

# 14.454 Problem set: Q1-Q2

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## 1 Bernanke and Gertler (1989)

Consider the Bernanke-Gertler (1989) model seen in class. Maintain the same notations and assumptions as in the lecture.

1. The financial friction in Bernanke-Gertler is the *asymmetric information* and *costly state verification*. Lenders cannot observe the outcome of entrepreneur's projects (state), so that entrepreneurs have incentives to misreport their outcome and obtain a benefit from it. However, lenders can audit entrepreneurs with certain cost, encouraging entrepreneurs to truthfully report. At the same time, auditing makes borrowing more costly, and "internal funds" better.

Internal and external funds are therefore not equivalent. If the entrepreneur contributes more of his own net worth, the required loan is smaller, the incentive problem is weaker, and expected agency costs fall. In the incomplete collateralization region, an extra unit of internal funds does two things at once: it replaces one unit of external funds and it lowers expected auditing costs. That is why the return to inside funds exceeds the safe return. The external finance premium is exactly this wedge between the cost of funds raised externally and the opportunity cost of internal funds.

2. (a) If the entrepreneur is fully collateralized, the probability of auditing when the bad state is realized is zero,  $p_1(\ell) = 0$ . If collateral is incomplete, then  $p_1(\ell)$  is increasing  $\ell$ . Intuitively, the higher the borrowing size, the lower the consumption to the entrepreneur required to compensate the lender (i.e., to verify the participation constraint), and, thus, the higher the incentives to misreport if the state is good. Then, the probability of auditing must increase to provide incentives to telling the truth.  
(b) The probability of auditing when the good state is realized,  $p_2(\ell)$ , is zero independently of  $\ell$ . That is, there is no need to audit if the entrepreneur reports the good state, as there is no incentive to misreport in the bad state.  
(c) Higher entrepreneurial net worth means a smaller required loan. That reduces the promised transfer that must be extracted from the entrepreneur, which weakens his incentive to misreport good outcomes as bad ones, reducing the likelihood of auditing and auditing costs. In short, more net worth means more skin in the game and less need for costly monitoring.

- Without friction,  $E[k_{t+1}]$  and  $E[k_{t+2}]$  remains unchanged (and any other periods). The stock of capital is state-independent in the perfect information equilibrium. The high productivity realization makes entrepreneurs wealthier, but they use the additional wealth to consume and/or save more (storage) without changing the investment level.

In an economy with asymmetric information, a high  $\theta_t$  increases wages, rising entrepreneur's net worth and savings at  $t$ . The higher entrepreneurial saving,  $S_t^e$ , the lower the agency costs (lending costs). This allows more entrepreneurs to invest, and the same entrepreneurs will need to borrow less. Graphically, this shock generates a rightward shift of the supply of capital at period  $t$ . Thus,  $\mathbb{E}[k_{t+1}]$  rises and  $\mathbb{E}[q_{t+1}]$  falls. A high shock on  $\theta_t$  also affects equilibrium at  $t + 1$  as the increment in  $\mathbb{E}[k_{t+1}]$  raises the level of wages at  $t + 1$ , generating a higher  $S_{t+1}^e$  and shifting the supply of capital rightwardly (again). Hence,  $\mathbb{E}[k_{t+2}]$  rises and  $\mathbb{E}[q_{t+2}]$  falls.

The amplification and persistence of shocks are generated by the negative relationship of agency costs and  $S^e$  (only active with financial friction) and the dependence of  $S^e$  on  $\mathbb{E}[k]$  (active with and without financial friction). The former generates shock amplification on the stock of capital, while the latter induces persistence. Furthermore, at each period, the capital price falls, lowering the expected return on the project and the investment rate in the economy at each period. Therefore, the accumulation of capital will be less than in a counterfactual setting where the price of capital is constant.

There is no anticipated effects of  $\mathbb{E}[k_{t+2}]$  at time  $t + 1$ .

- A permanent increase in productivity will generate a permanent rise in the stock of capital in an economy without frictions. When financial frictions are active, we observe a similar effect but with a proportional larger increment in capital. In fact, the rise of capital increases over time. Thus, the shock from  $t$  persisting, and the new shock at  $t + 1$  amplify one another. There is no anticipated effects of  $\mathbb{E}[k_{t+2}]$  at time  $t + 1$ .
- Negative shocks can have stronger real effects than positive shocks because the agency problem may bind mainly on the downside. If the economy starts near the perfect information allocation, sufficiently favorable realizations of productivity may have no extra effect because efficient entrepreneurs are already fully collateralized. But an adverse shock can push them back into the constrained region, raise agency costs sharply, and shift the capital supply curve left. So the downside is nonlinear in a way the upside need not be.
- Due to the potential presence of asymmetric information in the real economy, taxing entrepreneur's income (even via lump-sum taxes) may have detrimental consequences for the economy. In particular, that policy would reduce entrepreneur's net worth and, consequently, their capability to obtain loans to finance their projects. As a result, this policy is a bad idea as it would generate a persistent (and large) decline in investment, wages, output and, then, social welfare.

## 2 Kiyotaki and Moore (1997)

Let

$$u(K_t)K_t = (a + q_t)K_{t-1} - RB_{t-1} \quad (1)$$

1. The friction comes from the *inalienability of human capital* and *limited commitment*. Once production starts, only the farmer can make the land produce, and he cannot commit in advance to keep supplying that labor. So if debt is too high, he can threaten to walk away and renegotiate. Because of that, lenders will only lend against collateral. If the farmer defaults, they can seize the land, but without the farmer's specific skill the land is worth less to outsiders than to the farmer. So borrowing is limited by the collateral value of land. Land commands a premium beyond its productive use because it does two things at once: it produces output and it relaxes borrowing constraints.
2.  $u(K_t)$  denotes the user cost of capital and  $(a + q_t)K_{t-1} - RB_{t-1}$  is the net worth of the farmer at period  $t$ .
3. By perfect foresight,  $q_t K_{t-1} = RB_{t-1}$ , reducing (1) to  $u(K_t)K_t = aK_{t-1}$ . Log-linearizing this equation yields

$$\begin{aligned} a\bar{K}\hat{K}_{t-1} &= \bar{K}(\bar{K}u'(\bar{K}) + u(\bar{K}))\hat{K}_t \\ \hat{K}_{t-1} &= \left( \frac{\bar{K}u'(\bar{K})}{u(\bar{K})} + 1 \right) \hat{K}_t \\ \hat{K}_{t-1} &= (\eta^{-1} + 1) \hat{K}_t \end{aligned}$$

where the second equality derives from the definition of steady-state  $u(\bar{K}) = a$ , and the third equality from the definition of the elasticity of user cost. Rearranging,

$$\hat{K}_{t+1} = \alpha \hat{K}_t \quad (2)$$

where  $\alpha \equiv \frac{\eta}{1+\eta} \in [0, 1]$ . Note that this implies that

$$\hat{K}_{t+s} = \alpha^s \hat{K}_t \quad \text{for } s = 0, 1, \dots \quad (3)$$

4. Log-linearizing the user cost  $u(K_t) = q_t - q_{t+1}/R$  yields

$$\begin{aligned} \bar{q}\hat{q}_t - \frac{\bar{q}}{R}\hat{q}_{t+1} &= \bar{K}u'(\bar{K})\hat{K}_t \\ \frac{R\hat{q}_t - \hat{q}_{t+1}}{R-1} &= \frac{\bar{K}u'(\bar{K})}{u(\bar{K})}\hat{K}_t \\ R\hat{q}_t - \hat{q}_{t+1} &= (R-1)\eta^{-1}\hat{K}_t \end{aligned}$$

Thus,

$$\hat{q}_t = \frac{(R-1)}{R\eta}\hat{K}_t + \frac{1}{R}\hat{q}_{t+1} \quad (4)$$

Iterating forward (4), and using (3), we have

$$\begin{aligned}\hat{q}_t &= \frac{(R-1)}{R\eta} \sum_{s=0}^{\infty} R^{-s} \hat{K}_{t+s} \\ &= \frac{(R-1)}{R\eta} \sum_{s=0}^{\infty} (\alpha/R)^s \hat{K}_t \\ &= \hat{K}_t \frac{(R-1)}{R\eta} \frac{R}{R-\alpha}\end{aligned}$$

Thus,

$$\hat{q}_t = \frac{R-1}{\eta(R-\alpha)} \hat{K}_t. \quad (5)$$

5. Consider an unexpected temporary shock at 0 that increases productivity to  $a(1+\epsilon)$ . At 0, the shock generates an unexpected increment in the capital price,  $q_0 - \bar{q}$ .

(a) Then,  $u(K_0)K_0 = a(1+\epsilon)\bar{K} + q_0\bar{K} - \bar{q}\bar{K}$ , and log-linearizing yields

$$\begin{aligned}a\bar{K}\epsilon + \bar{q}\bar{K}\hat{q}_0 &= \bar{K} (\bar{K}u'(\bar{K}) + u(\bar{K})) \hat{K}_0 \\ \epsilon + \frac{R}{R-1}\hat{q}_0 &= (\eta^{-1} + 1)\hat{K}_0 \\ \alpha\epsilon + \frac{\alpha R}{R-1}\hat{q}_0 &= \hat{K}_0\end{aligned} \quad (6)$$

For  $t > 0$ , dynamics are given by (2). Note that (4) and (5) are still valid for  $t \geq 0$ .

(b) Therefore,  $(\hat{q}_0, \hat{K}_0)$  are determined by

$$\begin{aligned}\hat{K}_0 &= \alpha\epsilon + \frac{\alpha R}{R-1}\hat{q}_0 \\ \hat{q}_0 &= \frac{R-1}{\eta(R-\alpha)}\hat{K}_0\end{aligned}$$

Since  $\frac{\alpha}{\eta(R-\alpha)} = \frac{1}{(1+\eta)(R-\alpha)} = \frac{1}{R+\eta(R-1)}$ ,

$$\hat{K}_0 = \frac{R+\eta(R-1)}{\eta(R-1)}\alpha\epsilon \quad \text{and} \quad (7)$$

$$\hat{q}_0 = \frac{\alpha(R-1)}{\eta(R-\alpha)} \frac{\hat{K}_0}{\alpha} = \eta^{-1}\epsilon. \quad (8)$$

(c) The unexpected change in the capital price allows farmers to renegotiate an increment in loans, triggering a “multiplier effect” over the demand of capital and capital price through the collateral constraint. Note that the response of the stock of capital is higher with collateral constraint:  $\hat{K}_0 > \alpha\epsilon$ , where  $\alpha\epsilon$  approximates the response without collateral constraint.

- (d) After an unexpected rise in productivity, farmers collect the capital gains and pay their financial obligations without a problem. Instead, an unexpected decline in productivity would call for a violation of the financial contract. We need to adjust the model to manage these situations in an internally-consistent way.
6. Suppose  $q_1 = \bar{q}$ .
- Then the price equation becomes purely static:  $\hat{q}_0 = \frac{R-1}{R}\eta^{-1}\epsilon$  and  $\hat{K}_0 = \epsilon$ .
  - The dynamic multiplier makes the price response larger by a factor  $R/(R-1)$ , and it also makes the quantity response strictly larger.
  - The static multiplier is purely within period. The shock changes net worth, this changes constrained demand for land, that moves the land price, and the price movement feeds back immediately onto net worth. The dynamic multiplier adds the forward looking part: because the shock affects future net worth and future land demand, it affects the whole future path of user costs, and therefore current land price moves much more today. That stronger current price movement tightens or relaxes current credit constraints even further.
7. There are two key differences between these models. First, the *demand curve is upwardly sloped in Kiyotaki-Moore*. This feature creates a larger static multiplier of productivity on the stock of capital than in the Bernanke-Gertler. Second, the *forward-lookingness of gatherers creates a dynamic multiplier* that, again, is not present in the Bernanke-Gertler model. All in all, these two features make the stock of capital more sensitive to productivity shocks.
8. Once we tighten the collateral constraint, the effect will be dampened. The key amplification mechanism in Kiyotaki-Moore is the dual purpose of land as a factor of production and collateral: a higher productivity shock leads to an increase in net worth, which means more could be put in as collateral, so future prices of land are going to increase, which feed back into contemporaneous price. Tightening the collateral constraint will dampen this channel. An extreme case would be when  $\phi \rightarrow 0$ ; there would be no amplification channels at all.

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