Collateral and Amplification

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Economic Crises

References

1. Bernanke B. and M.Gertler, “Agency Costs, Net Worth, and Business Fluctuations,” *American Economic Review*, 79(1), 14-31, March 1989.
2. Kiyotaki, N. and J.Moore, “Credit Cycles,” *Journal of Political Economy*, 105(2), 211-248, April 1997.

Basic Idea

- Most models of financial constraints have an equation of the kind:

$$f'(K) = r + \lambda; \qquad \lambda > 0,$$

  where λ results from some financial friction.
  - New investment: underinvestment
  - Saving existing K: inefficient destruction.

Basic Idea

- Micro: λ could take the form of credit rationing or high lending rate.
  - Adverse selection: Rise in r^L means bad selection, thus keep r^L low.
  - Moral hazard: if too leveraged, wrong incentives
- Macro: micro-solutions such as collateral, self-financing, create problems during recessions
  - Amplification (rise in λ)
  - Persistence (constrained operation limits earnings, etc.)

Bernanke-Gertler

- OLG (simpler) with $t : 1, ..., \infty$
- η: fraction of population that have access to investment technology (entrepreneurs). The rest are lenders
- Entrepreneurs are heterogenous: building a project takes $x(\omega)$ units of output with $\omega \sim U[0,1]$ and $x'(\omega) > 0$
- Project (indivisible): yields k_i units of capital at $t+1$ (it depreciates after that): $\mathrm{E}[k_i] = k$ independent of ω
- Output (note: $L = 1$): $y_t = \tilde{\theta}_t f(k_t)$ with $\tilde{\theta}_{t+i}$ i.i.d. (aggregate shock)
- Storage technology (alternative for savings): $r \geq 1$. Linear preferences:

$$(*) \quad s_t^e = w_t \qquad s_t = w_t - z_t$$

Equilibrium with Perfect Information: Supply

- Let q be the price of capital, $\hat{q}_{t+1} = \mathrm{E}[q_{t+1}]$ and $k = \mathrm{E}[k_i]$. Free entry implies that there is a critical $\overline{\omega}$ such that

$$\hat{q}_{t+1} k = r x(\overline{\omega}_t)$$

- Since $\omega \sim$ U[0,1], the number of projects i (investment) and the stock of capital (no aggregate risk) are:

$$i_t = \eta \overline{\omega}_t; \quad k_{t+1} = k i_t$$

- Combining these results, the capital supply curve is:

$$\hat{q}_{t+1} = \frac{r}{k} x(\overline{\omega}_t) = \frac{r}{k} x\left(\frac{i_t}{\eta}\right) = \frac{r}{k} x\left(\frac{k_{t+1}}{k\eta}\right)$$

Equilibrium with Perfect Information: Demand

- Demand at $t+1$ is:

$$q_{t+1} = \theta_{t+1} f'(k_{t+1})$$

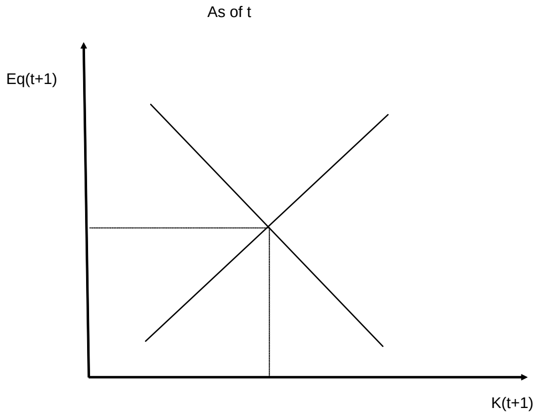
- Since shocks θ_{t+1} is i.i.d., *expected* demand (and eq) is

$$\hat{q}_{t+1} = \widehat{\theta} f'(k_{t+1}) \quad \left(= \frac{r}{k} x \left(\frac{k_{t+1}}{k\eta} \right) \right)$$

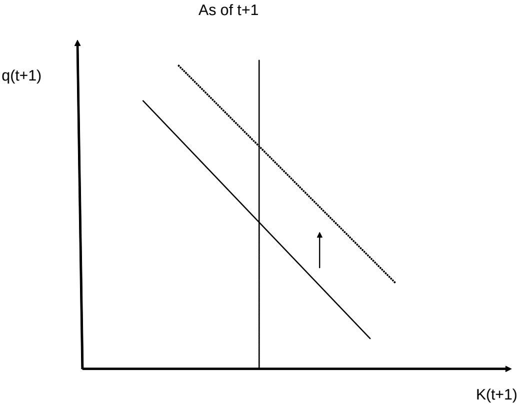
thus k_{t+1} is a function of $\widehat{\theta}$ but not of θ_{t+1} or θ_t

- By the same token, the shock θ_{t+1}affects y_{t+1}, q_{t+1}, consumption and wages (saving) but *not* investment, k_{t+2}, or y_{t+2}

Equilibrium with Perfect Information (Ex-ante)



Equilibrium with Perfect Information (Ex-post)



Equilibrium with Asymmetric Information

- **Goal:** To build a model where θ_t affects investment and *next* period's output (persistence)

- Townsend's costly state verification: k_i is costlessly observed by entrepreneurs only. Others can learn by auditing: costs γ k-goods. If h_t projects are audited

$$k_{t+1} = (k - h_t \gamma) i_t$$

- Two states: (1,2), k_1 is bad; k_2 is good

- Benefit of under-reporting: More consumption. Cost of under-reporting and getting caught: A fine (you will see the details of the contracting problem in rec.)

- **Basic features of contract:** No auditing in good state. Auditing with probability p in bad state.

Characterization

- $p = 0$ if

$$\hat{q}k_1 \geq r(x(\omega) - s^e),$$

i. e. if the expected value of the low output $\hat{q}k_1$ is larger than the repayment $r(x(\omega) - s^e)$, where $x(\omega) - s^e$ is the size of the loan (cost of project - entrepreneur's wealth)

- If not, $0 < p < 1$. p is chosen such that the entrepreneur reports honestly when the good state occurs.
- Characterization:
  - Good project even if $p = 1$ (i. e. if $\omega \leq \underline{\omega}$, ω is so low that the project is built even if $p = 1$.)

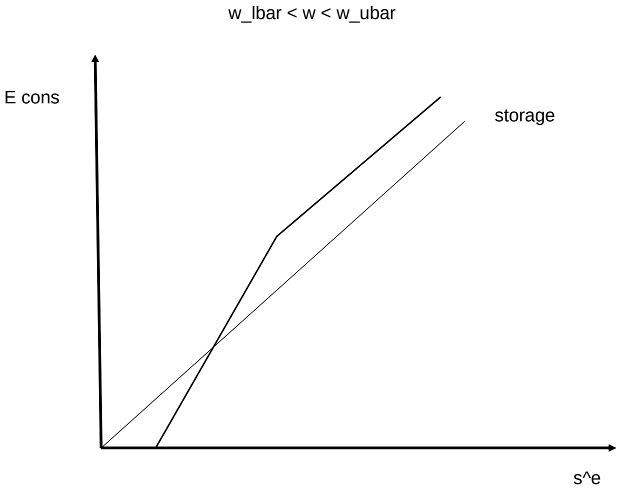
$$\hat{q}k - rx(\underline{\omega}) - \hat{q}\pi_1\gamma = 0$$

  - Positive return only if $p = 0$ (i. e. if $\omega = \overline{\omega}$, the project is built only if $p = 0$):

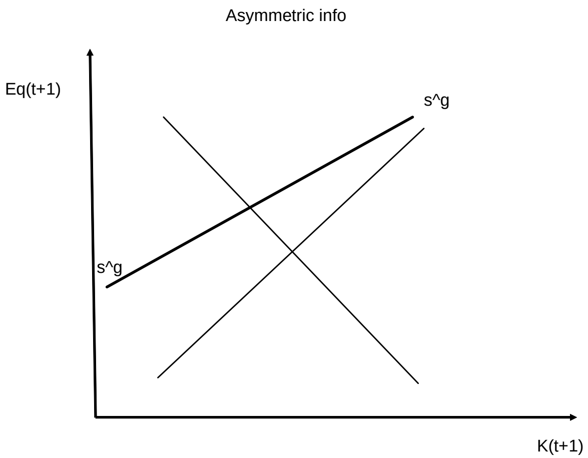
$$\hat{q}k - rx(\overline{\omega}) = 0$$

- The intermediate case $\omega \in [\underline{\omega}, \overline{\omega}]$ is illustrated in the following figure.

Equilibrium with Asymmetric Information



Equilibrium with Asymmetric Information

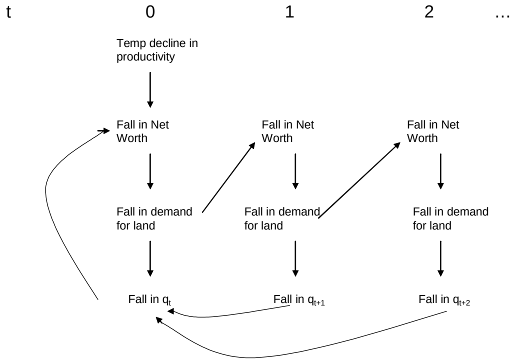


Equilibrium with Asymmetric Information

- An increase in θ_t increases s_t^e, so that more entrepreneurs can invest and the s^g-curve shifts down
- Hence, we get more investment and k_{t+1} increases (even though θ_t is i.i.d.)... and higher k_{t+1} implies higher wages tomorrow, and so on...
- We have amplification/persistence+
- However, the multiplier is "limited" (asset price movement dampens the effect).... next model...

Kiyotaki-Moore

- One group can't borrow as much as it wants. If it did, it would behave opportunistically (holdup/incomplete contracting)
- Land: factor of production *and collateral* (substitutes for commitment)



Kiyotaki-Moore

- $t = 0, 1, 2, ...$
- Two goods: A non-durable commodity (fruit), and land, with total supply $\bar{K}$
- Two types of agents (both produce and consume fruit): farmers (mass of one) and gatherers (mass of m)
- $\beta^F < \beta^G$ (linear preferences) plus other assumptions to rule out corners. Since farmers are more impatient, they are borrowers in equilibrium
- One period credit market: $R = 1/\beta^G$.

Farmers

- CRS technology: out of k_t units of land, farmers produce ak_t units of tradeable fruit and ck_t units of nontradeable fruit

$$y_{t+1} = (a+c)k_t; \qquad \frac{a}{a+c} < \beta^F$$

- (Important) Assumption: After production starts, only specific farmer can complete it. Inalienability of human capital (farmer can withdraw effort). Moreover, farmers can get the entire surplus, hence specificity/appropriability imply reluctance to lend. Collateral is needed for lending:

$$Rb_t \leq q_{t+1}k_t, \tag{1}$$

  where b_t is the farmer's debt at t and q_{t+1} the price of land at $t+1$.

- The flow of funds constraint is

$$q_t(k_t - k_{t-1}) + Rb_{t-1} + (x_t - ck_{t-1}) = ak_{t-1} + b_t, \tag{2}$$

where x_t is consumption. Investment in land and consumption must be financed by output and net borrowing.

Gatherers

- DRS technology: $\tilde{k}_t$ units of time t land produce $G(\tilde{k}_t)$ units of time $t+1$ fruit

$$\tilde{y}_{t+1} = G(\tilde{k}_t) \qquad G' > 0, G" < 0.$$

- No specificity / no credit constraint. The gatherers' flow of funds constraint is

$$q_t(\tilde{k}_t - \tilde{k}_{t-1}) + R\tilde{b}_{t-1} + \tilde{x}_t = G(\tilde{k}_{t-1}) + \tilde{b}_t \tag{3}$$

Characterization of Equilibrium

- Farmers: Only consume nontradeable fruit and invest as much as they can:

$$x_t = ck_{t-1} \qquad Rb_t = q_{t+1}k_t$$

- Substituting this in (2) yields:

$$k_t = \frac{1}{q_t - q_{t+1}/R}\left[(a + q_t)k_{t-1} - Rb_{t-1}\right], \tag{4}$$

where $1/(q_t - q_{t+1}/R)$ is the multiplier and $\left[(a + q_t)k_{t-1} - Rb_{t-1}\right]$ is the farmers' net worth. Since everything is linear, we can aggregate (4)

$$K_t = \frac{1}{u_t}\left[(a + q_t)K_{t-1} - RB_{t-1}\right] \tag{5}$$

with $u_t \equiv q_t - q_{t+1}/R$ and (1) becomes

$$B_t = \frac{1}{R}q_{t+1}K_t. \tag{6}$$

- An increase in $q_t = q_{t+1}$ raises K_t (when collateral effect dominates).

Market Clearing

- The gatherers solve

$$\max_{\tilde{k}_t} \frac{1}{R} G(\tilde{k}_t) + \frac{1}{R} q_{t+1} \tilde{k}_t - q_t \tilde{k}_t$$

with FOC

$$\frac{1}{R} G'(\tilde{k}_t) = \frac{1}{R} \left[(R-1) q_t - (q_{t+1} - q_t) \right] = u_t.$$

- Market clearing implies $\tilde{K}_t = (\bar{K} - K_t)/m$ and hence

$$u(K_t) = \frac{1}{R} G' \left(\frac{1}{m} (\bar{K} - K_t) \right). \qquad (7)$$

- With perfect foresight / no bubbles, we can use the definition of user cost

$$u(K_t) = q_t - \frac{1}{R} q_{t+1}$$

  and solve forward

$$q_t = \sum_{s=0}^{\infty} R^{-s} u(K_{t+s}). \qquad (8)$$

Steady State

- In steady state, (6) implies $qK = RB$. Substituting in (5) yields

$$\frac{R-1}{R}q^* = u^* = a < a + c. \tag{9}$$

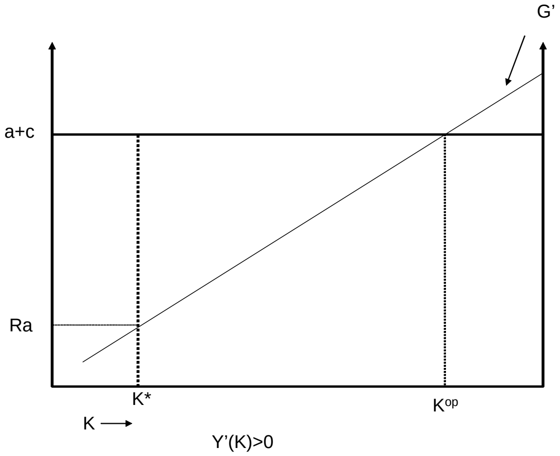
(7) becomes

$$\frac{1}{R}G'\left(\frac{1}{m}(\bar{K} - K^*)\right) = u^*. \tag{10}$$

- Combining (6) with (9) yields

$$B^* = \frac{a}{R-1}K^*. \tag{11}$$

Steady State



Dynamics

- Start from (K^*, B^*, q^*). Temporary increase in farmers' productivity a by Δ (surprise, followed by perfect foresight)
- First best: $\Delta Y_t = \Delta$; no further action
- Kiyotaki-Moore economy: By (5),

$$u(K_t)K_t = [a(1+\Delta) + q_t - q^*]K^*,$$

$$u(K_{t+s})K_{t+s} = aK_{t+s-1} + 0.$$

- From eq (8) we get a positive feedback since:

$$q_t = \sum_{s=0}^{\infty} R^{-s} u(K_{t+s}).$$

Final Remarks

- The feedback between asset prices and optimal investment/allocation is pervasive, especially during severe crises
- Collateral damage implies wasted opportunities
- Recoveries are retarded by collateral damage