MIT 14.41 – Problem Set 6

QUESTION 1: Redistribution and moral hazard [40 points]

In this problem, we will consider multiple types of redistribution in a society in which there are three types of families:

- Families of type H can earn a market wage of $45 per hour to support three people – a working parent, a stay-at-home parent, and a child. They have a family utility function $U_H = 3ln(C) + ln(L)$.
- Families of type P can earn a market wage of $15 per hour to support three people – a working parent, a stay-at-home parent, and a child. They have a family utility function $U_P = 4ln(C) + ln(L)$.
- Families of type S can earn a market wage of $15 per hour to support two people – a working parent and a child, but have to pay $10 per hour for childcare when working. They have a family utility function $U_S = 4ln(C) + ln(L)$ (but note that unlike the other families who have $C = Y$, their $C = Y - 10H$ if Y is income).

where C is all non-childcare consumption of the family which has a unit price of $1, L is units of leisure (time spent not working) of the worker, and H is hours the worker spends working. All working parents have a maximum of 2,000 hours per year of work/leisure, and there is no saving.

1. (6 points) Calculate the choice over consumption and leisure that each family will make. What is the per-person consumption in each family?

Solution: Each family chooses L and C to maximize their utility subject to the budget constraint $C = wH = w(2000 - L)$. Specifically, families of type H solve

$$\max_L 3ln(45(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-3}{2000 - L} + \frac{1}{L} = 0 \rightarrow L = 500, H = 1500, C = 67500$$

So per-capita consumption in family H is $22500.

Families of type P solve

$$\max_L 4ln(15(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4}{2000-L}+\frac{1}{L}=0 \rightarrow L=400, H=1600, C=24000$$

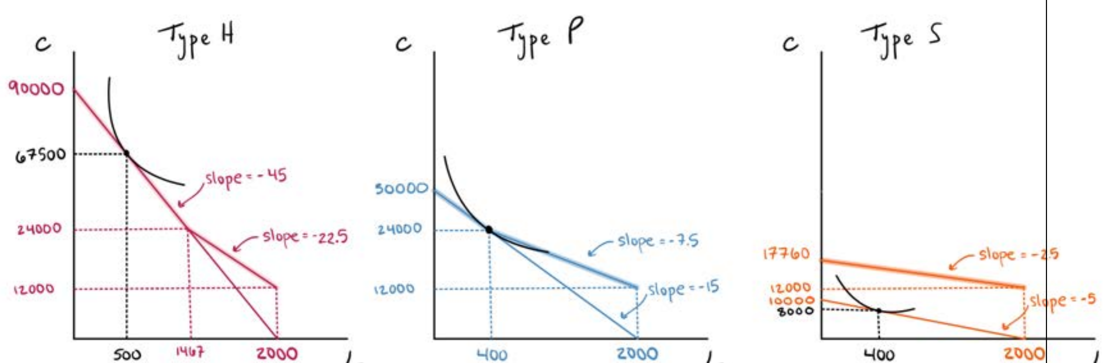
So per-capita consumption in family P is \$8000.

Families of type S solve the same problem as those of type P, but their effective wage is \$5. So they have $L=400$, $H=1600$, and $C=8000$, with per-capita consumption of \$4000.

Grading notes: 2 point per family type leisure and consumption choices. Minus 1 point if do not correctly translate to per-capita consumption.

Suppose that a transfer program is introduced that provides any family a guaranteed benefit level (G) of \$12,000 with a benefit reduction rate (τ) of 0.5. Childcare spending is tax-deductible, and the benefit reduction rate is based on taxable income.

2. (6 points) Draw the budget constraints faced by each type under this new transfer program and the point at which they originally optimized their utility from the previous part. Label each point, and make sure to label any kinks in your budget constraint on both axes. (You probably want to put each type of family on a separate graph)



Solution:

Grading notes: 1 point for each original budget constraint and 1 point for each new budget constraint. Half credit if they do not label relevant points/slopes.

3. (6 points) What effect does this policy have on each family type's labor supply decision? (Hint: contrast their leisure/labor supply decisions with and without the transfer program)

Solution: Now, each family has a kinked/piecewise budget set. On the new section introduced by the transfer program, the line is $C=12000+\frac{w}{2}(2000-L)$. Specifically, families of type H solve

$$\max_{L} 3ln(12000+\frac{45}{2}(2000-L))+ln(L)$$

Taking FOCs:

$$\frac{-3*45}{24000+45(2000-L)}+\frac{1}{L}=0 \rightarrow L=183, H=1817, C=40875$$

which is (1) not on this section of the budget set, and (2) which is clearly dominated by their choice they made absent the policy. This means any choice that is on the allowable portion of the budget set with the transfer will have even lower utility. So, they keep the same L, H, and C as in part 1.

Families of type P solve

$$\max_L 4ln(12000+\frac{15}{2}(2000-L))+ln(L)$$

Taking FOCs:

$$\frac{-4*15}{24000+15(2000-L)}+\frac{1}{L}=0 \rightarrow L=720, H=1280, C=21600$$

Comparing their utility under this choice with the utility they get if they chose absent the policy:

$$U(400, 24000 = 4ln(24000)+ln(400)=46.3$$

$$U(720, 21600 = 4ln(21600)+ln(720)=46.5$$

So family P chooses $L=720$, $C=21600$ and works less than they did before.

Families of type S solve

$$\max_L 4ln(12000+\frac{5}{2}(2000-L))+ln(L)$$

Taking FOCs:

$$\frac{-4*5}{24000+5(2000-L)}+\frac{1}{L}=0 \rightarrow L=1360, H=640, C=13600$$

Comparing their utility under this choice with the utility they get if they chose absent the policy:

$$U(400, 8000=4ln(8000)+ln(400)=41.9$$

$$U(1360, 13600=4ln(13600)+ln(1360)=45.3$$

So family S chooses $L=1360$, $C=13600$ and works less than they did before.

Grading notes: 2 points per family type solution

4. In this part only, imagine that the benefit reduction rate was zero.

   (a) (2 points) What is the popular name for this type of redistribution policy?

Solution: This is universal basic income.

Grading notes: 2 points for UBI

(b) (6 points) Draw another graph with each type's original budget constraint and their new budget constraint under this policy. Relative to their original labor supply decisions, what effect does this policy have on each type's labor supply decision? Add the families' new leisure-consumption choices to your graphs. (Again, you should put each family type on a different graph).

Solution: Now, each family maximizes their utility subject to the constraint that $C = 12000 + w(2000 - L)$ for all L. Specifically, for families of type H,

$$\max_L 3ln(12000 + 45(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-3 * 45}{12000 + 45(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 567, H = 1433, C = 76500$$

For families of type P,

$$\max_L 4ln(12000 + 15(2000 - L)) + ln(L)$$

Taking FOCs:

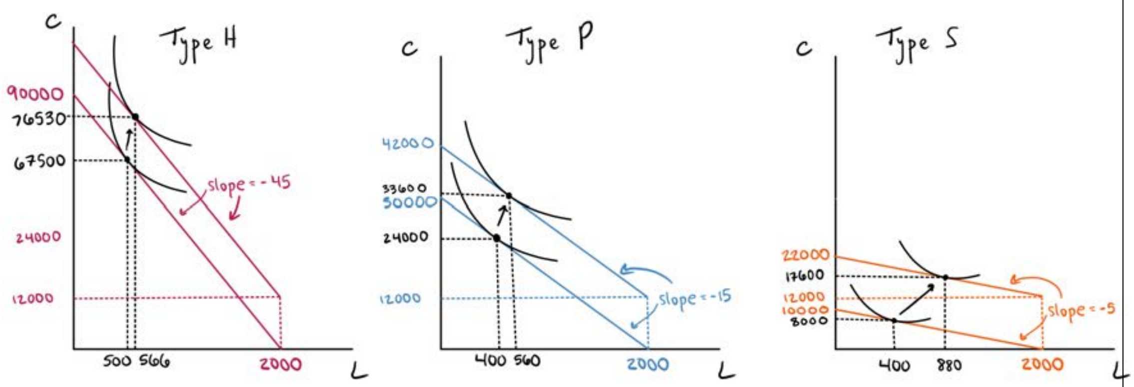
$$\frac{-4 * 15}{12000 + 15(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 560, H = 1440, C = 33600$$

And for families of type S,

$$\max_L 4ln(12000 + 5(2000 - L)) + ln(L)$$

Taking FOCs:

$$\frac{-4 * 5}{12000 + 5(2000 - L)} + \frac{1}{L} = 0 \rightarrow L = 880, H = 1120, C = 17600$$



Grading notes: 1 point for each family type's new choice of consumption and leisure, 3 points for the correct graphs.

(c) (5 points) How does the effect of the policy on labor supply change when $\tau = 0$ compared to when $\tau > 0$, and why?

Solution: For families of type P and S, When $\tau = 0$, there are only income effects (people are richer, so they work less, since leisure is a normal good). When $\tau > 0$, there is an income effects plus a substitution effects: leisure is relatively less expensive, and people work even less than when $\tau = 0$. For families of type H, there is no income or substitution effect when $\tau > 0$ because they are unaffected by the policy, but an income effect when $\tau = 0$, so they work less when $\tau = 0$, the opposite of the other families.

Grading notes: 3 points for correct discussion of income and substitution effects. 2 points for P and S work less when $\tau > 0$ than when $\tau = 0$, 1 point for H works more when $\tau > 0$ than $\tau = 0$.

(d) (2 points) Why might the government prefer to have $\tau > 0$? Why might they prefer $\tau = 0$?

Solution: When $\tau = 0$, everyone gets a transfer. Especially if the government cares about getting families above some absolute level of poverty, they do not want to give a transfer to the H type families. However, $\tau > 0$ distorts the labor supply decision, especially for families who earn low wages. While $\tau = 0$ does change the labor supply decision of families relative to the world with no policy, it does not implicitly tax wages, meaning that workers are still efficiently trading off between leisure and consumption (it's just that at a higher basic income level, they get more value from a bit more leisure).

Grading notes: 1 point for $\tau = 0$ gives money to high-income families, 1 point for $\tau > 0$ distorts the labor supply decision.

5. In this part only, suppose that the government instead provides a child tax credit of \$15 per hour

worked that is restricted to single-parent families.

(a) (3 points) Relative to their original labor supply decisions, what effect does this policy have on each family type's labor supply decision and consumption?

Solution: Families of type H and P are unaffected, because they are not single-parent families. Families of type S face the same problem as in part 1, except their effective wage is now $20 ($15-$10+$15). So, they choose L = 400/H=1600 as above and have consumption of $32,000.

Grading notes: 1 point for families of type H and P being unaffected, 1 point for single parents now having an effective wage of $20, 1 point for the correct labor supply and consumption.

(b) (4 points) Besides the effects on labor supply you found above, what are two other economic reasons the government might prefer this policy to their original transfer program?

Solution: On reason could be that they care particularly about helping single-parent families (i.e. if the social returns to increased consumption are larger for single-parent families, or if the social welfare function has a larger weight on these families, or if the society cares about everyone having equal opportunities to work regardless of child-having and marital status). Another reason is that they may think that being in high-quality formal child care/preschool may be beneficial for these low-income children's long run outcomes (consistent with evidence discussed in the textbook).

Grading notes: 2 point per reason, full credit for any plausible economic (not political) reason.

Question 2: Taxes on Savings [40 points]

Your friend Sarah currently works at Google and has decided to complete a one-year master's degree starting next year. Sarah doesn't have any savings yet, but she will earn an income y_1 before entering grad school. With her earnings, Sarah can choose to consume some while she is working and save in the bank, which earns an annual rate of interest return r. During grad school, she will have no income, so she plans to sell her shares and consume the savings. We will assume that Sarah's utility function is

$$U = c_1^{1-\frac{1}{\gamma}} + \beta c_2^{1-\frac{1}{\gamma}},$$

where c_1 is consumption this year and c_2 is consumption during grad school.

For now, assume that $\gamma = 5$, so that Sarah's utility function is $U = c_1^{4/5} + \beta c_2^{4/5}$.

(a) (2 point) Write down Sarah's budget constraint.

Solution: Sarah's budget constraint is $c_1 + c_2/(1 + r) = y_1$,

(b) (4 points) Solve Sarah's optimization problem and obtain her optimal consumption c_1.

Solution: Sarah's optimization problem is

$$\max_{c_1,c_2} \left\{ c_1^{4/5} + \beta c_2^{4/5} \right\} \quad \text{s.t.} \quad c_1 + c_2/(1+r) = y_1.$$

We can rewrite the budget constraint as

$$c_2 = (1+r)(y_1 - c_1)$$

and then substitute it in, yielding the simpler optimization problem

$$\max_{c_1} \left\{ c_1^{4/5} + \beta[(1+r)(y_1 - c_1)]^{4/5} \right\}.$$

Take the first order condition with respect to c_1:

$$c_1^{-1/5} - \beta(1+r)[(1+r)(y_1 - c_1)]^{-1/5} = 0,$$

which implies that

$$c_1 = \frac{1}{1 + \beta^5(1+r)^4} y_1.$$

Grading: 2 point for optimization problem, 2 points for optimal consumption

(c) (2 point) Now we will plug in some values for the parameters. Let the discount parameter $\beta = 1$ and the rate of return be $r = 0.04$. What share of her income does Sarah consume this year?

Solution: Sarah's consumption as a share of income is

$$\frac{c_1}{y_1} = \frac{1}{1 + (1+0.04)^4} = \frac{1}{1 + 1.17} = 0.461,$$

and so she will consume about 46 percent of her income.

(d) (2 points) Would Sarah save more or less of her income if $\beta < 1$? Explain.

Solution: Sarah's savings fall when β is smaller. This is because when β is small, each dollar consumption during her MBA is less valuable in present value terms. We can show this formally by taking the partial derivative of c_1/y_1:

$$\frac{\partial\left(\frac{c_1}{y_1}\right)}{\partial \beta} = -\frac{\beta^5}{(1 + \beta^5(1+r)^4)^2} < 0,$$

which implies that savings $(y_1 - c_1)/y_1$ are increasing in β.

Grading: 2 points for correct answer with explanation (no math needed)

Suppose that the government introduces a 20-percent tax the interest gains from bank deposits.

(e) (2 point) Write down Sarah's new budget constraint.

Solution: Let $\tau = 0.2$ be the capital gains tax rate. Then the new budget constraint is

$$c_2 = [1 + (1-\tau)r](y_1 - c_1) = (1 + 0.8r)(y_1 - c_1).$$

(f) (2 point) Using your answer to part (b) of this problem, find Sarah's new c_1.

Solution: Proceeding exactly as above,

$$c_1 = \frac{1}{1 + \beta^5[1 + 0.8r]^4} y_1.$$

(g) (4 points) Does Sarah choose to increase or reduce her savings in response to the tax? Explain why, referring to both the income effect and the substitution effect.

Solution: From the result above, we can see that, as τ increases, more of Sarah's consumption occurs while working, and thus there is less savings, as y_1 is fixed. Here, the income effect means that any given amount of saving gives Sarah less consumption during her MBA, which is a force for saving more because Sarah wants to smooth her consumption over time. The substitution effect is that the tax raises the price of consumption during her MBA, so she responds by allocating towards the now-cheaper good, which is consumption today.

Grading: 1 point for reduction, 1 point for correct income effect, 1 point for correct substitution effect

(h) (4 points) Suppose that r was the real return, but there was also an inflation rate π so that the nominal return was $r + \pi$. Find the real after-tax rate of return $\widetilde{r}$. Then explain whether Sarah save more or less today if inflation were higher.

Solution: An increase in inflation raises nominal capital gains, but not real capital gains. As the capital gains tax is not inflation-adjusted, an increase in π holding r fixed is equivalent an increase in the tax rate on real capital gains. We can see this by computing the real after-tax rate on Sarah's savings account:

$$\widetilde{r} = \frac{1 + (r+\pi)(1-\tau)}{1+\pi} - 1 \approx r(1-\tau) - \tau\pi,$$

which is decreasing in π:

$$\frac{\partial \widetilde{r}}{\partial \pi} \approx -\tau < 0.$$

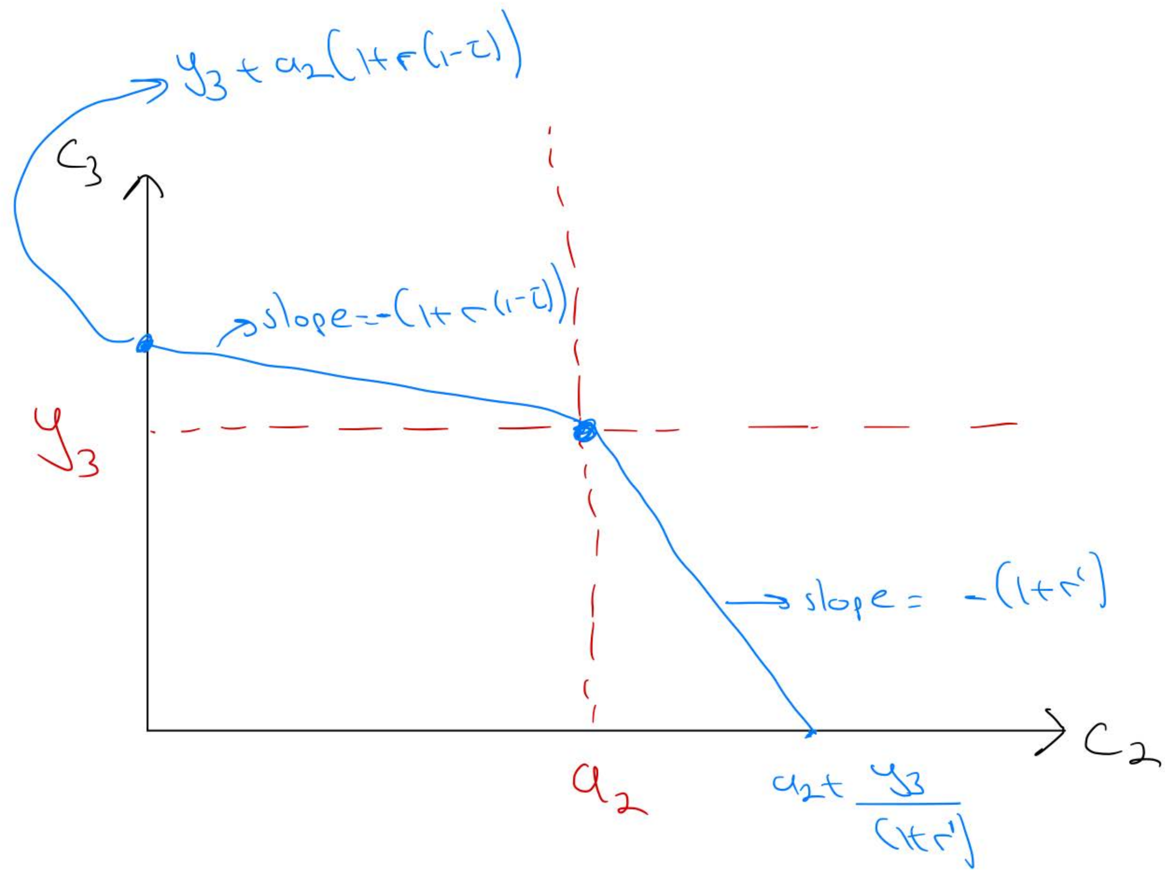
We have now shown that Sarah saves more when the interest rate is higher, and also that inflation reduces the economically-relevant interest rate to Sarah. Thus inflation reduces saving.

Grading: 1 point for after-tax rate of return, 1 point for reduction, 1 point for justification

Sarah is now entering her second semester, with a job at Tesla lined up for next year that will pay her an income of y_3. Grad-student life has proved expensive, however, and she only has a_2 in savings left. Sarah is considering borrowing money to pay for her consumption, but the interest rate r' on her credit card (at which she can borrow) is higher than r, her savings interest rate.

(i) (2 points) Draw a diagram with Sarah's intertemporal budget constraint between this year and next year.

Solution:

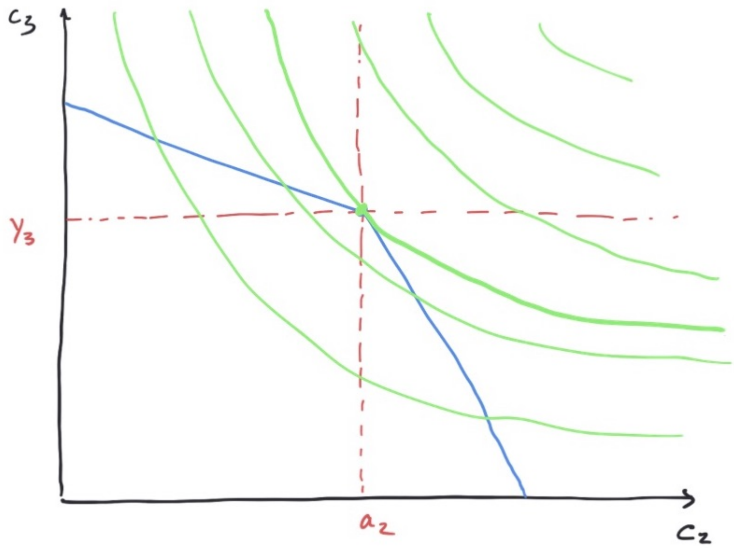


(j) (4 points) Using indifference curves, show why Sarah is particularly likely to consume exactly a_2 this year and borrow nothing. Explain.

Solution: Sarah's budget constraint is kinked at $c_2 = a_2$ because the interest rate on savings is not the same as the interest rate on credit card debt. Such kinks cause "bunching," because in equilibrium, Sarah will consume until her MRS equals MRT, which is determined by the interest rate. She will consume exactly $c_2 = a_2$ if her MRS at this point falls between the MRTs that are fixed by r' and r.

Grading: 1 point for indifference curve, 1 point for bunching explanation

(k) (4 points) Suppose there is inflation rate π. So, if the price of the final good is \$1 today, it will be \$(1+π) next period. Draw a diagram with Sarah's new intertemporal budget constraint between this year and next year in the presence of inflation. How does the budget set in periods 2 and 3 change? Note: Suppose Tesla pays y_3 real income in the next period.



Solution: Budget set shrinks in period 3 and expands in period 2. Inflation increases the cost of borrowing, and hence improves the consumption capabilities today while decreasing it tomorrow because real interest goes down. Grading: 2 point for the correct-looking graph. 2 points for the correct explanation.

(l) (4 points) Explain how Sarah's borrowing decision is impacted by the presence of inflation. Hint: Absent inflation, Sarah is either borrowing, saving, or doing neither. Analyze the change in Sarah's decision in each case.

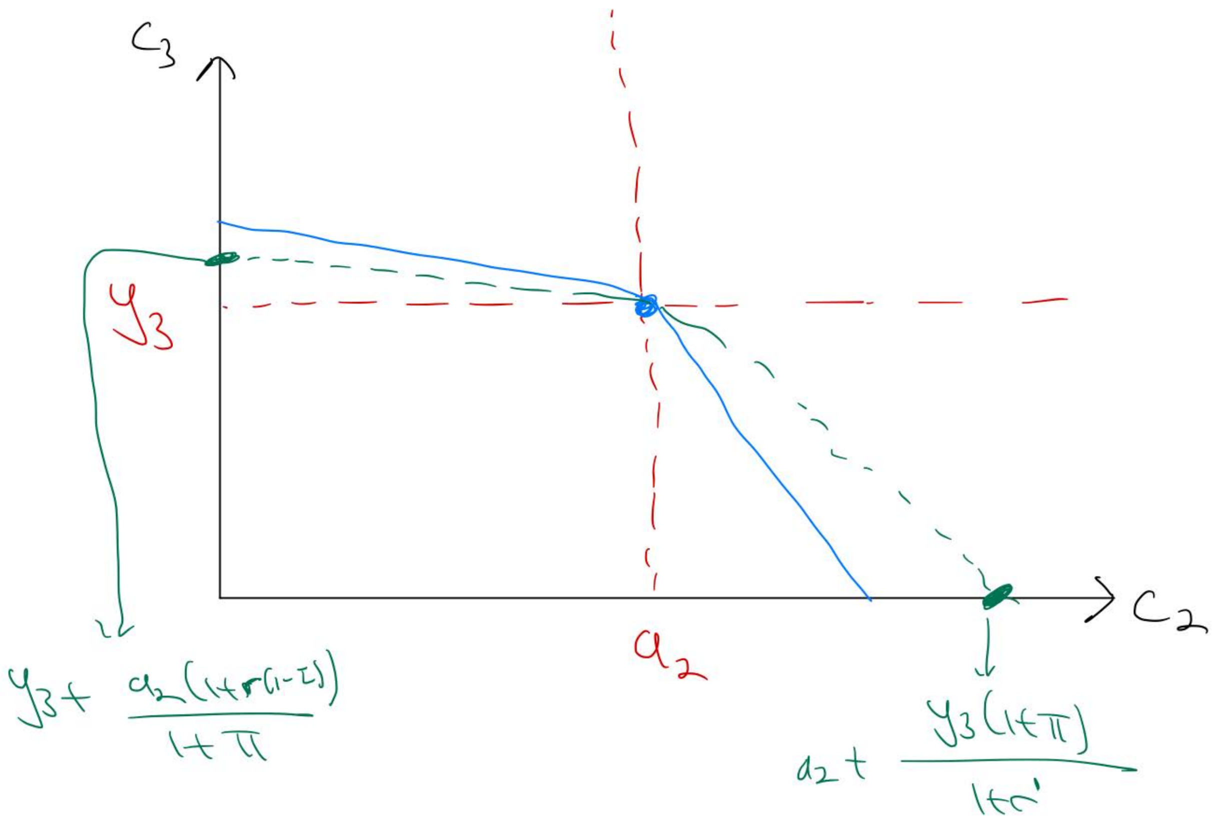
Solution: If Sarah is saving without inflation, her saving rate goes down. If Sarah is borrowing absent inflation, her borrowing goes up. If Sarah is at the kink, her borrowing weakly goes up (meaning, she can stay in the kink if the inflation is not too strong). Grading: 1 point for the effect in the saving state, 1 point for the effect in the saving state, 2 points for the effect in the kink state.

(m) (4 points) Sarah has started her job at Tesla. She expects she will eventually retire with a large nest egg, so that her taxable income will be higher even in retirement than it is now. Should she save her first dollar in a 401(k) or using a Roth IRA? Explain any advantages to Sarah of either option.

Solution:

- Many companies partially match employee contributions to 401(k)s. This may make the effective return on her first $1 of saving in the 401(k) higher than in the Roth IRA.
- Both the 401(k) and the Roth IRA are tax-advantaged. However, in the 401(k), Sarah will pay taxes upon withdrawing the money, and since she expects to be rich by then, she will face a higher marginal tax rate then than she does now. She can exploit this tax-rate differential by saving in her Roth IRA, as she pays taxes before depositing income into the Roth IRA, and thus at her *current* tax rate as a young worker.

Grading: 2 point for advantage to each option



Question 3: Rate Change vs. Taxation on Accrual for Capital Gains [20 points]

A new policy debate has emerged regarding the taxation of capital gains. Two proposals are under consideration: 1. **A change in the capital gains tax rate** (e.g., increasing or decreasing the rate applied when gains are realized). 2. **Taxation on accrual**, where gains are taxed annually based on their estimated increase in value, even if the asset is not sold.

Each approach has distinct advantages and trade-offs. Answer the following questions by comparing these two options.

(a) Lock-in Effect (5 points)

How does each policy influence the *lock-in effect*, where taxpayers delay selling assets to minimize taxes? Which policy is more likely to reduce this effect, and why?

Solution:

- A rate change generally worsens the lock-in effect. Higher rates incentivize taxpayers to hold onto assets longer to defer taxes. Lower rates may reduce this effect temporarily but not eliminate it.
- Taxation on accrual eliminates the lock-in effect since gains are taxed annually regardless of whether

the asset is sold. This could encourage more frequent asset turnover, improving market liquidity.

(b) Liquidity Concerns (5 points)

What are the liquidity implications of each policy for taxpayers? Which policy creates greater challenges, and why?

Solution:

- A rate change does not impose liquidity concerns directly, as taxes are only due upon realization (when the asset is sold). Taxpayers can use the proceeds from the sale to pay taxes.
- Taxation on accrual creates significant liquidity challenges, especially for taxpayers holding illiquid or appreciating assets (e.g., real estate, art). They may need to sell assets or borrow to pay taxes on gains they haven't realized in cash.

(c) Infinite Loss Offset (5 points)

Capital gains tax systems often limit how much of a loss can offset taxable gains. How does each policy handle situations where taxpayers face large or infinite losses, and what are the trade-offs?

Solution:

- Under a rate change, losses can be used to offset realized gains, often with limits (e.g., $3,000 per year in the U.S.). This creates a delay in the tax benefit for taxpayers with large losses.
- Taxation on accrual may account for losses annually, potentially providing more immediate offsets. However, accurate annual valuation of losses could be administratively challenging.

(d) Administrative Feasibility (5 points)

What are the administrative challenges associated with each policy? Which is likely to be easier to implement and enforce?

Solution:

- A rate change is administratively simpler because the tax liability is calculated when the asset is sold, using clear market prices. However, it may incentivize tax avoidance strategies, like holding assets indefinitely.
- Taxation on accrual is complex to administer because it requires annual valuation of all assets, including those without active markets (e.g., private businesses, art). It also introduces disputes over valuation and potential compliance costs for taxpayers.

Grading Notes:

- Award **5 points each** for logical reasoning and clarity. Partial credit for mentioning relevant considerations but failing to fully elaborate on trade-offs.
- Encourage intuitive responses based on understanding of efficiency, equity, and practical challenges rather than technical or mathematical details.