# 14.462 Lecture Notes Commitment, Coordination, and Expectation Traps 

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Spring 2004

## 1 Kydland Prescott/Barro Gordon

A large number of private agents play against a government.
The government solves

$$
\min L=\left\{\left(y-y^{*}\right)^{2}+\beta \pi^{2}\right\}
$$

subject to the Philips curve

$$
y=\bar{y}+\alpha\left(\pi-\pi_{e}\right)-\varepsilon .
$$

The shock $\varepsilon$ is distributed uniform over $[-e,+e]$
The agents solve

$$
\min \mathbb{E}\left(\pi_{e}-\pi\right)^{2}
$$

The agents move first, setting $\pi^{e}$ without observing $\varepsilon$. Nature then draws $\varepsilon$. The government moves last, setting $\pi$ after observing $\varepsilon$ and taking $\pi^{e}$ as given.

The best response of the government is given by

$$
\pi=g\left(\pi_{e}, \varepsilon\right)=\frac{\alpha\left(y^{*}-\bar{y}+\varepsilon\right)+\alpha^{2} \pi_{e}}{\alpha^{2}+\beta}
$$

Note that $g(0,0)>0$ and $g_{\pi}(\pi, \varepsilon)=\frac{\alpha^{2}}{\alpha^{2}+\beta} \in(0,1)$. The best response for the agents is

$$
\pi_{e}=\mathbb{E} \pi
$$

Hence, in equilibrium,

$$
\pi_{e}=\mathbb{E} g\left(\pi_{e}, \varepsilon\right)=\frac{\alpha\left(y^{*}-\bar{y}\right)+\alpha^{2} \pi_{e}}{\alpha^{2}+\beta} \equiv G\left(\pi_{e}\right)
$$

By the properties of $g$, we have $G(0)$ and $G^{\prime} \in(0,1)$. Hence, there is a unique fixed point with $\pi_{e}>0$. Indeed, this is given by

$$
\pi_{e}=\frac{\alpha\left(y^{*}-\bar{y}\right)}{\beta}
$$

It follows that equilibrium inflation is

$$
\pi=\pi_{e}+\frac{\alpha}{\alpha^{2}+\beta} \varepsilon
$$

and equilibrium output is

$$
y=\bar{y}+\frac{\alpha^{2}}{\alpha^{2}+\beta} \varepsilon
$$

## 2 Obstfeld (1994)

We now reinterpret $\pi$ as the rate of devaluation. We also modify the preferences of the government so that the government solves

$$
\min L=\left\{\left(y-y^{*}\right)^{2}+\beta \pi^{2}+\theta R\right\}
$$

subject to the Philips curve, where $R$ is an indicator that takes the value 1 if $\pi \neq 0$ and 0 if $\pi=0$. The variable $\theta$ represents the value of maintaining the peg.

The timing is the same. The government chooses $\pi$ after observing $\varepsilon$ and after agents have set $\pi_{e}$.

If the government sets $\pi=g\left(\pi_{e}, \varepsilon\right)$, then welfare losses are given by

$$
L=L_{f l e x}\left(\pi_{e}, \varepsilon\right)=\frac{\beta}{\alpha^{2}+\beta}\left(y^{*}-\bar{y}+\varepsilon+\alpha \pi^{e}\right)^{2}+\theta
$$

If instead the government sets $\pi=0$, then welfare losses are given by

$$
L=L_{\text {fixed }}\left(\pi_{e}, \varepsilon\right)=\left(y^{*}-\bar{y}+\varepsilon+\alpha \pi^{e}\right)^{2}
$$

Define $\underline{\varepsilon}=\underline{\varepsilon}\left(\pi_{e}\right)$ and $\bar{\varepsilon}=\bar{\varepsilon}\left(\pi_{e}\right)$ as the lowest and highest solution to

$$
L_{f l e x}\left(\pi_{e}, \varepsilon\right)=L_{\text {fixed }}\left(\pi_{e}, \varepsilon\right)
$$

Whenever $\varepsilon \in[\underline{\varepsilon}, \bar{\varepsilon}]$, the government finds it optimal to set $\pi=0$ (fixed). Whenever $\varepsilon \notin[\underline{\varepsilon}, \bar{\varepsilon}]$, the government prefers to set $\pi=g\left(\pi_{e}\right)$ (flexibility).

Now consider the equilibrium. In equilibrium,

$$
\pi^{e}=\mathbb{E} \pi=0 \cdot \operatorname{Pr}(\varepsilon \in[\underline{\varepsilon}, \bar{\varepsilon}])+\mathbb{E} g\left(\pi_{e}, \varepsilon\right) \cdot \operatorname{Pr}(\varepsilon \notin[\underline{\varepsilon}, \bar{\varepsilon}]) \equiv G\left(\pi_{e}\right)
$$

Note that as $\pi_{e}$ increases, the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$ shifts down. It can be shown that $G(0)>$ $0, G^{\prime}>0$ and, over some range, $G^{\prime}>1$. Hence, $G$ may possibly have either a unique of multiple fixed points, depending on the value of $\theta$.

The graph of $G$ is illustrated in Figure 1. For $\theta$ either small enough or large enough, the unique equilibrium is unique. But for intermediate value of $\theta$, there are multiple equilibria. In particular, for intermediate $\theta$, there are three equilibria, represented by points $A, B$, and $C$ in the figure. In point $C$, the peg is always abandoned. In point $A$, the peg is abandoned only for very extreme shocks. The equilibrium in $C$ thus represents an "expectations trap". On the other hand, only the "bad" equilibrium $(C)$ survives for $\theta$ small enough, whereas only the "good" equilibrium $\left(A^{\prime}\right)$ survives for $\theta$ high enough.

Discuss the role of coordination and the role of commitment.
Discuss Albanesi, Chari and Christiano (2000).
Discuss Fisher (19??).


Figure 1:

