MIT 14.41 – Problem Set 3

QUESTION 1: Social Security [33 points]

Tegan makes decisions about two periods in her life: In her working period, she earns a wage w and can save an amount s at interest rate r (with $s \geq 0$). In her retirement period, she doesn't earn anything but consumes her savings (plus interest), and she cares some fraction $\delta \in [0, 1]$ about her retirement period utility relative to her working period utility. She knows that she won't have children and doesn't want to leave any money behind. So, she chooses c_w, an amount to consume during her working period, and c_r, an amount to consume during her retirement period, to maximize her lifetime utility:

$$U = ln(c_w) + \delta ln(c_r)$$

subject to the constraints that:

$$w = c_w + s$$

$$c_r = s(1 + r)$$

1. (5 points) How much does Tegan optimally save? What is Tegan's optimal consumption in each period?

> **Solution:** Tegan solves
>
> $$\max_s ln(w - s) + \delta ln(s(1 + r))$$
>
> FOC wrt s:
>
> $$\frac{-1}{w - s} + \frac{\delta(1 + r)}{s(1 + r)} = 0$$
>
> Rearranges to:
>
> $$s = \frac{\delta w}{1 + \delta}$$
>
> Plugging into the expressions for c_w and c_r, $c_w = \frac{w}{1+\delta}$ and $c_r = \frac{\delta w(1+r)}{1+\delta}$
>
> **Grading notes:** 1 point for correct maximization problem, 1 point for FOC, 1 point for correct c_w, 1 point for correct c_r, 1 point for correct c_s

2. (3 points) Why might the government be concerned that Tegan isn't saving enough?

> **Solution:** The rationale that is consistent with this setup is that the government is paternalistic: they don't think Tegan will save enough for retirement. For example, they may think that she is myopic and her δ is too low. Note that while market failures in the annuity market is a rationale for government intervention in the real world, the problem as it is described here does not feature any market failures.

Grading notes: 3 points for "paternalism" plus intuition behind it or for saying the government might think Tegan is using too small a δ. 1 point if just say "paternalism." Minus 0.5 points for discussing market failures in annuities, since that is not a rationale for this specific case.

3. For the reason(s) you described above, the government decides to help Tegan save for retirement. They require her to put some amount b in a retirement account during her working period, and they return back $b(1+r)$ during her retirement period. She can still choose to put some amount s into a personal savings account, which still returns $s(1+r)$ in the retirement period.

   (a) (2 points) Write down the problem that Tegan will solve now

   Solution: Tegan solves:

   $$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$

   $$\text{s.t. } w = c_w + s + b$$

   $$c_r = (s+b)(1+r)$$

   Grading notes: 2 point for correct set-up

   (b) (5 points) How does this government policy affect Tegan's chosen consumption and **private** savings levels, relative to the case without government intervention? What about her **total** savings? Why?

   Solution: The problem above can also be written as:

   $$\max_{s} ln(w-s-b) + \delta ln((s+b)(1+r))$$

   FOC wrt s:

   $$\frac{-1}{w-s-b} + \frac{\delta(1+r)}{(s+b)(1+r)} = 0$$

   Rearranges to:

   $$s = \frac{\delta w}{1+\delta} - b$$

   Plugging into the expressions for c_w and c_r, $c_w = \frac{w}{1+\delta}$ and $c_r = \left(\frac{(\delta w)}{1+\delta}\right)(1+r)$.
   Tegan puts less into her private savings account (a dollar of b reduces her personal savings by a dollar, until she sets $s = 0$). When $\frac{\delta w}{1+\delta} > b$, her outcomes are unchanged. When $\frac{\delta w}{1+\delta} < b$, the policy forces her to save more and consume less in the working period and consume more in the retirement period (i.e. smooths consumption).

   Grading notes: 1 point for correct s, 1 point for correct c_w and c_r, 1 point for government policy having no effect unless she is saving less than b to begin with, 2 points for in which case it increases saving, and smooths consumption.

   (c) (2 points) What is this type of social security called, and what is its main advantage?

Solution: This is funded social security. It's primary benefit is that it pays retirees using the money that they contributed to the system, so it is always solvent (unless the money was invested badly).

Grading notes: 1 point for funded social security, 1 point for solvency

4. Now, assume that there are 2 otherwise identical people alive at any time, except one is in their working period and the other is in their retirement period. Instead of forcing individuals to save for the own retirement, the government has instituted the following: in their working years, an individual faces a social security tax at rate x, the proceeds of which is paid out to the individual currently in their retirement years. When that person retires, they receive the social security tax proceeds collected from the subsequent generation. Assume that wages are growing at the same rate as savings, so the workers who work when Tegan is retired are earning $w(1+r)$, and Tegan knows this will be true when she makes her initial consumption and savings decisions.

   (a) (2 points) Write down the problem that Tegan will solve now

   Solution: Tegan solves:

$$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$

$$\text{s.t. } w - wx = c_w + s$$

$$c_r = s(1+r) + xw(1+r)$$

   Grading notes: 2 point for correct set-up

   (b) (4 points) Is this policy economically different from the previous part? Why? What is the effect of this policy on Tegan's savings and consumption rates?

   Solution: No – we've just replaced b with xw. Therefore, the policy has the same effect as above.

   Grading notes: 1 point for no, 1 point for replacing b with xw, 2 point1 for policy having the same effect as before.

   (c) (5 points) Now imagine that wage growth was slower than the rate of returns on financial investments (though both are strictly greater than 0). In particular, wages are growing at a rate $\tilde{r} < r$. How would this affect Tegan's saving levels and consumption levels, relative to part (b)?

   Solution: With the rate of wage growth $\tilde{r} < r$, Tegan solves:

$$\max_{c_r, c_w} ln(c_w) + \delta ln(c_r)$$

$$\text{s.t. } w - wx = c_w + s$$

$$c_r = s(1+r) + xw(1+\tilde{r})$$

   Or equivalently:

$$\max_{s} ln(w - wx - s) + \delta ln(s(1+r) + wx(1+\tilde{r}))$$

FOCs:

$$\frac{-1}{w - wx - s} + \frac{\delta(1+r)}{s(1+r) + wx(1+\tilde{r})} = 0$$

Rearranging,

$$s = \frac{\delta w}{1+\delta} - \left(\frac{\delta(1+r) + 1 + \tilde{r}}{(1+\delta)(1+r)}\right) wx$$

The coefficient on xw is strictly less than 1, since it can be written as $\frac{\delta}{1+\delta} + \frac{1+\tilde{r}}{(1+\delta)(1+r)}$, which converges to 1 as $\tilde{r} \to r$, but with $\tilde{r} < r$ there is a smaller numerator in the right side fraction, so the total is less than one. So now a dollar of benefits crowds out less than a dollar of private savings.
Tegan consumes less in period 1 and period 2 than before:

$$c_w = \frac{w}{1+\delta} - wx\left(1 - \frac{\delta(1+r) + 1 + \tilde{r}}{(1+\delta)(1+r)}\right)$$

which is strictly less than $\frac{w}{1+\delta}$ since as we showed above, $\frac{\delta(1+r)+1+\tilde{r}}{(1+\delta)(1+r)} < 1$, and similarly:

$$c_r = \frac{\delta w(1+r)}{1+\delta} + wx\left(1 + \tilde{r} - \frac{\delta(1+r) + 1 + \tilde{r}}{1+\delta}\right)$$

which is strictly less than $\frac{\delta w(1+r)}{1+\delta}$ since the second term is always negative since $1 + \tilde{r} < 1 + r$.

Grading notes: 2 points for correct s. 2 points for now having less than full crowd-out. 1 point for lower consumption in both periods.

(d) (2 points) What is this type of social security called, and intuitively (no math necessary), how does it impact total social savings relative to funded social security?

Solution: This is unfunded social security. Savings are lower relative to a funded system. This is because, with a funded system, the savings effect varies from zero (if there is full crowd out, i.e. $\frac{\delta w}{1+\delta} > b$) to positive (if $\frac{\delta w}{1+\delta} < b$ so the government actually gets citizens to save more). On the other hand, unfunded social security simply transfers resources within the economy (from the young to old), yet private savings are crowded out. This leads to lower savings under unfunded SS compared to funded SS.

Grading notes: 1 point for unfunded social security. 1 point for savings decreasing relative to funded social security. Extra credit for discussing how crowd out is lower under unfunded social security.

Now let's consider another decision affected by social security – the decision to retire. Sara lives in the Netherlands, and is deciding when to retire. Each year, she can earn a wage w if she works, of which she pays t in social security taxes, or, if she retires, she can receive αw in social security benefits for each year going forward. In year 0, she chooses whether or not to retire by maximizing her remaining lifetime income for the next N years:

$$U = \sum_{t=0}^{N} \delta^t ln(c_t)$$

5. (1 point) What is α called?

Solution: α is the replacement rate

Grading notes: 1 point for replacement rate

6. (4 points) Sara is deciding whether to retire in year 0. What inequality should she solve to make this decision? Solve this inequality for an expression in terms of α,w, t, and/or δ, and explain the intuition behind the condition you get. Discuss how each parameter in the condition affects Sara's decision.

Solution: Sara retires in year 0 when her lifetime utility from doing so is greater than her lifetime utility from waiting a year.

$$\sum_{t=0}^{N} \delta^t ln(\alpha w) > ln(w - t) + \sum_{t=1}^{N} \delta^t ln(\alpha w)$$

$$\delta^0 ln(\alpha w) > ln(w - t)$$

$$\alpha w > w - t$$

Alternatively, if students interpreted the tax t as a progressive tax:

$$\sum_{t=0}^{N} \delta^t ln(\alpha w) > ln(w(1 - t)) + \sum_{t=1}^{N} \delta^t ln(\alpha w)$$

$$\delta^0 ln(\alpha w) > ln(w(1 - t))$$

$$\alpha w > w(1 - t)$$

$$\alpha > 1 - t$$

A high payroll tax or a high replacement rate both make it more likely that Sara retires in year 0. Sara will retire when she could earn more in benefits than she could working.

Grading notes: 1 point for set-up, 1 point for correct condition, 2 points for intuition.

QUESTION 2: Education [38 points]

Consider a society with four types of families. Each family chooses s, an amount of quality schooling, and c, all other consumption. Assume that c is a numeraire good with price 1 and the price of one more unit of school quality is $p = \frac{1}{4}$. The four types of families are as follows:

$$y_a = 100; u_a = ln(c) + 4ln(s)$$

$$y_b = 100; u_b = ln(c) + ln(s)$$

$$y_c = 40; u_c = ln(c) + ln(s)$$

$$y_d = 40; u_d = ln(c) + 4ln(s)$$

There are 400 families of type A, 100 of type B, 200 of type C, and 300 of type D.

Assume that the future income of a child is $y^1 = \frac{y^0}{2} + \left(\frac{s}{4}\right)^{\frac{3}{2}}$, where y^0 is their household income when they are a child.

1. (2 points) Do parents' preferences take children's earnings fully into account when they choose their children's level of schooling? How can you tell?

> **Solution:** No, their utility functions do not include terms that accurately reflect the effect of schooling on their children's future income. Heterogeneity in how much parents care about s could come from how much they take this benefit into account, i.e. how "selfish" they are, or from a variety of other things.
> **Grading notes:** 1 point for "no," 1 point for the children's income term not showing up in parents' utility functions.

2. (4 points) First, let's solve for each family's decision when there is no public schooling. Find each type of family's chosen c, s.

> **Solution:** Maximization problems:
>
> - A: $\max_s ln(100 - \frac{s}{4}) + 4ln(s)$
> - B: $\max_s ln(100 - \frac{s}{4}) + ln(s)$
> - C: $\max_s ln(40 - \frac{s}{4}) + ln(s)$
> - D: $\max_s ln(40 - \frac{s}{4}) + 4ln(s)$
>
> FOCs and rearranging:
>
> - A: FOC yields $\frac{-p}{100-ps} + \frac{4}{s} = 0 \rightarrow s = 320, c = 20$
> - B: FOC yields $\frac{-p}{100-ps} + \frac{1}{s} = 0 \rightarrow s = 200, c = 50$
> - C: FOC yields $\frac{-p}{40-ps} + \frac{1}{s} = 0 \rightarrow s = 80, c = 20$
> - D: FOC yields $\frac{-p}{40-ps} + \frac{4}{s} = 0 \rightarrow s = 128, c = 8$
>
> **Grading notes:** 1 point for each solution. Half credit for correct FOCs but math mistake.

3. Now, assume the government introduces public schools which provide the same quality schooling as choosing $s = E$, for free. If a family chooses to send their child to public school, they must consume exactly E education.

(a) (2 points) Under what conditions on E will each type of family choose to send their child to public school?

Solution:

- A: $ln(100) + 4ln(E) > ln(20) + 4ln(320) \rightarrow E > 214$
- B: $ln(100) + ln(E) > ln(50) + ln(200) \rightarrow E > 100$
- C: $ln(40) + ln(E) > ln(20) + 4ln(80) \rightarrow E > 40$
- D: $ln(40) + 4ln(E) > ln(8) + 4ln(128) \rightarrow E > 86$

Grading notes: 0.5 points for each solution

(b) Suppose the government set $E = 45$

i. (2 points) Which families send their children to public school?

Solution: Only family C

Grading notes: 2 points for correct answer; full credit if answer is correct given their expressions in the previous part.

ii. (4 points) What does this policy do to total productivity (measured by total future income of children)?

Solution: Children's income from families A, B, and D are unaffected. Children from families of type C would have $y_1 = 20 + \left(\frac{45}{4}\right)^{\frac{3}{2}} = 57$ when there are low-quality public schools compared to $y_1 = 20 + \left(\frac{80}{4}\right)^{\frac{3}{2}} = 109$ when there are no public schools. So, total productivity decreases by $52 * 200 = 10,400$ as an outcome of the government policy to implement low-quality public schools.
Grading notes: 2 points for correct solution, 1 point if say that total productivity decreases but don't have mathematical solution.

iii. (2 points) What happens to inequality in this society? Here and for the rest of the problem, think about inequality in terms of the range of **children's future incomes**.

Solution:

- The children of family type A have income $y_1 = 50 + \left(\frac{320}{4}\right)^{\frac{3}{2}} = 766$ in either case
- The children of family type B have income $y_1 = 50 + \left(\frac{200}{4}\right)^{\frac{3}{2}} = 404$ in either case
- The children of family type D have income $y_1 = 20 + \left(\frac{128}{4}\right)^{\frac{3}{2}} = 201$ in either case

The range of incomes in society is increased by the introduction of low-quality public schools; those whose parents are at the bottom of the income distribution and care less about education have their relative incomes fall compared to what would have happened in the absence of public schools.

Grading notes: 2 points for the correct answer; 1 point if they just say it increases but don't give rationale or intuition.

(c) Now instead suppose the government set $E = 150$

i. (2 points) Which families send their children to public school?

Solution: Families B, C, D

Grading notes: 2 points for correct answer; full credit if answer is correct given their expressions in the previous part.

ii. (5 points) What does this policy do to total productivity (measured by total children's future income), relative to when there were no public schools?

Solution:

- Children's income from families A are unaffected.
- Children from families of type B would have $y_1 = 50 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 280$ when there are high-quality public schools compared to $y_1 = 50 + \left(\frac{200}{4}\right)^{\frac{3}{2}} = 404$ when there are no public schools.
- Children from families of type C would have $y_1 = 20 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 250$ when there are high-quality public schools compared to $y_1 = 20 + \left(\frac{80}{4}\right)^{\frac{3}{2}} = 109$ when there are no public schools.
- Children from families of type D would have $y_1 = 20 + \left(\frac{150}{4}\right)^{\frac{3}{2}} = 250$ when there are high-quality public schools compared to $y_1 = 20 + \left(\frac{128}{4}\right)^{\frac{3}{2}} = 201$ when there are no public schools.

So, total productivity increases by $(250-201)*300+(250-109)*200-(280-404)*100 = 55,300$ as an outcome of the government policy to implement high-quality public schools.
Grading notes: 4 points for correct calculation and finding that productivity increases.
1 point if say that total productivity increases but don't have mathematical solution. 3 points if the steps are correct but there is a mathematical error. Full credit if answers are correct given previous solutions.

iii. (2 points) What happens to inequality in this society?

Solution: Based on the results in the previous part, high-quality public schools have increased children's future incomes for those with parents at the bottom of the distribution, and decreased children's future incomes for some of those with parents at the top of the distribution (though this hasn't affected the top end of the range, since children of type A still earn the most). Thus, there is less inequality than in the case with no public schools (or low-quality public schools!). The overall income range is smaller, and more children who grow up at the bottom of the distribution are moved up than the number who grow up at the top move down.

Grading notes: 3 points for the correct answer; 1 point if they just say it decreases but don't give rationale or intuition.

(d) (4 points) Discuss how and why parts (b) and (c) of this question differ in terms of productivity and inequality.

Solution: In part (b), the government implements low-quality public schools and only children from the poorer families whose parents don't care as much about education go to those schools. The provision of these schools actually crowds out schooling that these kids would have gotten absent the public schools, so total productivity goes down and inequality increases. However, when the government implements high-quality public schools and children from the top and bottom of the income distribution attend them, overall productivity increases and inequality decreases. There is still crowd-out (kids from families of type B get less schooling) but it is offset but large increases in the amount of schooling that kids from the bottom of the income distribution get.

Grading notes: 2 points for discussing crowd out (1 point for crowd out in (b), 1 point for how this compares to crowd-out in (c)), 2 points for intuition about who goes to public school in each case and relating this to the effect on productivity/inequality.

4. Instead of introducing public schools, suppose that in addition to their income, each family gets voucher of value V dollars from the government that they can use on education.

   (a) (4 points) Write down each family's utility maximization problem, and solve for each family's choice of s and c (given V).

Solution:

- A: $\max_s ln(100 + V - ps) + 4ln(s)$ s.t. $ps \geq V$
- B: $\max_s ln(100 + V - ps) + ln(s)$ s.t. $ps \geq V$
- C: $\max_s ln(40 + V - ps) + ln(s)$ s.t. $ps \geq V$
- D: $\max_s ln(40 + V - ps) + 4ln(s)$ s.t. $ps \geq V$

Taking FOCs and solving:

- A: FOC yields $\frac{-p}{100+V-ps} + \frac{4}{s} = 0 \rightarrow s^\star = \frac{4(400+4V)}{5}, c^\star = \frac{100+V}{5}$
- B: FOC yields $\frac{-p}{100+V-ps} + \frac{1}{s} = 0 \rightarrow s^\star = \frac{4(100+V)}{2}, c^\star = \frac{100+V}{2}$
- C: FOC yields $\frac{-p}{40+V-ps} + \frac{1}{s} = 0 \rightarrow s^\star = \frac{4(40+V)}{2}, c^\star = \frac{40+V}{2}$
- D: FOC yields $\frac{-p}{40+V-ps} + \frac{4}{s} = 0 \rightarrow s^\star = \frac{4(160+4V)}{5}, c^\star = \frac{40+V}{5}$

Note: these are all conditional on $ps^\star \geq V$. If that condition for each family is not met, then they set $s = 4V$ and $c = y$ to max utility: i.e. if

- A: $V \geq 400 \rightarrow s^\star = 4V$ and $c^\star = 100$
- B: $V \geq 100 \rightarrow s^\star = 4V$ and $c^\star = 100$
- C: $V \geq 40 \rightarrow s^\star = 4V$ and $c^\star = 40$
- D: $V \geq 160 \rightarrow s^\star = 4V$ and $c^\star = 40$

Grading Notes: 0.5 points for each family's maximization problem and 0.5 points for each family's choice of s and c. Minus 1 point if they don't discuss the conditions under which they set $s = 4V$ (but they don't need to explicitly calculate in each case).

(b) (4 points) What level of schooling would each family choose if $V = 11.25$ *dollars*? How would this affect productivity and inequality, relative to the world without any public education or voucher system? (Note that, in terms of units of schooling, the voucher is worth $4 \times 11.25 = 45$ units)

> **Solution:**
>
> - A would choose $s = 356$, up from $s = 320$ in a world without vouchers or public schools, spending most of the voucher to purchase 36 more units of education quality and the rest to purchase (45-36)/4 = 2.25 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 890$.
> - B would choose $s = 222.5$, up from $s = 200$ in a world without vouchers or public schools, spending half of the voucher to purchase 22.5 more units of quality education and the other half to purchase 5.625 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 465$.
> - C would choose $s = 102.5$, up from $s = 80$ in a world without vouchers or public schools, spending half of the voucher to purchase 22.5 more units of quality education and the other half to purchase 5.625 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 150$.
> - D would choose $s = 164$, up from $s = 128$ in a world without vouchers or public schools, spending most of the voucher to purchase 36 more units of education quality and the rest to purchase 2.25 more units of consumption. Plugging in, children of these families go on to earn $y_1 = 283$.
>
> Since all students are getting more education, productivity goes up unambiguously. It goes up by $(890 - 766) * 400 + (465 - 404) * 100 + (150 - 109) * 200 + (283 - 250) * 300 = 73,800$. Inequality has also increased, if we look at the range of the incomes in society, but mobility is greater (i.e. those at the bottom can get relatively closer to the top by investing more in education, compared to under public provision of education where they are more "stuck.")
> **Grading Notes:** 2 points for correct calculations of the four levels of equilibrium schooling. 1 point for productivity going up unambiguously (or the calculation, don't need both). 1 point for saying that inequality has increased by the metric of the range of incomes in society OR that mobility is greater.

5. Now, let's think about the differences you found in the effects of the policies in part 2 and part 3.

(a) (2 points) If the government is politically or financially constrained and can only offer $E = 45$ or $V = 11.25$ (worth 45 units of schooling), which should they do? Comment on the different effects of these policies on productivity and inequality.

> **Solution:** Public education and vouchers of the same size (45) have very different effects on productivity and inequality: this is because at this level, the government can only provide low-quality public schools that only the poorest children whose parents don't care as much about education will attend. And, because of the increase in consumption available, children from these families actually get less education than they would in the private market – public education crowds out private education. On the other hand, spending the same amount of money on vouchers leads all families to invest more in education, increasing productivity. Vouchers solve this crowd-out problem, holding the benefit offered to families fixed. Both increase inequality, though vouchers do so by more because all families get the benefit regardless of their income, and the government ends up paying some of the private school costs that families would have incurred in the absence of the program.

Grading notes: 1 point for correct observations about levels of schooling and productivity, and 1 points for correct observations about inequality. 1 point if they draw the right conclusion but don't provide logic.

(b) (2 points) If the government is unconstrained, should they offer the the policy you recommended in (a) or $E = 150$? Comment on the different effects of these policies on productivity and inequality.

Solution: High-quality education can increase productivity (though not as much as lower-cost vouchers) and lower inequality, whereas vouchers increase inequality as described above. Which policy is preferred will depend on the government's social welfare function.

Grading notes: 1 point for correct observations about levels of schooling and productivity, and 1 points for correct observations about inequality. 1 point if they draw the right conclusion but don't provide logic.

QUESTION 3: Unemployment Insurance [29 points]

In this question, we consider the optimal design of unemployment insurance (UI). To do this, we'll consider a model with a single representative agent, Julie, who spends fraction $f(e)$ of her time employed, and the remaining $1 - f(e)$ of their time unemployed, where e is the amount of effort she puts into job search. To simplify the problem, we will use $f(e) = e$.

Julie earns w before taxes when employed, and nothing when unemployed. She receives UI of b when unemployed, so her income when unemployed is b. This UI is financed by a tax on Julie when she is employed: $\tau(b) = b\frac{1-e}{e}$.

Julie is hand-to-mouth, meaning that income is immediately consumed. This means there is no self-insurance or informal insurance: she cannot save in the good period (employment) for the bad (unemployment), or seek support from anyone including family. Consumption after all taxes and transfers in the two states of the world are therefore given by:

$$c_e = y_e = w - \tau(b)$$
$$c_u = b$$

Julie gets utility $u(c)$ from consumption, where $u(0) = 0, u'(c) > 0, u"(c) < 0$. As mentioned earlier, she can increase the amount of time they spend in employment by spending more effort e on job search (notice more effort corresponds directly to more time spent in employment), but this gives her disutility $\phi(e) = e^2$. Therefore, her expected utility is

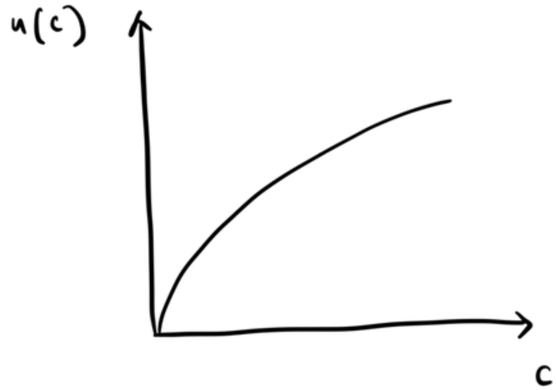
$$W(b,e) = eu(c_e) + (1-e)u(c_u) - \phi(e)$$

1. (3 points) While this model features a representative agent, more generally, labor markets feature heterogeneous workers. A private market for unemployment insurance may not exist due to adverse selection. Briefly explain the information asymmetry between workers and private insurers, and how it drives adverse selection such that the private sector fails to provide UI.

Solution: Workers know their risk of unemployment much better than private insurers. Workers who face a higher risk of unemployment are more likely to purchase UI, while those with lower risks are likely to opt out as insurance prices rise so that insurers charge higher premiums to break even. This creates a situation where the insurance pool becomes disproportionately filled with high-risk indviduals, driving up premiums until it is unprofitable or unsustainable for the private sector to provide UI coverage.

2. (2 points) Draw $u(c)$ against c. What can you infer about Julie's risk preference and demand for insurance?

Solution:



1 point for correct concave graph. 1 point for pointing out that Julie is risk averse and arguing she will demand full insurance because $MU_u > MU_e$, so smoothing consumption makes her better off.

3. (4 points) Suppose that due to adverse selection, there is no private market. However, the government is able to observe and control Julie's search effort e.

   (a) (2 points) What level of effort and UI benefits will the government choose to maximize Julie's utility? Leave your answer in terms of the first order conditions.

Solution:

$$\max_{b,e} eu(c_e) + (1-e)u(c_u) - \phi(e)$$
$$\Rightarrow \max_{b,e} eu(w - b\frac{1-e}{e}) + (1-e)u(b) - e^2$$
$$\frac{\partial W(b,e)}{\partial e} = u(w - b\frac{1-e}{e}) + eu'(w - b\frac{1-e}{e})b\frac{1}{e^2} - u(b) - 2e = 0$$
$$\frac{\partial W(b,e)}{\partial b} = -e\frac{1-e}{e}u'(w - b\frac{1-e}{e}) + (1-e)u'(b) = 0$$

   (b) (2 points) Simplify your first order condition with respect to b. What does this imply about the government's optimal provision of UI when it can observe effort?

Solution: The government provides UI to achieve full consumption smoothing by Julie across both states of the world.

$$u'(w - b\frac{1-e}{e}) = u'(b)$$
$$w - b\frac{1-e}{e} = b \iff c_e = b$$

4. (10 points) In practice, the government cannot observe or control Julie's search effort e. However, it can design the level of UI to maximize Julie's utility, anticipating Julie's optimal choice of effort in response to benefits. Let's solve for the optimal level of UI in this more realistic case in steps.

(a) (3 points) What level of effort e^* will Julie choose, conditional on the benefit level b, as a function of c_e and c_u? Note that Julie takes taxes as fixed, i.e. she does not internalize the fact that her choice of effort affects the tax she ends up paying.

Solution:

$$\max_e eu(c_e) + (1-e)u(c_u) - e^2$$
$$\Rightarrow u(c_e) - u(c_u) - 2e = 0$$
$$\Rightarrow e^* = \frac{u(c_e) - u(c_u)}{2}$$

(b) (3 points) Now suppose $u(c) = \sqrt{c}$ and $w = 1$. Solve for Julie's optimal e: (a) when $b = 0$ and (b) when $b = 0.1$. Interpret in words what this means for how increases in benefits change the probability of unemployed people finding a job.

Solution:

When $b = 0$, $e = \frac{\sqrt{1}-\sqrt{0}}{2} = 0.5$.

When $b = 0.1$, $e = \frac{\sqrt{1-0.1\frac{1-e}{e}}-\sqrt{0.1}}{2} \Rightarrow e = 0.130$ or 0.268. Plugging these values for e into the utility function, we can see that $e = 0.268$ results in higher utility than $e = 0.130$.

Higher benefits reduce the probability of unemployed people finding a job.

Grading notes: 1 point for correct e when $b = 0$, 1 point for correct e when $b = 0.1$ (even if they do not argue 0.268 is the unique optimal level of e), 1 point for higher benefits reducing search effort.

(c) (1 point) What do we call the effect you demonstrated in part (b)?

Solution: This is an example of moral hazard: providing more insurance makes the bad state (unemployment) relatively more desirable, and so makes people take behaviour that increases their risk of being in the bad state (searching for jobs less).

(d) (3 points) Calculate Julie's expected utility: (a) when $b = 0$ and (b) when $b = 0.1$. How has the provision of social insurance affected her expected utility and why?

Solution: When $b = 0, EU = 0.5\sqrt{1} + 0.5\sqrt{0} - 0.5^2 = 0.25$

When $b = 0.1, EU = 0.268\sqrt{1 - 0.1\frac{1-0.268}{0.268}} + (1 - 0.268)\sqrt{0.1} - 0.268^2 = 0.388$

Julie's expected utility has increased because the provision of UI transfers utility from her high marginal utility of consumption state (employment) to her low marginal utility of consumption state (unemployment).

Grading notes: 1 point for correct calculation of EU when $b = 0$, 1 point for correct calculation when $b = 0.1$, and 1 point for explaining that her EU has increased because UI helps her smooth her consumption. Extra credit for pointing out that when $b = 0$, her marginal utility of consumption when unemployed is $\frac{1}{2\sqrt{c}} = \infty$.

5. (4 points) Using your answers to 3(b), 4(b) and 4(d), what can you conclude about the government's key tradeoff in providing unemployment insurance?

Solution: The key tradeoff the government must manage in designing UI is providing *consumption smoothing* for workers by transferring resources from times when they have low marginal utility (employment) to times when they have high marginal utility of consumption (unemployment), versus creating *moral hazard* as people reduce search effort in response to more generous UI.

Grading notes: One point for mentioning consumption smoothing, and another for explaining it. One point for mentioning moral hazard, and another for explaining it.

6. (6 points) During the peak of the COVID-19 pandemic, the federal government substantially increased unemployment insurance payments as part of the Coronavirus Aid, Relief and Economic Security (CARES) Act, in many cases paying people as much as they earned in their previous jobs (so that $b = w$). Before the pandemic, unemployment insurance payments were much lower so that usually $(w - \tau) - (b)$ was reasonably large. Suggest and explain two possible reasons why the US government might have temporarily increased the size of its unemployment insurance payments during the pandemic.

Solution:

- Reduced moral hazard in a weak labor market: The moral hazard effects of UI may have been lower in the pandemic, meaning that workers could not get hired regardless of the amount of effort that they put into search. This implies that more consumption smoothing – i.e. increasing UI benefits – is desirable.
- Aggregate demand effects: If product demand is inefficiently low in a recession like it was during the pandemic, higher UI generosity can increase unemployed workers' purchasing power, raise aggregate demand and speed up economic recovery.
- Public health compliance: By temporarily raising benefits to levels close to/exceeding previous wages, the government can reduce the urgency for people to find work, especially in sectors where remote work was not feasible. This could have helped reduce the spread of Covid-19.

Grading notes: 3 points per reasonable answer.