Bubbles and Speculation

14.454

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Economic Crises

References

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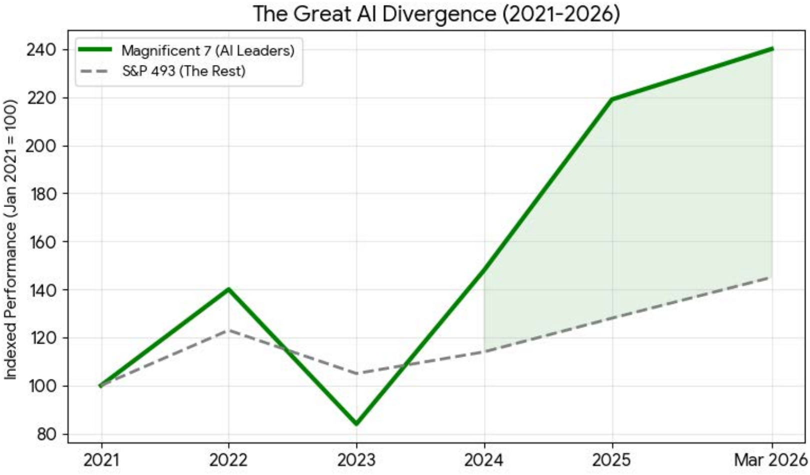
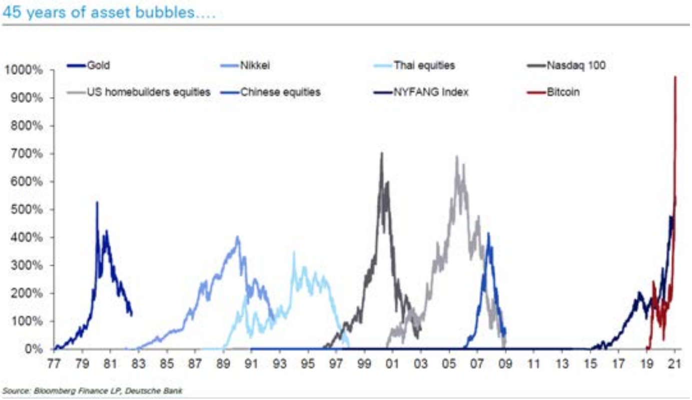
Recent events...



Introduction

- Historical: Dutch Tulipmania, South Sea... Great Crash of 1929
- South Sea Bubble (1710-1720)
  - Isaac Newton: 04/20/1720 sold shares at £7,000, profiting £3,500. Re-entered the market later – ended up losing £20,000
  - "I can calculate the motions of the heavenly bodies, but not the madness of people"
- Japan boom-bust (a lost decade); EMEs, Nasdaq, real estate (all around the developed world), commodities?
- Where do they come from? What to do about them?

A "Conservation Law" of Bubbles...



Main Points

- Two broad (and polar) views:
  - There is a shortage of store of value – bubbles help fixing this problem
  - Agents misbehave (either an agency problem or a behavioral problem)
- My view: These views are more intertwined than it may seem
  - The former is about macro environments where there is shortage of assets
  - The latter is about the location of bubbles
    - "Irrational exuberance" and agency problems are more likely to arise when the macro conditions for bubbles are present
    - E.g., Fed flooded the market with liquidity during Covid, which led to all sort of weird asset appreciations (even for assets created as a joke!.. Dogecoin), supported by amateur social media narratives

Asset shortages

- Read Tirole's 1982 "On the possibility of speculation under RE" (EMA).... so you realize that *rational* bubbles are not easy to get...
- However, we know from Samuelson's (1958) consumption-loan model that "bubbles" (i.e. assets with positive price but no intrinsic value) can exist in OLG structures (infinite new traders in the horizon) and that they can be *good*
  - "Money" in Samuelson's model, but not for its transaction service but to store value. Pareto gain from solving dynamic inefficiency (no capital wasted to store value).

A barebones version of Samuelson's model

- OLG. Individuals live for two periods, they are born with an endowment w_t
- Which they save in its entirety and only consume when old (hence we can index the generations welfare by $c_{t,t+1}$)
- There is no population growth, but the endowment grows at a rate γ.

$$\begin{aligned} w_{t+1} &= (1+\gamma)w_t \\ c_{t,t+1} &= (1+r_t)w_t \end{aligned}$$

- What is the interest rate in this economy?

Mother nature...

- The answer depends on which assets are available to store value.
- Samuelson first observed that the young could not save by lending to the old since the latter will not be around to repay them later (financial market incompleteness). The only option of the young is to trade with "mother nature," i.e. to invest in physical capital.
- Let's simplify the technology side and assume that it has constant returns: π. That is, one unit of savings at t produce $1+\pi$ at $t+1$ (we could have a more standard $f(k)$... but main insights would be unchanged). It follows that the interest rate in this economy must be:

$$r_t = \pi$$

and utility is:

$$U_t^{MN} = (1+\pi)w_t$$

Social contract

- Is there any other solution to this model? Consider a social contract by which the young give the entire endowment to their parents who then consume it. Under this social contract the welfare of generation t is:

$$U_t^{SC} = (1+\gamma)w_t$$

- If $\gamma > \pi$, the social contract provides a higher utility than the market!
- How is this possible? In each period, the resources that the market economy devotes to investment, w_t, exceed the resources that it obtains from such activity, $(1+\pi)w_{t-1}$, wasting:

$$(\gamma - \pi)w_{t-1}$$

- The social contract stops this waste, and raises welfare for all

- More broadly: the market economy is overaccumulating capital to facilitate store of value
- Does this mean that the market economy is suboptimal? Not necessarily. Naturally, if $\gamma < \pi$ the market outperforms the social contract. But even if $\gamma > \pi$, the market can reach the same allocation as the social contract, provided we enlarge the saving options of the young to include one irreproducible and useless object with price B_t such that:

$$B_{t+1} = (1 + r_t)B_t$$

Welfare

- Let $x \equiv B/w$. Then

$$x_{t+1} = \frac{B_{t+1}}{w_{t+1}} = \frac{(1+r_t)B_t}{(1+\gamma)w_t} = \frac{1+r_t}{1+\gamma}x_t$$

- If $x < 1$, then $r_t = \pi$ and the bubble vanishes asymptotically

- However, if $x = 1$, then $r_t = \gamma$ and we reproduce the social contract! That is, not only a bubble can exist, but it is also welfare enhancing.

Final remarks

- There are two Pareto-rankable stationary equilibria (bubble better than fundamental); and a continuum of non-stationary equilibria that converge to the fundamental equilibrium that provide intermediate welfare (note: all these equilibria contain bubbles, but these become small relative to the economy

- Rational bubbles arise as a result of coordination across different "generations." They complete markets, and as such they are welfare enhancing. There is an extensive literature that replaces intergenerational frictions for more standard financial frictions – pockets of dynamic inefficiency and bubbles can relax financial constraints (collateral)

Final remarks

- Bad rap: Basic theory sounds weird:
  - Investment declines when bubbles arise... no longer so in models with, e.g., financial frictions (more on this later)
  - Bubbles are welfare enhancing.... Yes, but most of the concern for bubbles is with the cost of crashes, which can perfectly well happen here since the bubbly equilibrium is just one of the possible equilibria, and hence the possibility of a crash is latent (more on this later)
- Next: other, less benign, reasons for bubbles...

Risk shifting

- Allen-Gale (2000) – Bubbles and crises
- There is a pattern:
  - Phase 1: financial liberalization or some expansionary policy fuels a bubble
  - Phase 2: the bubble bursts and asset prices collapse
  - Phase 3: widespread defaults by leveraged asset buyers, leading to a banking and/or exchange rate crisis, and a persistent recession
- Main ingredient (this is all we'll discuss here): Uncertainty about payoffs (real or financial sector) can lead to bubbles in an intermediated financial system (risk shifting)

Model

- Two dates, $t = 1, 2$ and a single consumption good
- Two assets:
  - Safe and in variable supply at a rate r
  - Risky and in fixed supply. Stochastic return is R per unit, with density $h(R)$ and support $[0, R_{MAX}]$
- The return on the safe asset is determined by marginal product of capital: $r = f'(x)$ where x are units of the consumption good (standard assumptions on f)
- Non-pecuniary convex cost of investing in risky assets $c(x)$ (to restrict portfolio sizes)

Model

- There is a continuum of small, risk neutral *investors*; idem for *banks*
- Investors have no wealth while banks have a fixed amount B (which they supply inelastically). Only investors know how to invest, so banks' only choice is to lend to investors
- Banks and investors are restricted to use simple debt contracts
- Since investors can borrow as much as they want at the going rate, in equilibrium the contracted rate on loans must be equal to the riskless interest rate
- Symmetric eq. All investors are identical ex-post. X_s and X_R are the representative investor's holdings of the safe and risky assets

Risk shifting

- Because banks use debt contracts and cannot observe investment decisions by borrowers, the latter do not bear the full cost of investment if the outcome is bad, while they get the benefit if the outcome is good
- If representative investor buys X_s and X_R, it borrows $X_s + PX_R$ (where P is rel. price of risky asset) and the repayment (if not bankrupt) is $r(X_s + PX_R)$
- The liquidation value of the portfolio is $rX_s + RX_R$, so the payoff for the investor is:

$$\max\{(R - rP)X_R, 0\}$$

- and the decision problem is:

$$\max_{X_R \geq 0} \; X_R \int_{R^*=rP}^{R_{MAX}} (R - rP)h(R)dR - c(X_R)$$

Equilibrium

- Market clearing conditions:

$$\begin{aligned} X_R &= 1 \\ X_S + P &= B \\ r &= f'(X_S), \end{aligned}$$

- the focs evaluated at the equilibrium are:

$$\int_{R^*=rP}^{R_{MAX}} (R - rP)h(R)dR = c'(1)$$

$$r = f'(B - P)$$

from which we can solve for (r, P)

P

- We can re-write the foc to get P:

$$P = \frac{1}{r} \frac{\int_{R^*=rP}^{R_{MAX}} Rh(R)dR - c'(1)}{\Pr[R \geq R^*]}$$

$$= \frac{1}{r} \left(E[R|R \geq R^*] - \frac{c'(1)}{\Pr[R \geq R^*]} \right)$$

- Define the *fundamental* as the price an agent would be willing to pay in the absence of risk shifting, then:

$$P^f = \frac{1}{r} \left(E[R] - c'(1) \right)$$

- It is easy to show that, as long as $\Pr[R \leq R^*] > 0$,

$$P > P^f$$

Proof

$$
\begin{aligned}
rP\,\Pr[R &\geq R^*] = \int_{R^*=rP}^{R_{MAX}} Rh(R)dR - c'(1) \\
&= rP^f - \int_0^{R^*} Rh(R)dR \\
&> rP^f - (rP)(1 - \Pr[R \geq R^*])
\end{aligned}
$$

$$\Rightarrow$$

$$rP > rP^f$$

Mean-Preserving Spread in h(R)

- Consider the case where the tails gain mass (and all the reallocation of the mass is within each region around R^*). Then from the first order condition:

$$\int_{R^*=rP}^{R_{MAX}} (R - rP)h(R)dR = c'(1)$$

- we have that rP must rise (since otherwise LHS>RHS). And from:

$$r = f'(B - P)$$

- we have that r and P move in the same direction, so P rises, while P^f drops (since r rises), so the "bubble" increases (more risk shifting)

Final remarks

- Due to risk shifting, P is higher than fundamental (bubble)
- The counterpart of the bubble is the bank losses... massive default if return below R^*
- The "bubble" increases with risk itself, and so does the probability of a crisis (mean preserving spread)
- In a sense it is not a GE bubble, as the price of banks should go down... but it may well be that households are stuck... which takes us back to the standard model of RE bubbles in macro that highlights the shortage of assets..

Information/"Behavioral"

- One example: Abreu and Brunnermeir
- Behavioral biases lead to bubbles (they take this as given)
- Assuming that rational arbitrageurs understand that the market will eventually collapse, will they still ride the bubble?
- *Delayed arbitrage model* (riding the bubble for a while may be optimal)
- A model of *market timing*
    - Dispersion in exit strategies makes the bubble possible ("bad news travel slowly")
    - At some random time t_0 price surpasses the fundamental value. Thereafter, rational arbitrageurs become sequentially aware that the price has departed from fundamentals. They don't know whether they are early or late relative to others
    - Bubble bursts when a sufficient mass of arbitrageurs have sold out (coordination)

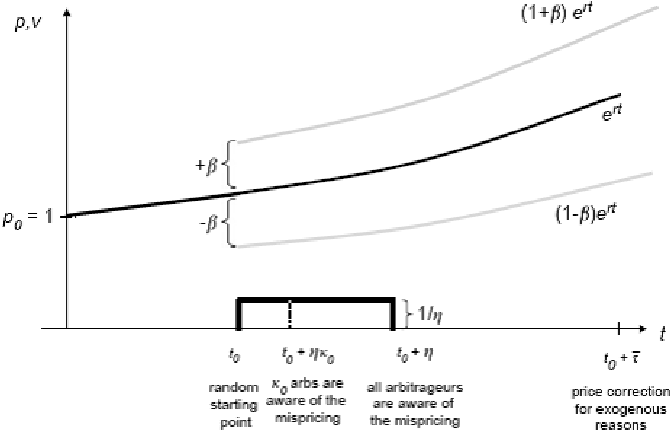
The Setup

- In "Bubbles and Crashes," they discuss an "irrational exuberance" episode where after same random date t_0 the price continues to rise at some rate $g > r$, while the fundamental only rises at r
- The main economic forces in their EMA paper are also found in their simpler, JFE, paper: "*Synchronization Risk* and Delayed Arbitrage" (we will develop this one)
- There is a single risky asset with price p_t and fundamental v_t. Prior to the arrival of a shock at a random time t_0, the fundamental value is e^{rt} and after that $(1+\tilde{\beta})e^{rt}$, with $\tilde{\beta}$ taking values β and $-\beta$ with equal prob., and $F(t_0) = 1 - e^{-\lambda t_0}$
- Prior to the shock at t_0, $p_t = v_t$. After t_0 the price deviates from fundamentals until full arbitrage takes place (the crash if $-\beta$, which we assume henceforth)

The Setup

- There are two types of agents: rational arbitrageurs and behavioral traders
- The only role of the latter agents is to support the mispricing and maintain the price at $p_t = e^{rt}$ as long as the selling pressure by rational arbitrageurs lies below a threshold $\kappa(.)$
- The focus of the paper is on the former agents. Arbitrageurs are ex-ante identical but receive information about the deviation sequentially (uniformly) between t_0 and $t_0 + \eta$
- An individual arbitrageur who learns about the change in fundamental at t_i (denoted by $\hat{t}_i$) thinks that t_0 is distributed between t_i and $t_i - \eta$

The Setup



p,v
(1+β) e^rt
e^rt
+β
p0 = 1
-β
(1-β)e^rt
1/η
t
t0
t0 + ηκ0
t0 + η
t0 + τ̄
random starting point
κ0 arbs are aware of the mispricing
all arbitrageurs are aware of the mispricing
price correction for exogenous reasons

The Setup

- Arbitrageurs are risk neutral but the maximum short position is $x_i = -1$. The "normal/neutral" position is $x_i = 0$. Departing from this benchmark generates ("large") holding costs of $cp_t|x_i|$

- The price correction occurs as soon as the aggregate order imbalance of all arbitrageurs exceeds $\kappa(t - t_0)$, with (reduced form from behavioral agents)

$$\kappa(t - t_0) = \kappa_0 \left[1 - (1/\bar{\tau})(t - t_0)\right]$$

- Motivation: The longer the mispricing persists, the smaller is the mass of behavioral traders that remain confident that the "price is right"

- Since there are no price changes, arbitrageurs cannot infer t_0 from them while pressure is below $\kappa(.)$

Market Timing and Delayed Arbitrage

- Arbitrageur $\hat{t}_i$ specifies a trading strategy as function of $\tau_i = t - t_i$. A-B focus on trigger strategies such that the arbitrageur sets $x_i = 0$ until a date $t_i + \tau_i^*$ and $x_i = -1$ after that (until the correction takes place)

- An arbitrageur that trades just before the correction achieves the highest payoff. By postponing the trade she reduces holding costs but risks missing the arbitrage opportunity (Keynes: "beat the gun" terminology)

- Let $h(t|\ \hat{t}_i)$ be arbitrageur $\hat{t}_i$'s perceived hazard rate that the price correction occurs in the next instant t. Thus, her estimate of a correction in the next (small) time interval Δ is $h(t|\ \hat{t}_i)\Delta$, while the holding cost is $cp_t\Delta$

- The arbitrageur will trade if the expected benefit $\beta p_t h(t|\ \hat{t}_i)\Delta$ exceeds the expected cost of holding a short position $(1 - h(t|\ \hat{t}_i)\Delta)cp_t\Delta$

- Of course the hazard rate depends on other arbitrageurs' trading strategies. A-B restrict attention to symmetric trigger strategy equilibria

Abreu-Brunnermeier: Market Timing and Delayed Arbitrage

- If all arbitrageurs trade with a delay τ', then the price correction occurs at $t_0 + \varphi(\tau')$, where the latter is defined implicitly from

$$\frac{\varphi(\tau') - \tau'}{\eta} = \kappa(\varphi(\tau'))$$

- Using the linear expression for $\kappa(.)$, we have

$$\varphi(\tau') = \bar{\tau}\frac{\tau' + \eta\kappa_0}{\bar{\tau} + \eta\kappa_0}$$

Market Timing and Delayed Arbitrage

- Arbitrageur $\hat{t}_i$ knows that, in equilibrium, the price correction will occur no later than $t_i + \varphi(\tau')$ but after $t_i + \varphi(\tau') - \eta$
- Given the prior distribution on t_0, the latter observation yields a simple posterior:

$$h(t|\hat{t}_i) = \left\{ \begin{pmatrix} 0 & \text{for} \quad t < t_i + \varphi(\tau') - \eta \\ \frac{\lambda}{1-\exp\{-\lambda(t_i+\varphi(\tau')-t))\}} & \text{for} \quad t \geq t_i + \varphi(\tau') - \eta \end{pmatrix} \right.$$

Market Timing and Delayed Arbitrage

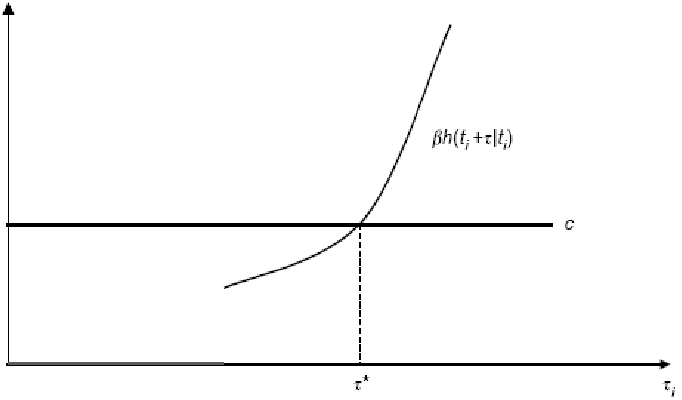


Fig. 2. Arbitrageur $\hat{t}_i$'s hazard rate.

Final Remarks

- Arbitrage is delayed. This is possible because mispricing is never common knowledge, which preserves the disagreement about the timing of price corrections
    - The arbitrageur who becomes immediately aware of the mispricing at t_0 knows that at $t_0 + \eta$ everyone knows about the mispricing. However, the trader who only becomes aware at $t_0 + \eta$ thinks that he might be the first to hear of it and he does not know that all traders already know it. Hence, even if everybody knows of the mispricing at $t_0 + \eta$, only the first trader knows that everybody knows
    - At $t_0 + 2\eta$, even the last trader knows that everybody knows, but he does not know that everybody knows that everybody knows of the mispricing, and so on

- The main distinction with noise-traders is that most of the action comes from the rational traders. It is the uncertainty about the behavior of other rational traders that leads to delayed arbitrage (synchronization risk).

Behavioral – Belief disagreements and overvaluation

- Two steps:
  - Static: Belief disagreement and overvaluation
  - Dynamic: Speculative bubbles (resale option value)
- Based on Alp's notes on:
  - Miller (1977)
  - Harrison-Kreps (1978)

Basic two period environment

- Consider a model with two periods, $t \in \{0, 1\}$ single good (dollar)
- Traders have endowments at date 0, consume at date 1.

Resources invested in two ways:

- Safe asset (storage): 1 dollar gives $1 + r$ dollars at $t = 1$.
- **Risky asset:** Supplied inelastically.
- Normalized supply of 1 unit (by other agents that sell).
- Yields s dollars, where s is distributed $N\left(\mu^{true}, \sigma^2\right)$.(

Benchmark with common beliefs

- First consider mass 1 of identical traders
- **Optimism**: Traders believe $s \sim N\left(\mu, \sigma^2\right)$, where:
$$\mu > \mu^{true}.$$
- They each have n dollars (sufficiently large)
- They have CARA preferences, so mean-variance optimization:
$$\max_x \underbrace{n\left(1+r\right) + x\left(\mu - p\left(1+r\right)\right)}_{\text{expected wealth at date 1}} - \frac{1}{2}\theta x^2 \sigma^2.$$

Benchmark with common beliefs

- Traders' demand for the asset is given by:

$$x = \frac{\mu - p\,(1+r)}{\theta\sigma^2}.$$

- The equilibrium requires, $x = 1$, which implies:

$$\underbrace{p = \frac{\mu - \theta\sigma^2}{1+r}}_{\text{market price}} \quad > \quad p^{true} = \underbrace{\frac{\mu^{true} - \theta\sigma^2}{1+r}}_{\text{rational price}}.$$

- **Average optimism generates overvaluation.**

Benchmark with heterogeneous beliefs

- Suppose there are two groups, $j \in \{0, 1\}$
- **Optimists,** $j = 0$, and **pessimists**, $j = 1$, with mass $1/2$ each
- They have beliefs $N(\mu^j, \sigma^2)$, where
$$\mu^0 = \mu^{true} + \Delta \text{ and } \mu^1 = \mu^{true} - \Delta.$$
- Note that we shut down average optimism
- As before, mean-variance preferences with θ
- What is the equilibrium price in this case?

Benchmark with heterogeneous beliefs

- Traders' demand is given by:

$$x^j = \frac{\mu^j - p\,(1+r)}{\theta\sigma^2}.$$

- Aggregate demand is:

$$x = \frac{\mu^{true} - p\,(1+r)}{\theta\sigma^2}.$$

- The equilibrium price is the rational price:

$$p = \frac{\mu^{true} - \theta\sigma^2}{1+r}.$$

Heterogeneity, by itself, doesn't generate overvaluation.

Heterogeneity and short-selling constraints

- Suppose $\Delta > \theta\sigma^2$ so that

$$\mu^{true} - \theta\sigma^2 > \mu^0 = \mu^{true} - \Delta$$

- What are the positions of optimists and pessimists?
  - Recall that: $x^1 = \left(\mu^1 - p\,(1+r)\right)/\theta\sigma^2$ and $p(1+r) = \mu^{true} - \theta\sigma^2$

$$x^1 = \frac{\theta\sigma^2 - \Delta}{\theta\sigma^2} < 0$$

- What if pessimists cannot short sell the asset?

Heterogeneity and short-selling constraints

- With short selling constraints, demand is given by:

$$x^j = \max \left(0, \frac{\mu^j - p\,(1+r)}{\theta\sigma^2} \right)$$

- In equilibrium, pessimists constrained and out of the market:

$$x^1 = 0.$$

- All of the supply is purchased by optimists
- What is the equilibrium price in this case?

Equilibrium with short-selling constraints

- Market clearing requires, $x^0/2 = 1$, which implies

$$
\begin{aligned}
p &= \frac{\mu^0 - 2\theta\sigma^2}{1+r} \\
&= \frac{\mu^{true} + \Delta - 2\theta\sigma^2}{1+r} \\
&= \frac{\mu^{true} - \theta\sigma^2 + (\Delta - \theta\sigma^2)}{1+r} > p^{true}.
\end{aligned}
$$

- Miller (1977): **Heterogeneity + shorting restrictions $\Longrightarrow$ overvaluation**

- Natural assumptions (for some markets)
  - Policy: Restrictions on shortselling during crises.... (e.g., Naked CDS during European crisis)

Harrison and Kreps (1978): Speculative bubbles

- Harrison-Kreps (1978): Miller mechanism in dynamic setting
- Heterogeneity and shorting restrictions as before
- In addition, **identity of optimists changes over time**
- **Speculation:** Optimists buy to sell to future optimists
- Price exceeds the pdv of even the most optimistic investor
- Known as a **resale option value** or a **speculative bubble.**

Consider dynamic version of earlier model

- Consider the earlier model with more dates, $t \in \{0, 1, ..\}$
- There is new generation of traders at every date, $t \in \{0, 1, ..\}$
- At each date t, new traders (of mass 1) enter the market
- They have large endowments n, and preferences:

$$\sum_{\tilde{t}=t}^{\infty} \left(\frac{1}{1+r} \right)^{\tilde{t}-t} c_{\tilde{t}}.$$

- For simplicity, they are risk-neutral, i.e., $\theta = 0$.

Heterogeneous beliefs about the risky asset

- As before, single risky asset in fixed supply of 1 unit.
- As before, the asset cannot be short sold.
- (Log) dividends follow random walk:

$$a_{t+1} = a_t s_{t+1}.$$

- Suppose $a_0 = 1$ and $s_{t+1} \sim F^{true}$ with mean $\mu^{true} = 1$.
- Within generation t, optimists, $j = 0$, and pessimists, $j = 1$.
- Beliefs about next shock, s_{t+1}, given by F^j with mean:

$$\mu^0 = 1 + \Delta \text{ and } \mu^1 = 1 - \Delta.$$

- But beliefs for $\{s_{n+k}\}_{k=2}^{\infty}$ are identical and given by F^{true}.
- Optimism about only next shock. "This time is different."

Heterogeneous buy-and-hold valuations

- Each trader believes (on average) a_t will move to $a_t \mu^j$, and then randomly fluctuate around that new level
- Buy-and-hold valuations for each generation t trader:

$$v^{0,\text{hold}} \equiv \frac{a_t (1+\Delta)}{r} \text{ and } v^{1,\text{hold}} \equiv \frac{a_t (1-\Delta)}{r}.$$

- Is the equilibrium price the same as one of these?
- If yes, which one? If not, how do we find the equilibrium?

Capital gains increase price beyond buy-and-hold

- Price is not the same as $v^{0,\text{hold}}$ due to possibility of **capital gains**
- Under our assumptions **future optimists** that will enter at date $t+1$ will be more optimistic than **current optimists** will be at $t+1$
- Current optimists can, and will, sell the asset to future optimists
- So optimists' valuation (and thus, the equilibrium price) satisfies:

$$p_t = \frac{1}{1+r}\left(\underbrace{E_t^0\left[a_{t+1}\right]}_{\text{dividend gains}} + \underbrace{E_t^0\left[p_{t+1}\right]}_{\text{capital gains}}\right).$$

- Capital gains likely to be important. How do we find them?

Resale option value

- In this model, the current dividend, a, is a sufficient statistic
- Let's conjecture a recursive equilibrium with price $p(a)$
- At each date, the asset is priced by optimists at that date
- Recursive pricing equation:

$$
\begin{aligned}
p(a) &= \frac{1}{1+r}\left(a(1+\Delta) + \int_{\mathcal{S}} p(as)\, dF^1\right) \\
&= \frac{1}{1+r}\left(a(1+\Delta) + p(a)\int_{\mathcal{S}} s dF^1\right) \\
&= \frac{1}{1+r}\left(a(1+\Delta) + p(a)(1+\Delta)\right)
\end{aligned}
$$

Equilibrium features a resale option value

- To imply:

$$p(a) = \frac{a(1+\Delta)}{r-\Delta} > v^{0,\text{hold}} = \frac{a(1+\Delta)}{r}.$$

- The difference captures **the resale option value**
- Could be quite large, e.g., if r is low and Δ is high
- Higher price level than buy-and-hold valuation of all current traders!

Taking Stock

- There is a variety of mechanisms that can lead to asset overvaluations
- These overvaluations may have macroeconomic consequences through wealth effects and credit expansion (e.g. by relaxing collateral constraints). This is our next topic.
- For the same reason, their crashes can have dire macroeconomic consequences (more on this later in the course)

Appendix: Bubble Episodes (Brunnermeir - Schnabel)

	Event	Time	Place
1	Tulipmania	1634-37 (crisis: Feb. 1636)	Netherlands
2	Mississippi bubble	1719-20 (crisis: May 1720)	Paris
3	Crisis of 1763	1763 (crisis: Sept. 1763)	Amsterdam, Hamburg, Berlin
4	Crisis of 1772	1772-73 (crisis: June 1772)	England, Scotland
5	Latin America Mania	1824-25 (crisis: Dec. 1825)	England (mainly London)
6	Railway Mania	1840s (crises: April/Oct.1847)	England
7	Panic of 1857	1856-57 (crisis: Oct.1857)	United States
8	Gründerkrise	1872-73 (crisis: May 1873)	Germany, Austria
9	Chicago real estate boom	1881-83 (no crisis)	Chicago
10	Crisis of 1882	1881-82 (crisis: Jan. 1882)	France
11	Panic of 1893	1890-93 (crisis: Jan. 1893)	Australia
12	Norwegian crisis of 1899	1895-1900 (crisis: July 1899)	Norway
13	U.S. real estate bubble	1920-26 (no crisis)	United States
14	German stock price bubble	1927 (crisis: May 1927)	Germany
15	U.S. stock price bubble	1928-29 (crisis: Oct. 1929)	United States
16	"Lost decade"	1985-2003 (crisis: Jan. 1990)	Japan
17	Scandinavian crisis: Norway	1984-92 (crisis: Oct. 1991)	Norway
18	Scandinavian crisis: Finland	1986-92 (crisis: Sept. 1991)	Finland
19	Asian crisis: Thailand	1995-98 (crisis: July 1997)	Thailand
20	Dot-com bubble	1995-2001 (crisis: April 2000)	United States
21	Real estate bubble in Australia	2002-04 (no crisis)	Australia
22	Subprime housing bubble	2003-10 (crisis: 2007)	United States
23	Spanish housing bubble	1997-? (crisis: 2007)	Spain

Speculative Growth and the AI "Bubble"

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Speculative growth: Caballero, Farhi & Hammour (2006)

Motivation. US 1990s: Investment and output **boomed**; yet long-run real rates *fell* $\approx$ 200bp, explaining $>$ 50% of the equity rise.

Puzzle: How can expectations of a *low* long-run cost of capital be consistent with the high funding demand of a high-investment equilibrium?

Central ingredient: the growth-funding feedback.

Along the expansion path, the supply of effective funding rises endogenously, more than offsetting higher demand — lowering the cost of capital and validating high valuations ex ante.

Two of the funding channels that seemed relevant for the US in the 1990s:

- Procyclical fiscal surpluses *(surplus illusion)*
- Capital flows as relative foreign opportunities declined (US exceptionalism)

CFH: the capital gains mechanism

Model: Diamond (1965) OLG + adjustment costs + funding function $s(k, r)$ with a jump Δ at k^o

Multiple SS when feedback Δ is large enough:

- k^n: normal — low k, **high** r, low valuations
- k^s: speculative — high k, **low** r, high valuations

Without adj. costs: multiple SS but *unique equilibrium*. The adj. cost creates **capital gains**, decoupling the return from the MPK.

Intermediate adj. costs required (Prop. 2):
Too small $\Rightarrow q \approx 1$, no capital gains
Too large $\Rightarrow$ transition too costly

The speculative-growth path

From $(k^n, q = 1)$, coordinated optimism triggers $q \uparrow$. Economy rides the saddle path of k^s with $q > 1$ throughout. High q sustains investment; funding feedback delivers low r, validating the initial jump.

- **The crash is self-fulfilling:** pessimism returns to k^n with no commensurate fundamental shock
- **Vs. Tirole (1985):** investment and valuations comove *positively* — unlike standard rational bubbles

Today's debate: bubble or fundamentals?

The investment boom is real

- Big Four hyperscalers: $>$ \$600B capex committed for 2026 alone, up 62% YoY
- Tech equipment & software near 4.4% of GDP — matching dot-com peak
- AI stocks: dominant driver of S&P 500 returns since ChatGPT

Fragility is in the air

- CAPE $\approx$ 40: only the second time above this threshold in 155 years
- The Nvidia Paradox: record earnings beat — stock fell 5%
- Big tech issued \$100B in bonds in early 2026; investors demanded record CDS protection

This paper: the "bubble vs. fundamentals" framing is too rigid. Elevated valuations can be **rational** and **fragile** at the same time.

The thesis

AI technology can generate **speculative-growth equilibria**: elevated valuations support rapid capital accumulation and sustain a transition toward a high-capital steady state.

Two AI-specific features drive this:

- AI capital performs **labor tasks** $\Rightarrow$ expands effective labor, flattens MPK
- Gains accrue to **capitalists** whose saving rises with wealth $\Rightarrow$ funding feedback, $R \downarrow$

These equilibria are **rational** but **fragile**: they persist only as long as beliefs remain coordinated. A loss of confidence can precipitate a **self-fulfilling crash and reversal**.

A possibility argument: a coherent mechanism for the joint behavior of valuations, investment, wealth distribution, and interest rates — not a claim that current AI markets fully satisfy rational expectations.

Roadmap

1. Motivation *CFH + AI context*

Speculative-growth mechanism; AI as a natural new application

2. Model *how AI generates the feedback*

Production (flat MPK), distribution (saving feedback), asset pricing

3. The speculative-growth path *equilibrium and dynamics*

Multiple steady states, high-q transition, time paths, crash

4. Comparative statics *what can derail it?*

Higher γ, fiscal deficits, creative destruction

This paper: AI provides the feedback naturally

AI is a single primitive that can generate both the real-side and funding-side pieces of speculative growth.

1. Production side

AI substitutes for labor tasks. Effective labor expands with K, creating a **flat MPK region**.

$\Rightarrow$ **easier multiple steady states**

2. Distribution side

AI displaces labor income. Wealth shifts toward high and increasing-saving capitalists.

$\Rightarrow$ **saving $\uparrow$, $R\downarrow$**

3. Adjustment costs

Expensive investments and natural barriers to entry, but still plenty of investment.

$\Rightarrow$ **elevated q bridges transition**

Result: high valuations can be rational but fragile.

Labor-like AI creates a flat MPK region

Production: $Y = AK_c^{\alpha} N^{1-\alpha}$; $K = K_c + K_\ell$
$N = 1 + \gamma \min\{K_\ell, \bar{K}_\ell\}$ (e.g., Restrepo (2025))

Region I ($K < K_{\mathrm{AI}}$): no AI, $K_\ell = 0$

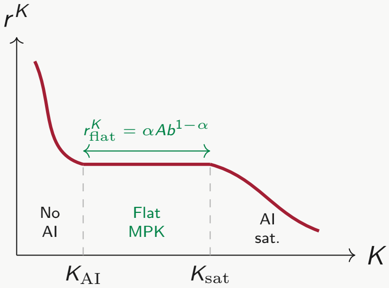
$$r^K = \alpha A K^{\alpha - 1}$$

Region II ($K_{\mathrm{AI}} \le K < K_{\mathrm{sat}}$): interior FOC

$$\frac{\alpha}{K_c} = \frac{(1-\alpha)\gamma}{N} \Rightarrow r^K = \alpha A b^{1-\alpha}; \quad b \equiv \frac{(1-\alpha)\gamma}{\alpha}$$

Region III ($K \ge K_{\mathrm{sat}}$): AI saturated, $K_\ell = \bar{K}_\ell$

$$r^K = \alpha A (K - \bar{K}_\ell)^{\alpha - 1} (1 + \gamma \bar{K}_\ell)^{1-\alpha}$$



In Region II: $K_\ell \uparrow$ keeps $K_c/N = 1/b$ constant $\Rightarrow$ **no diminishing returns**

Model: households and asset pricing

Workers: hand-to-mouth, $c_w = w$

Capitalists:

$$\max \int_0^\infty e^{-\rho t}[\ln c_t + \lambda W_t]\, dt$$

$$\Rightarrow \quad c = \kappa W^\phi, \quad 0 < \phi < 1$$

Key: saving rate *rises* with wealth (e.g., Straub 2019)

Capital: $\dot{K}/K = \psi \ln q - \delta$

Asset pricing:

$$\underbrace{\tfrac{\dot{q}}{q} - \delta}_{\text{cap. gain}} + \underbrace{\tfrac{r^K(K)}{q}}_{\text{div. yield}} = R$$

Euler equation: $\dfrac{\dot{c}}{c} = R - \rho + \lambda c$

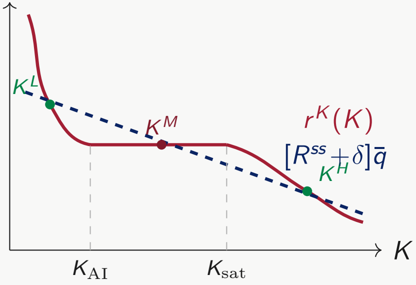
Equilibrium rate:

$$\phi(R - \kappa W^{\phi-1}) = R - \rho + \lambda \kappa W^\phi$$

$$R(W) = \tfrac{\rho - \phi\kappa W^{\phi-1} - \lambda\kappa W^\phi}{1-\phi}$$

Funding feedback: $K\uparrow \Rightarrow \; W = qK\uparrow \Rightarrow$ saving$\uparrow \Rightarrow \; R\downarrow \Rightarrow$ asset prices supported.

A flat MPK region facilitates three steady states



Steady-state condition:

$$r^K(K) = [R^{ss}(\bar{q}K) + \delta]\bar{q}, \quad R^{ss}(W) = \frac{\rho}{1+\lambda W}$$

- Left: "down-flat-down" r^K
- Right: strictly decreasing in K

Multiplicity condition (necessary):

$$[R^{ss}(\bar{q}K_{\mathrm{AI}}) + \delta]\bar{q} > r^K_{\mathrm{flat}} > [R^{ss}(\bar{q}K_{\mathrm{sat}}) + \delta]\bar{q}$$

- $q = \bar{q}$ at both stable SS, but market cap: $\bar{q}K^H \gg \bar{q}K^L$

→ Appendix: stability

Multiplicity needs a high-q transition path

The stable manifold W^s_ψ of $(K^H, \bar{q})$ must **reach back** to K^L at elevated but plausible q.

Too high adj. costs
ψ too small

Weak response to q.
Required q becomes too high.

No feasible reach-back.

Too low adj. costs
ψ too large

Capital adjusts too easily.
Capital gains are too small.

No $q > \bar{q}$ bridge.

Intermediate adj. costs
$\psi \in (\psi_{\text{min}}, \psi_{\text{max}})$

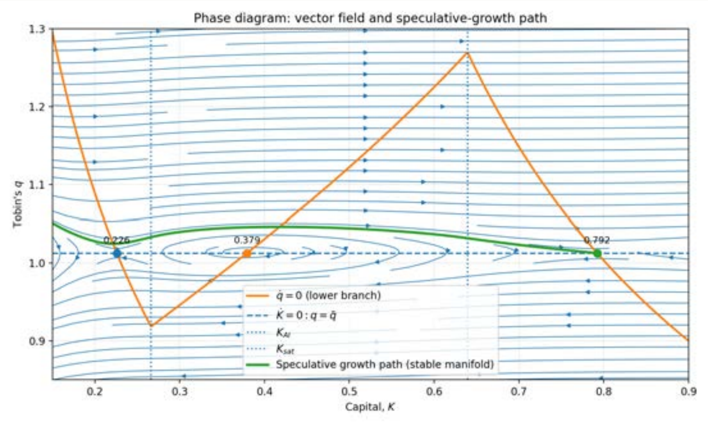
Capital gains sustain $q > \bar{q}$.
Accumulation remains gradual.

Reach-back is feasible.

Proposition 1. Reach-back requires intermediate adjustment costs: $\psi \in (\psi_{\text{min}}, \psi_{\text{max}})$.
Transverse reach-back at ψ^* persists locally.

▸ Formal analysis

The speculative-growth path



From $(K^L, \bar{q})$ **to** $(K^H, \bar{q})$

1. Optimism coordinates beliefs
2. q jumps to $q_0 > \bar{q}$
3. Investment booms
4. AI deploys as K crosses K_{AI}
5. Wealth concentration lowers R

High q is the mechanism, not a symptom: it makes the transition feasible.

Rates can be high early and low later

The same AI transition can generate **high rates early** and **low rates later**.

Early phase

Investment demand is strong.

Consumption growth is high.

Implication: R can remain elevated.

Late phase

Flat MPK raises wealth sharply.

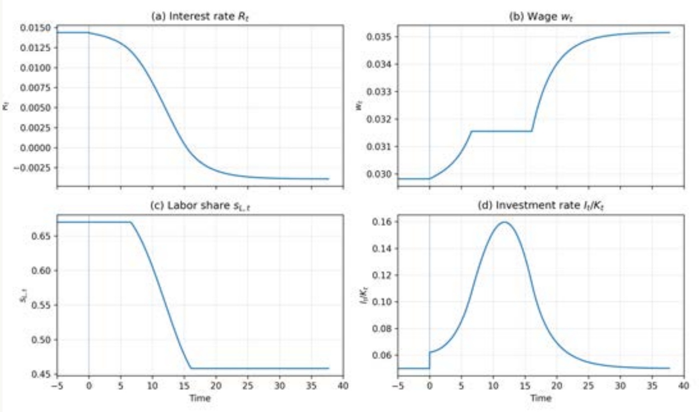
Saving rises with wealth.

Implication: R falls sharply.

Interpretation. An AI boom can coexist with high rates initially, even if the long-run implication is a lower funding rate.

Interestingly, Andrews–Farboodi (2025) find empirically that positive news about AI progress lowers medium-run interest rates — suggestive of the same pattern.

Time paths along the speculative-growth trajectory



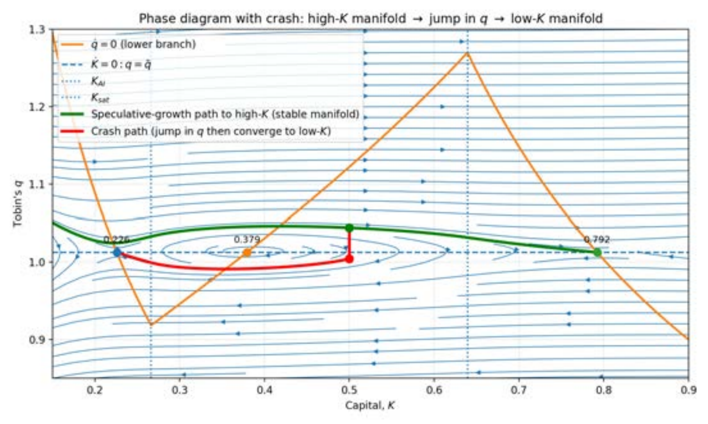
R_t: high early; **falls in Region II**.

Labor share: flat, then **compresses**.

Wages: stall in Region II; resume in Region III.

Investment: jump, hump, mean reversion.

The crash is belief-driven



At (K_t, q_t) with $q_t > \bar{q}$, confidence weakens.

q drops discretely to the low-K manifold.

Investment collapses; capital decumulates; the economy returns to $(K^L, \bar{q})$.

Self-fulfilling: beliefs $\downarrow \Rightarrow q \downarrow \Rightarrow$ beliefs are validated.

No fundamental shock is required.

Not a bubble — but fragile

Not a bubble

- Valuations consistent with fundamentals
- Growth and wealth can ultimately validate them
- $q = \bar{q}$ at both stable SS
- High q is the mechanism, not a symptom

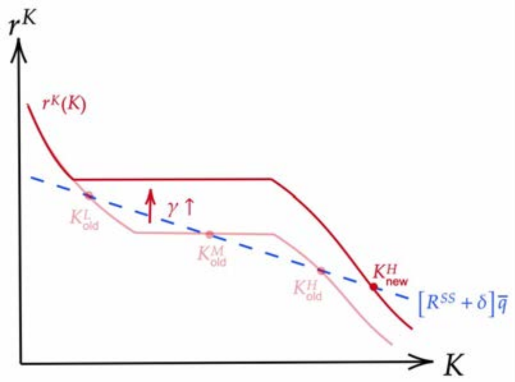
Fragile

- Validation requires sustained confidence throughout a lengthy transition
- A belief-driven drop in q can abort at any point
- Crash requires no fundamental shock

Key insight: A rational equilibrium can sustain the narratives that keep it alive.

Discussion: higher γ can remove multiplicity

As $\gamma\uparrow$: flat MPK rises $\Rightarrow$ the low equilibrium disappears



If γ keeps rising

r^K_{flat} eventually clears $[R^{ss}+\delta]\bar{q}$ everywhere.

K^L and K^M vanish; only K^H survives.

Speculative character disappears.

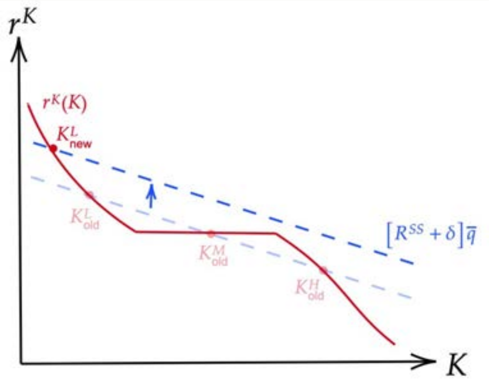
If γ disappoints

The flat MPK stays low. The high-K equilibrium may cease to exist.

The boom unravels and the speculative path is abandoned.

What can derail it?

$[R^{ss} + \delta]\bar{q}$ **shifts up** $\Rightarrow$ **the high equilibrium may disappear**



Large fiscal deficits

Wars, geopolitical shocks, or political redistribution compete for the saving that sustains low R at K^H.

Effect: $[R^{ss} + \delta]\bar{q}$ shifts up.

Too much creative destruction

Obsolescence acts like an increase in δ.

Effect: the high-K equilibrium may disappear.

Conclusion

1. Flat MPK

Labor-like AI forestalls diminishing returns and opens the door to multiple steady states.

2. Funding feedback

AI redistribution raises saving; wealth rises and R falls along the transition.

3. Capital gains

Intermediate adjustment costs let elevated q finance the path toward high K.

Main implication: the boom can be rational yet fragile. Interest rates may be high early, fall sharply later, and reversals can be self-fulfilling.

A possibility argument, not a claim that current AI valuations satisfy rational expectations.

Appendix

Appendix: local stability

Linearize around $(K^*, \bar{q})$ with K predetermined, q a jump variable:

$$J_{12} = \tfrac{\psi K^*}{\bar{q}} > 0, \quad J_{21} = -(r^K)'(K^*) + \bar{q}^2 R'(W^*), \quad J_{22} = R^{ss} + \delta + \bar{q} R'(W^*) K^*$$

$\mathrm{tr}(J) = J_{22} > 0; \quad \det(J) = -J_{12} \cdot J_{21}$

- **At K^L, K^H:** $(r^K)' < 0 \;\Rightarrow\; \det(J) < 0$. **Saddle points (stable).**
- **At K^M:** $(r^K)' = 0$ (flat) $\Rightarrow\; \det(J) > 0$, $\mathrm{tr} > 0$. **Unstable node.**
- Saddle + one predetermined variable $\Rightarrow$ unique convergent path (saddle-path stability).

Appendix: formal analysis of intermediate ψ

Definition. W^s_ψ *reaches back at elevated-plausible valuations* if $W^s_\psi \cap \{K = K^L,\ q \in [(1+\eta)\bar{q}, \bar{Q}]\} \neq \emptyset$.

Lemma 1. $\psi \leq \delta / \ln \bar{Q} \Rightarrow \bar{q} \geq \bar{Q}$. Reach-back impossible.

Lemma 2. For $\psi \geq \psi_{\text{max}} \equiv 2M(\bar{Q}) \ln(K^H/K^L)/[\eta \ln(1+\eta/2)]$, K reaches K^H before q falls to $\bar{q}$: overshoot contradiction. Reach-back impossible.

Proposition 1. Reach-back requires $\psi \in (\psi_{\text{min}}, \psi_{\text{max}})$. Transverse intersection at ψ^* persists locally (Stable Manifold Theorem + IFT).

Appendix: numerical example

Parameter		Value
A	TFP	0.0729
α	Capital share	0.33
γ	AI labor equiv.	1.85
$\bar{K}_\ell$	AI capacity	0.25
ρ	Discount rate	0.08
λ	Wealth in utility	20
δ	Depreciation	0.05
ψ	Inv. adjustment	3.0

$K_{\mathrm{AI}} = 0.266$; $K_{\mathrm{sat}} = 0.641$; $r^K_{\mathrm{flat}} \approx 0.058$; $\bar{q} \approx 1.017$

$K^L = 0.224$; $K^M = 0.384$; $K^H = 0.790$

Multiplicity condition:

$$\underbrace{0.062}_{[R^{ss}(K_{\mathrm{AI}})+\delta]\bar{q}} > \underbrace{0.058}_{r^K_{\mathrm{flat}}} > \underbrace{0.056}_{[R^{ss}(K_{\mathrm{sat}})+\delta]\bar{q}} \quad \checkmark$$