5 points for discussing positive externalities and provision at the privately optimal level, not socially optimal. (b) 2 points for if dog owners are a minority, they won't be able to win funding or political support for more dog parks. 1 point for political reasons

2. (5 points) In the US, the government largely funds education through publicly-provided free schools rather than through providing vouchers for students to attend public or private schools of their choice. Claim 1: Providing vouchers instead would increase the amount/quality of schooling that students on average receive. Claim 2: This means that the government should switch to a voucher system.

Please respond separately for whether Claim 1 and Claim 2 are true, false, or uncertain, and give a rationale for each.

Solution:

Claim 1: True. In the analysis we saw in class and on the problem set, providing any amount of funding in the form of vouchers unambiguously increases the educational quality for all types of families, whereas providing public schooling at the same level is not guaranteed to do so. This is because families have to consume a fixed amount of public education. Some families might choose to reduce their educational spending and send their kids to public schools so that they can consume more and be better off. This crowd-out problem is solved by vouchers. If a family was originally spending less than the voucher amount on schooling, they can increase to spending that amount and also consume more. If a family was originally spending more than the voucher amount on schooling, the voucher is an increase in income which will be spent proportionally on increased schooling and increased other consumption, where the proportion is given by the MRS. **Or**, an answer in general equilibrium is fine – Uncertain, if there are economies of scale and public schools get worse or private schools get more expensive after vouchers implemented, the kids left in public school are worse off.

Claim 2: False. There are other reasons not to use vouchers. Some examples include:

- Vouchers may lead to excessive school specialization if schools try to tailor themselves to specific tastes. But this undermines one of the reasons for public financing of education: positive externalities from providing a common educational program. (This could be solved through regulation, but holding schools accountable for student performance is difficult and may distort incentives)
- Vouchers may increase inequality if more motivated, skilled, and resourced students leave low-quality public schools
- Vouchers may be an inefficient and inequitable use of public funds: Vouchers are effectively a cash transfer to all families who already purchase more than the voucher amount of schooling. Replacing public school systems with vouchers would mean that the government would pay a portion of the private school costs that some families are already paying themselves, possibly only achieving a very small change in the amount of schooling purchased. Since these families are more likely to be more resourced to begin with, this is not achieving redistribution, another rationale for government provision of education
- Where there are large economies of scale in education (i.e. it is a natural monopoly), it may not be efficient for there to be lots of small schools competing for students' voucher money. If there are some schools that are "too big/important to fail," and schools know this, they won't be as susceptible to market pressures to provide a high-quality education
- Not all students cost the same amount to educate. If schools receive the same voucher amount for any student but some students are more expensive to educate, schools will try to avoid having those students come to their schools. (It is very difficult to adjust voucher amounts for each child)

Grading notes: 0.5 points for Claim 1 is True, 0.5 points for Claim 2 is False. Claim 1: 1 point for explaining crowd-out problem, 1 point for explaining how vouchers solve crowd-out problem. Claim 2: 2 points for one correct reason or two partially/mostly correct reasons

3. (5 points) Many local communities in the US have restrictive zoning rules that limit the construction of housing. These zoning rules mean that the government should be more willing to redistribute from communities that have high levels of public goods to communities with low levels of public goods.

Solution:

True. In the Tiebout model, public goods provision only varies across communities because of variation in people's desire for different levels of public goods, making community-based redistribution inefficient. But zoning restrictions inhibit people's ability to move to neighborhoods that provide their ideal level of public goods: there may be people who want a higher level of public goods and would be willing to pay the taxes for it, but are not able to pay the additional housing costs from zoning and so are stuck in low-tax, low-public good neighbourhoods despite that not being their preference. If this is the case then redistribution to the communities with lower levels of public goods might help to offset the inefficiently low level of public goods in these communities.

Grading notes: 1 point for 'true', 1 point for a reference to Tiebout, 1 point for thinking of zoning restrictions as limiting mobility counter to perfect mobility assumption in Tiebout, 1 point for arguing that government shouldn't redistribute with perfect mobility, 1 point for arguing that imperfect mobility means government should redistribute.

4. (5 points) In a setting where insurers **cannot** offer separate contracts to different types, (a) higher-risk types are always made better off than in a setting where insurers **can** offer separate contracts and (b) lower-risk types are always made worse off than a setting where insurers **can** offer separate contracts .

   Please respond separately for whether statements (a) and (b) are true, false, or uncertain, and give a rationale for each.

Solution: (a) False. Higher-risk types will be better off in a pooling equilibrium (they get more insurance or lower prices for full insurance), equally or better off in a separating equilibrium (similar to a pooling equilibrium or they get the same as if they contracts could be separated by type), and worse off if the market unravels due to adverse selection (because they have no insurance). (b) True. Lower-risk types are worse off in every equilibrium: in a pooling equilibrium, they either pay more for full insurance or get less than full insurance, in a separating equilibrium they get less or no insurance, and if the market unravels, they get no insurance.

Grading notes: (a) 0.5 points for False, 1 point for better off in pooling equilibrium and why (0.5 each), 1 point for equally or better off in separating equilibrium and why (0.5 each), 1 point for worse off in death spiral and why (0.5 each). (b) 0.5 points for True, 1 point for why.

QUESTION 2: Pollution reduction (30 points)

A city receives its electricity from two power plants, plant A and plant B. These power plants both produce air pollution, which is harmful to the people in the city, meaning that there is a social benefit from reducing air pollution. A unit of air pollution does the same damage no matter which power plant it comes from. The people who run the power plants live outside the city, and do not suffer from the air pollution, so they get no private benefits from reducing air pollution. Both plants can invest in technology to reduce the pollution they emit at some private cost; the costs at each plant might be different. There are no additional social costs for reducing emissions – the only social costs are the costs to the plants.

1. (3 points) Explain what the externality is in this context, and why.

> **Solution:** Negative production externality because producing power creates pollution, which has social costs but no private costs to the plants.
>
> **Grading notes**: 1 point for saying negative externality, 1 point for production externality, 1 point for reasonable explanation.

The government makes the following estimates:
The social benefits of carbon emissions reduction are constant at $50 per unit of carbon reduction: $SB(r) = 50r$.
The government estimates that one of the plants has a private cost of carbon emissions reduction of $C_1(r_1) = r_1^2$, and the other plant has a private cost of $C_2(r_2) = 10r_2 + r_2^2$. However, it does not know which plant has which cost function. The plants cannot choose a negative reduction quantity, so are constrained to set $r_1 \geq 0, r_2 \geq 0$.

2. (6 points) Draw a diagram illustrating the social marginal benefit curve, and the marginal cost curves for the two firms. Calculate the socially optimal quantity reduction for each of the firms, and label these quantities on the diagram.

> **Solution:** At the social optimum, both plants reduce until the marginal cost of reduction equals 50, which implies $r_1 = 25, r_2 = 20$.
>
> **Grading notes:** 1 point for each cost curve correct (linear with different intercepts and same slope), 1 point for SMB curve correct, 1 point for each quantity reduction setting SMB=MC, 1 point for the whole question being correct.

3. (5 points) Calculate the quantities of reduction chosen by each plant and the deadweight loss in the equilibrium where the government does not intervene.

> **Solution:** In private equilibrium, since plants get no private benefit from reducing emissions they will set $r_1 = 0, r_2 = 0$, implying
>
> $$SWF(r_1, r_2) = 50(r_1 + r_2) - r_1^2 - 10r_2 - r_2^2 = 0$$
>
> At the social optimum, both plants reduce until the marginal cost of reduction equals 50, which implies $r_1 = 25, r_2 = 20$. Social welfare is then
>
> $$SWF(r_1, r_2) = 50(r_1 + r_2) - r_1^2 - 10r_2 - r_2^2 = 50(45) - 25^2 - 10(20) - 20^2 = 1025$$

so the DWL is \$1025. DWL can also be calculated by summing up the areas of the DWL triangles for each plant:

$$\frac{40 \times 20}{2} + \frac{50 \times 25}{2} = 1025$$

Grading notes: 1 point for stating that private reduction quantities are both 0. 2 points for correctly calculating welfare at the social optimum and private equilibrium and 2 points for drawing correct conclusion about DWL, OR 1 point for using triangles to calculate the DWL, 1 point for adding the two triangles, 2 points for getting each triangle's area right.

4. (6 points) Describe (a) a tax or subsidy policy that achieves the social optimum, and (b) a command-and-control quantity regulation that achieves the social optimum. What information does the government need to know for each policy?

Solution: Social optimum can be achieved with a tax of \$50 per unit emissions or a subsidy of \$50 per unit emissions reduction: in both cases this induces firms to set their marginal costs of reduction to the marginal social benefit of \$50. For this policy the government needs to know the marginal externality at the optimum, but since the private and social benefit functions are both linear they just need to know the (constant) difference between these functions.

A command-and-control policy would just set $r_1 = 25, r_2 = 20$. For this the government needs to know the optimal quantity, which depends on the marginal social benefit function and the marginal cost functions for both firms.

Grading notes: 1 point for saying tax of \$50 or subsidy of \$50, 1 point for saying this depends on the marginal externality, 1 point for saying we need marginal externality *at the optimal quantity* (so shape of benefit and cost functions needed). 1 point for correct command-and-control policy for both firms, 1 point for saying this depends on optimal quantities, 1 point for saying what the government needs to know to get to optimal quantities.

5. (6 points) Suppose the government thinks that plant A has the cost function $C_A(r_A) = r_A^2$, and plant B has the cost function $C_B(r_B) = 10r_B + r_B^2$. However, in fact it is the other way round: plant A's cost function is actually $C_A(r_A) = 10r_A + r_A^2$, and plant B's cost function is actually $C_B(r_B) = r_B^2$. If the government uses command-and-control quantity regulations, what quantities will it make plant A and B reduce emissions by based on its mistaken beliefs? Label these $\hat{r}_A, \hat{r}_B$ respectively. Calculate the deadweight loss from this mistake.

Solution: Government will make A reduce 25 units and B reduce 20 units. Social welfare is then

$$SWF(r_A, r_B) = 50(r_A + r_B) - 10r_A - r_A^2 - r_B^2 = 50(45) - 10(25) - 25^2 - 20^2 = 975$$

Social optimum has welfare of 1025, so DWL of $1025 - 975 = 50$.

Grading notes: 2 points for identifying that quantities will just be swapped, 2 points for attempting a valid method to calculate DWL (either SWF or triangle areas), 2 points for correct numbers.

6. Suppose now that the government is worried about getting this wrong, so it switches to a tradable permits scheme. Under this scheme the government gives permits that mean plant A has to reduce emissions by $\hat{r}_A$ and

plant B has to reduce emissions by $\hat{r}_B$ (as calculated in the previous question), but it allows the plants to buy or sell some of their permits to each other.

(a) (2 points) What is the quantity that each plant would trade?

(b) (2 points) What price would the permits trade for?

(c) (1 point) What is the deadweight loss from the new equilibrium outcome?

Solution: In equilibrium both plants must be indifferent between reducing costs by paying for reduction and buying emissions permits. For this to be true for both plants they must both have the same marginal cost of reductions, and this marginal cost must be equal to the price that permits are traded at. In addition, total emissions reductions must be 45 for the market to clear. This will be the case when plant A buys 5 units of emissions permits, B sells 5 units of emissions permits, and the price per unit permit is $50. In this equilibrium the deadweight loss is 0.

Grading notes: 1 point in part (a) for getting that the plants must buy and sell the same quantity.

QUESTION 3: Funding Fire Departments (35 points)

Before 2013, the state of Victoria in Australia financed fire stations and services by a tax, $\tau > 0$, on home fire insurance policies. To understand the effects of a policy like this one, let's assume the following:

Households can purchase an amount of fire insurance coverage f at actuarially fair price of p in the absence of the tax, and an amount of all other consumption c at a price of 1. All households have an income y to spend on f and c. Assume that there are two households, and they choose f_1 and f_2, respectively.

Regardless of their purchase of fire insurance, households receive the services of the fire department. Denote the quantity of the fire department's services F which is equal to the total tax revenue collected to fund the fire department: $F = \tau(f_1 + f_2)$

A household's value of insurance is parameterized by α. α summarizes characteristics of the household, like risk of fire and risk aversion. Specifically, someone with a higher α may be more risk averse or be at a higher risk of their house catching fire. Therefore, they place more value on both how much funding the fire department has and on having insurance. *Until part 6* assume that every household has the same α. Specifically, we can write the expected utility of a household i as:

$$u_i = ln(c_i) + \alpha ln(F + f_i)$$

and their budget constraint as:

$$y = (1 + \tau)pf_i + c_i$$

1. (5 points) Find the optimal level of f_1 that the first household chooses when they are individually maximizing expected utility, taking as given the decisions of the other household as f_2. Directionally, how does f_1 depend on f_2 and why?

Solution:

$$\max_{f_1} ln(y - (1 + \tau)pf_1) + \alpha ln((\tau + 1)f_1 + \tau f_2)$$

Take FOCs:

$$\frac{-(1 + \tau)p}{y - (1 + \tau)pf_1} + \frac{\alpha(\tau + 1)}{(\tau + 1)f_1 + \tau f_2} = 0$$

Rearranges to:

$$f_1^\star = \frac{\alpha y - p\tau f_2}{(\alpha + 1)(\tau + 1)p}$$

f_1 is decreasing in f_2 – they are free-riding on others' contributions.
Grading notes: 1 point for correct maximization problem, 1 points for correct FOC, 1 point for solution, 1 point for f_1 decreasing in f_2, 1 point for discussing free-rider problem.

2. (3 points) What will be the private equilibrium value of F, $F^\star$?

Solution: Both households will make the same choice, so $f_1^\star = f_2^\star$. Call this value $\tilde{f}$. So, households solve the following:

$$\tilde{f} = \frac{\alpha y - p\tau\tilde{f}}{(\alpha + 1)(\tau + 1)p}$$

Rearranges to:

$$\tilde{f} = \frac{\alpha y}{p((1+\alpha)(1+\tau)+\tau)}$$

So in equilibrium,

$$F^{\star} = \frac{2\tau\alpha y}{p((1+\alpha)(1+\tau)+\tau)}$$

Grading notes: 1 point for they all make the same choice, 1 point for correct $\tilde{f}$, 1 point for correct $F^{\star}$. Minus 0.25 points if they forget to multiply by τ.

3. (5 points) Define social welfare as the sum of both households' expected utilities. If the social planner was choosing everyone's insurance policies to maximize social welfare, how much fire insurance would they mandate each household to buy? What is the optimal level of funding for the fire department, F^s?

Solution:

$$\max_f 2ln(y-(1+\tau)pf) + 2\alpha ln((2\tau+1)f)$$

FOC w.r.t. f:

$$\frac{-2(1+\tau)p}{y-(1+\tau)pf^s} + \frac{2\alpha(2\tau+1)}{(2\tau+1)f^s} = 0$$

Rearranges to:

$$f^s = \frac{\alpha y}{p(1+\alpha)(1+\tau)}$$

$F^s = 2\tau f^s$

Grading notes: 1.5 point for correct maximization problem, 2 point for correct FOC, 1 points for correct f^s, 0.5 points for correct F^s.

4. (2 points) Compare $F^{\star}$ and F^s. Is the privately chosen level of funding for the fire-department higher, lower, or the same as the the socially optimal level?

Solution: Since $\tau > 0, \tilde{f} < f^s$ and the fire-department is under-funded relative to the social optimum.

Grading notes: 2 points for under-funded/$F^{\star} < F^s$.

5. In this part of the problem only, suppose that the government decided to provide a lump-sum T to the fire department, in addition to the tax revenue generated by households' purchases of fire insurance. The lump-sum T comes out of other government revenue: it is *not* generated through any additional tax on households.

   (a) (4 points) What new level of f does each household choose when they are individually maximizing expected utility?

   Solution:

   $$\max_{f_1} ln(y-(1+\tau)pf_1) + \alpha ln((\tau+1)f_1 + \tau f_2 + T)$$

   Take FOCs:

   $$\frac{-(1+\tau)p}{y-(1+\tau)pf_1} + \frac{\alpha(\tau+1)}{(\tau+1)f_1+\tau f_2+T} = 0$$

Plugging in $f_1 = f_2$ this rearranges to:

$$\tilde{f}' = \frac{\alpha y - p\tau\tilde{f} - pT}{(\alpha+1)(\tau+1)p} \implies \tilde{f}' = \frac{\alpha y - pT}{p(\alpha+1)(\tau+1) + p\tau}$$

Grading notes: 1 point for correct maximization problem, 1.5 point for correct FOC, 1.5 point for correct solution

(b) (4 points) Imagine the government sets T equal to the difference between the socially optimal level of fire department funding and the level obtained when individuals are maximizing utility that you found in part 2 (i.e. $T = |F^s - F^\star|$). Given the results you found above, will the government achieve the social optimum using this policy? Why? (No additional math required).

Solution: No, because when $\tilde{f} > 0$ (as above, given $y > 0$ there is crowd-out: for each unit of fire department services provided, households buy less insurance, thus contributing less to the fire department. So, assuming households are buying positive amounts of fire insurance ($y > 0$), they do not reach the socially optimal level of funding for the fire department.
Grading notes: 3 points for no, 1 point for crowd-out

6. Finally, assume that the insurance company has gone out of business (i.e. no one can purchase any private insurance) and the government has decided that they should finance the fire department through a lump-sum tax t on each household (and the government will no longer make the lump-sum payment T out of other government revenue). They will put to a vote what the level of that tax should be. Now, let's assume that there are 80 households of type 1 ($\alpha = \alpha_1$) and 20 households of type 2 ($\alpha = \alpha_2$), with $\alpha_2 > \alpha_1$. So, $F = 100t$.

(a) (2 points) What is a household of type i's utility function?

Solution:

$$ln(y - t) + \alpha_i ln(100t)$$

Grading notes: 2 points for correct utility function

(b) (4 points) Using the same social welfare function as above, what is the socially efficient level of the lump-sum tax t?

Solution:
SWF:

$$100ln(y - t) + (80\alpha_1 + 20\alpha_2)ln(100t)$$

FOC:

$$\frac{-100}{y - t} + \frac{80\alpha_1 + 20\alpha_2}{t} = 0$$

Rearranges to:

$$t^\star = \frac{(80\alpha_1 + 20\alpha_2)y}{100 + 80\alpha_1 + 20\alpha_2}$$

Grading notes: 1.5 points for correct SWF, 1.5 points for correct FOC, 1 point for correct $t^\star$

(c) (3 points) What level of the lump-sum tax does each voter vote for? Who votes for a higher tax?

Solution: Similarly as above, maximizing each individual utility function w.r.t. t yields $t_1 = \frac{\alpha_1 y}{1+\alpha_1}$ and $t_2 = \frac{\alpha_2 y}{1+\alpha_2}$

Since $\alpha_2 > \alpha_1$, $t_2 > t_1$ **Grading notes:** 1 point for correct t_1 and t_2, 2 points for $t_2 > t_1$

(d) (3 points) Will the vote lead to an efficient level of the tax? Why or why not?

Solution: No, the type 1's will win the vote and the tax will be t_1, which is less than $t^{\star}$. This is because decisions by majority rule do not account for intensity of preferences/a minority group can never win, regardless of the strength of their preferences.
Grading notes: 1.5 points for correct inefficiency, 1.5 points for discussion of intensity of preferences or how a minority group cannot win.

14.41 Public Finance and Public Policy
Fall 2024