Problem Set 5

14.41 Public Finance and Public Policy

Grading notes: All effort should be made not to punish the same mistake twice and to allow for 'follow-on' marks. So if an error is made in one sub-question but subsequent answers are correct *given* the error, full marks should be awarded, unless the error has made subsequent questions substantially easier.

1 Incidence of Air Taxes [25 points]

Most countries levy significant taxes on air travel. Who bears the burden of these taxes? Suppose that the demand for air travel, $D(p)$, is a function of the ticket price p, where

$$D(p) = Ap^{-\alpha},$$

where $A > 0$ and $\alpha > 0$. All airlines have the same production function

$$y = f(k) = k^{1-\sigma}/(1-\sigma),$$

where $0 < \sigma < 1$ and capital k (i.e., airplanes) is supplied perfectly elastically at a price r.

(a) (4 points) Find the demand elasticity for air travel.

Solution: The demand elasticity is defined as

$$\eta_d = \frac{p}{D(p)} \frac{\partial D(p)}{\partial p}.$$

The derivative $D'(p) = -\alpha A p^{-1-\alpha}$, and so we obtain,

$$\eta_d = \frac{p}{Ap^{-\alpha}} \left(-\alpha A p^{-1-\alpha}\right) = -\alpha.$$

Grading: 2 points for correct definition of demand elasticity, 2 points for correct result

(b) (5 points) Find the supply elasticity for air travel. (*Hint:* You may find the fact that $\frac{p}{x(p)} \frac{\partial x(p)}{\partial p} = \frac{\partial \log x(p)}{\partial \log p}$ useful here.)

Solution: The airline's problem is

$$\max_k \{pf(k) - rk\},$$

which has the first order condition with respect to k of

$$pf'(k) = r \implies pk^{\sigma} = r \implies k^*(p) = \left[p/r\right]^{1/\sigma}.$$

The supply of flights is thus

$$S(p) = f(k^*(p)) = \left[p/r\right]^{\frac{1-\sigma}{\sigma}}/(1-\sigma).$$

To compute the supply elasticity, we note that

$$\eta_s = \frac{p}{S(p)} \frac{\partial S(p)}{\partial p} = \frac{\partial \log S(p)}{\partial \log p},$$

and then write the log-supply equation:

$$\log S(p) = \frac{1-\sigma}{\sigma}\left[\log p - \log r\right] - \log(1-\sigma),$$

and so taking the derivative with respect to $\log p$, we obtain

$$\eta_s = \frac{1-\sigma}{\sigma}.$$

Grading: 2 points for airline problem solution, 1 points for supply function, 2 points for elasticity

(c) (4 points) How much would after-tax ticket prices rise in response to a new per-ticket tax of τ?

Solution: We know that the incidence formula is

$$\frac{\Delta p}{\tau} = \frac{\eta_d}{\eta_s - \eta_d}.$$

Substituting our results from (4a) and (4b), and simplifying, we obtain that

$$\frac{\Delta p}{\tau} = -\frac{\alpha\sigma}{\alpha\sigma + \sigma - 1}.$$

Grading: 2 points for incidence formula, 2 points for answer

(d) (4 points) Give an example of a travel destination that is likely to have a low α (in absolute value) and one that is likely to have a high α (in absolute value). Explain why.

Solution: One reason a destination could have a low (high) α is that it has few (many) close substitutes. A plausible example of such a low-α destination is London. If it became more expensive to fly to London, few people would choose to go to Paris or Berlin instead. By contrast, Caribbean islands are plausibly close substitutes, and so they should have high αs. If it became more expensive to fly to one Caribbean island, many people would simply choose to travel to other Caribbean islands instead.

Grading: 4 points for reasonable answers with explanations

(e) (4 points) Imagine a more-complicated model with two types of customers, business and vacation travelers. Which is likely to have the higher α? If Zoom meetings make most business travel unnecessary, without changing the demand for vacations, how will this change the incidence of taxes on air travel?

Solution: Vacation travelers are likely to be more price-elastic than business travelers, as vacation travelers can plan the times/dates of their flight to minimize travel cost, whereas business travelers are typically less flexible. If Zoom reduces business travel, then the marginal traveler is more likely to be a vacation traveler, and so the demand elasticity will rise. As a result, we know that travelers will bear less of the tax burden and airlines will bear more.

Grading: 2 points for travelers more elastic, 2 points for lower traveler incidence after Zoom

(f) (4 points) Governments assess taxes on both travelers (such as ticket taxes) and airlines (such as jet-fuel taxes and overflight fees). Explain why this might puzzle an economist. Propose one reason why governments might tax both sides of this market, other than a lack of sophistication among travelers.

Solution: It is puzzling because taxing both sides of the market for air travel is redundant from the perspective of economic incidence.

Students could propose many resolutions of this puzzle. One idea is that different levels of government are taxing different sides of the market as "user fees" for the types of infrastructure they provide. For example, in the U.S., taxes on airlines are collected by the federal government as user fees for air traffic control, whereas taxes on travelers are primarily collected by local governments as user fees for airport infrastructure.

Another idea is that it may be easy for airlines to pay the federal government, but a hassle to pay thousands of different airports. By contrast, airports find it easy to collect from travelers who pass through their facilities.

Grading: 2 points for why taxing both sides is puzzling, 2 points for reasonable explanation (be generous)

2 Deadweight Loss and Progressive Taxation [30 points]

Suppose there are two types of workers, high-wage workers and low-wage workers, which each represent half the population. Before the introduction of a tax, high-wage workers earn an hourly wage $w_H = \$100$, and low-wage workers earn $w_L = \$10$. Both workers supply 2,000 hours of labor each year. The elasticity of labor supply is $\eta_{s,H}$ for high-wage workers and $\eta_{s,L}$ for low-wage workers. The elasticity of labor demand is η_d for both types of workers.

For now, assume the following: The government applies a flat tax rate of $\tau = 0.3$ on earnings to each worker. The elasticities of labor supply are $\eta_s = 0.2$ for both types of workers and the elasticity of labor demand is $\eta_d = -0.3$.

(a) (2 points) As a result of the tax, by how much do annual hours of labor fall for both types of workers?

Solution: The percentage change in quantity in response to a tax rate τ is

$$\frac{\Delta h_j}{h_j} = \frac{\eta_d \eta_{s,j}}{\eta_{s,j} - \eta_d}\tau,$$

for both worker types j. We can now use this formula to find the changes in hours:

$$\frac{\Delta h_j}{2000} = \frac{(0.2)(-0.3)}{0.2 - (-0.3)}(0.3) = -0.036 \implies \Delta h = -72,$$

so annual labor hours should fall by about 72 for both high- and low-wage workers.

Grading: 1 point for correct formula, 1 point for calculation

(b) (2 points) How much do the high- and low-wage workers now earn per hour, in terms of their pre-tax wage? (*Hint:* Is the income tax paid by consumers or producers of labor?)

Solution: The percentage change in price in response to a tax rate τ is

$$\frac{\Delta w_j}{w_j} = \frac{\eta_{s,j}}{\eta_{s,j} - \eta_d}\tau$$

for both worker types j. We can now use this formula to find the changes in wages for both types:

$$\frac{\Delta w_H}{100} = \frac{0.2}{0.2 - (-0.3)}(0.3)$$
$$\frac{\Delta w_L}{10} = \frac{0.2}{0.2 - (-0.3)}(0.3),$$

and we find that

$$\Delta w_H = \$12.71,\ \Delta w_L = \$1.27,$$

so the new wages are

$$w'_H = \$112.71,\ w'_L = \$11.27.$$

Grading: 1 point for correct formula, 1 point for calculation

(c) (2 points) How much revenue does the government raise on average per capita?

Solution: The government's average per-capita revenue is

$$R = \frac{\tau_H w'_H h'_H + \tau_L w'_L h'_L}{2} = \frac{1}{2}(0.3)(112.71 + 11.27)(1924) = \$35,855.02.$$

Grading: 2 points for calculation

(d) (5 points) What is the average per-capita deadweight loss? (*Hint:* Is the income tax a per-unit tax or an ad-valorem tax?)

Solution: The deadweight loss triangle formula for a type-j worker is

$$\text{DWL}_j = -\frac{\eta_{s,j}\eta_d}{2(\eta_{s,j} - \eta_d)}\tau^2 w_j h_j,$$

and so substituting in the appropriate values, we obtain

$$\text{DWL}_H = -\frac{(0.2)(-0.3)}{2(0.2 - (-0.3))}(0.3)^2(100)(2000) = \$1,080$$
$$\text{DWL}_L = -\frac{(0.2)(-0.3)}{2(0.2 - (-0.3))}(0.3)^2(10)(2000) = \$108.$$

Then the average per-capita deadweight loss is

$$\text{DWL} = \frac{\text{DWL}_H + \text{DWL}_L}{2} = \frac{\$1,080 + \$108}{2} = \$594.00.$$

Grading: 2 points for correct formula, 3 points for correct calculations

Suppose now that the government changes its mind and wants to implement a progressive tax schedule instead. In particular, there will be a standard deduction of \$20,000 per year and then a tax rate of τ'. The elasticities are the same as above.

(e) (2 points) How many hours of labor do low-wage workers supply? Explain why your answer does not depend on τ'.

Solution: Let's conjecture that the standard deduction will mean that the low-wage worker does not pay any income tax. If the flat-tax rate was $\tau = 0$, then the low-wage worker supplies 2,000 hours of labor and earns an hourly wage of \$10. Thus, the low-wage worker earns \$20,000 a year, or exactly the standard deduction. So this confirms our conjecture. The rate τ' is irrelevant to the low-wage worker's decision because he will never earn more than \$20,000, the level of income at which that rate kicks in.

Grading: 2 points for correct explanation

(f) (2 points) How many hours of labor do high-wage workers supply? (*Hint:* Your answer should be a linear function of τ'.)

Solution: By our logic above, he must face the positive marginal tax rate τ'. His hours therefore fall by

$$\frac{\Delta h_H}{2000} = \frac{(0.2)(-0.3)}{0.2-(-0.3)}\tau' \implies \Delta h_H = -240\tau',$$

and so he supplies

$$h'_H = 2000 - 240\tau'.$$

Grading: 2 points for correct calculation

(g) (2 points) How much do the high- and low-wage workers now earn per hour, in terms of their pre-tax wage? (*Hint:* For at least one of type of worker, your answer should be a linear function of τ'.)

Solution: We showed above the low-wage worker earns \$10 an hour. For the high-wage worker, we have

$$\frac{\Delta w_H}{100} = \frac{(0.2)}{0.2-(-0.3)}\tau' \implies \Delta w_H = 40\tau',$$

and so the high-wage worker now earns

$$w'_H = 100 + 40\tau'.$$

Grading: 2 points for correct calculation

(h) (5 points) What tax rate τ' is required to raise the same per-capita amount of revenue as in part (c)? (*Hint:* Feel free to use a solver, like WolframAlpha, to obtain τ'. Your tax rate should be positive but less than 100%.)

Solution: We have already determined that this progressive tax only raises revenue from the high-wage workers, so the per-capita revenue simplifies to

$$R = \frac{1}{2}\left[\tau'(w'_H h'_H - 20000)\right] = \frac{1}{2}\left[\tau'((100+40\tau')(2000-240\tau') - 20000)\right].$$

We want to raise the same amount of revenue as before, so we want to solve the following equation for τ':

$$\frac{1}{2}\left[\tau'(w'_H h'_H - 20000)\right] = \frac{1}{2}\left[\tau'((100+40\tau')(2000-240\tau') - 20000)\right] = 35,855.02$$
$$\implies \tau' = 0.360,$$

which is intuitive in that the tax rate on the high-wage workers must rise in order to offset the revenue loss from exempting low-wage workers.

Grading:

(i) (2 points) How much per-capita deadweight loss does this tax schedule create?

Solution: The per-capita DWL is

$$\text{DWL} = -\frac{(0.2)(-0.3)}{4(0.2-(-0.3))}(0.360)^2(100)(2000) = \$777.60.$$

Grading: 3 points for correct setup of revenue equation, 2 points for correct calculation

(j) (2 points) Is the per-capita deadweight loss larger under the progressive tax or the flat tax? Explain why or why not. How does this relate to the equity-efficiency trade-off that we have previously discussed?

Solution: Yes, the per-capita deadweight loss is higher under the progressive tax than under the flat tax. This is because deadweight losses are increasing in the square of the tax rate. Just as in commodity taxes we want a "low rate" and "broad base," progressive taxation shrinks the base and creates higher rates, so we know it will involve more deadweight loss per dollar of revenue than the flat tax will.

The connection to the equity–efficiency trade-off is that a progressive tax may be viewed as more equitable, in that it avoids burdening the poor. However, progressivity has efficiency costs, as we have shown in this problem.

Grading: 2 points for correct calculation

Now suppose that $\eta_{s,H}$ is larger than $\eta_{s,L}$

(k) (2 points) Would the additional deadweight loss from progressive taxation, relative to the flat tax, be larger or smaller than in part (d)? Explain intuitively (no math).

Solution: The additional deadweight loss from progressivity will be higher, because we know that deadweight losses are increasing in both the elasticity of supply and demand. The progressive tax would then shift more of the tax burden onto the more-elastic agent, which is a recipe for a big deadweight loss.

Grading: 1 point for DWL, 1 point for equity-efficiency

(l) (2 points) Can you think of any reason why, in the real world, $\eta_{s,H}$ might be larger than $\eta_{s,L}$?

Solution: High-wage earners likely have more ways to reduce their taxable income than low-wage earners. If there are some fixed costs to tax avoidance, like hiring an accountant or a tax lawyer, then those with larger tax burdens (i.e., high-wage workers) will be more likely to pay these fixed costs, whereas they won't be worthwhile for low-wage workers. As a result, high-wage workers' labor supply may be more responsive to taxes than that of low-wage workers.

Grading: 2 points for reasonable answer

3 Q3 [25 points]

There are two individuals in a society: a high-income person and a low-income person. There is a public healthcare clinic run by the government that anyone can go to for basic medical services. The clinic's operation is funded by taxing each individual to cover half the cost of total clinic services used. The cost per unit of healthcare service is 1 dollar.

The high earner's utility, a function of healthcare services used h_H and h_L by the high-income (H) and low-income (L) individuals, is given by:

$$u_H = \sqrt{h_H} - \underbrace{\frac{h_L + h_H}{2}}_{\text{tax to fund clinic}} .$$

The low-income person, who values healthcare services more than the high-income person, has utility:

$$u_L = 2\sqrt{h_L} - \frac{h_L + h_H}{2}.$$

1. (3 points) How much healthcare service will each person choose to use?

   Solution: The first-order condition for the high-income individual is

   $$\frac{1}{2\sqrt{h_H}} = \frac{1}{2} \implies h_H = 1.$$

   Similarly for the low-income individual:

   $$\frac{2}{2\sqrt{h_L}} = \frac{1}{2} \implies h_L = 4.$$

   Grading: 1 for the correct utility functions, 1 for each of the correct answers.

2. (3 points) If welfare is defined as $Welfare = u_H + u_L$, what are the welfare-maximizing quantities of healthcare services used for each person?

   Solution: Welfare is

   $$W = \sqrt{h_H} + \sqrt{h_L} - h_L - h_H.$$

   The first-order conditions are

   $$\frac{1}{2\sqrt{h_H}} = 1 \quad \text{and} \quad \frac{1}{2\sqrt{h_L}} = 1,$$

   which gives

   $$h_L = 1, \quad h_H = \frac{1}{4}.$$

   Grading: 1 for the correct welfare function, 1 for the correct FOCs, 1 for the correct answers.

3. (4 points) Are the welfare-optimal levels of healthcare service use higher or lower than the individually chosen levels? Explain intuitively why this is so.

   Solution: The individually chosen levels are higher than socially optimal, as the fiscal externality is not taken into account by the individuals.

   Grading: 2 for noting they are higher than optimal, 2 for clearly explaining the negative externality.

Now suppose the government can impose a waiting time of w minutes per unit of healthcare service used (assume any fraction of a unit can be used). The utilities of each type are now

$$u_H = \sqrt{h_H} - wh_H - \frac{h_L + h_H}{2}$$

and

$$u_L = 2\sqrt{h_L} - \alpha w h_L - \frac{h_L + h_H}{2}.$$

Note that α represents the difference in sensitivity to waiting times: if $\alpha > 1$, the low-income person dislikes waiting more than the high-income person; if $\alpha < 1$, the opposite is true; and if $\alpha = 1$, their disutility from waiting is equal.

4. (4 points) Suppose $\alpha = 1$. Show that the government can set a $w > 0$ so that the individually optimal choices will be socially optimal. What is the w that achieves this? Hint: you can do this without math.

   Solution: If $w = \frac{1}{2}$, then the externality will be internalized, and private choices will align with socially optimal choices.

   Grading: 4 points for showing that private choices with $w = 1/2$ align with the planner's FOCs. Alternatively, give 2 points if they redefine welfare using the new utility functions and find no such w.

The government will set w after which the individuals will privately optimize.

5. (3 points) Find the individuals' privately optimal choices of h_L and h_H as functions of w and α.

   Solution: For a given w, the individual FOCs will be

$$\frac{1}{2\sqrt{h_H}} - w - \frac{1}{2} = 0$$

   and

$$\frac{2}{2\sqrt{h_L}} - \alpha w - \frac{1}{2} = 0.$$

   Solving yields

$$h_H = \frac{1}{(2w+1)^2} \quad \text{and} \quad h_L = \frac{4}{(2\alpha w + 1)^2}.$$

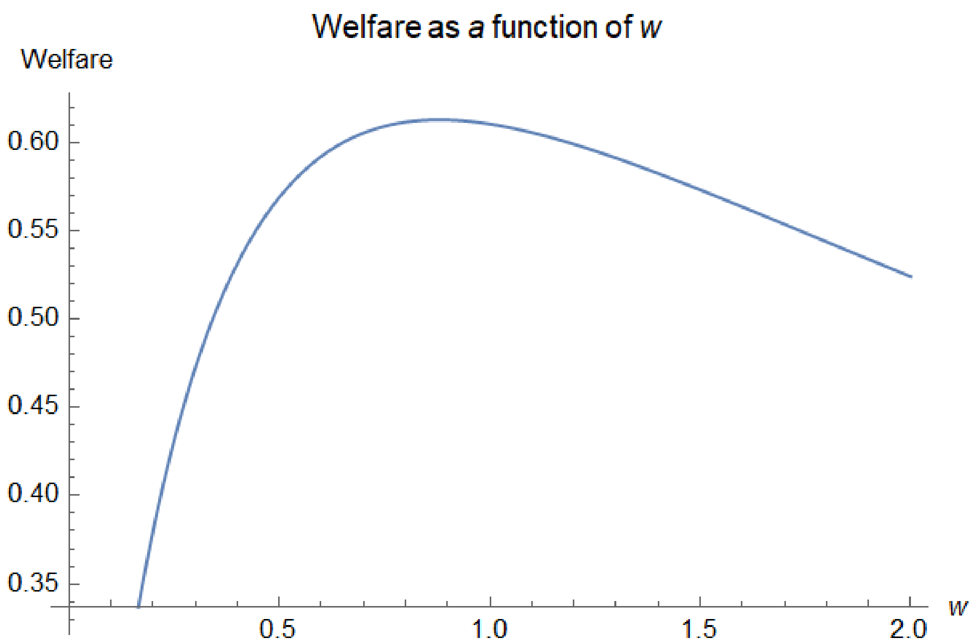
   Grading: 1 point for each correct FOC, 1 for correct answers.

6. (3 points) Suppose $\alpha = \frac{1}{2}$. Write down welfare as a function of w only (after individuals have made their private choices of h_H and h_L). This will not be a simple equation. Show that welfare is optimized at some w greater than the welfare-optimizing level from part 4 (when $\alpha = 1$). You can do this in two ways: (1) use calculus, but be aware that the math is not easy. (2) Evaluate this numerically. Use your favorite programming tool to graph the welfare as a function of w, and visually see around which values of w you see a local maximum.

   Solution: Plugging in the private choices from the preceding part and $\alpha = \frac{1}{2}$, welfare can be written as

$$Welfare = -\frac{2w}{(w+1)^2} - \frac{w}{(2w+1)^2} + 4\overline{\frac{1}{(w+1)^2}} + \overline{\frac{1}{(2w+1)^2}} - \frac{4}{(w+1)^2} - \frac{1}{(2w+1)^2}.$$

   Plotting welfare, we have:



   The maximum is numerically verified to be at $w^* \approx 0.88$, which is greater than the optimal w of $1/2$ from part 4.

   Grading: 1 for the correct expression, 1 for a plot, 1 for noting the new optimal w is greater than $1/2$. Hint: think about the role of α described between questions 3 and 4.

7. (5 points) Explain intuitively why the socially optimal waiting cost is higher when $\alpha = 1/2$ than when $\alpha = 1$.

   Solution: Waiting cost here plays two roles: it corrects the fiscal externality, as in part 4, but $w = 1/2$ is sufficient for this. The waiting cost also serves as a screening mechanism between the two people. The low-income person, who values healthcare services more, dislikes waiting less when $\alpha = 1/2$. By increasing w beyond $1/2$, the high-income person reduces their healthcare use by more than the low-income person does. Although the low-income person also gets less healthcare and pays a waiting cost, the benefit from the reduced cost of the high-income person's healthcare outweighs this.

Grading: 2 for noting waiting cost corrects fiscal externality, 3 for discussing its redistributive role, especially as it relates to targeting low-income individuals.