## 14.462 Lecture Notes Commitment, Coordination, and Expectation Traps

George-Marios Angeletos MIT Department of Economics

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## 1 Kydland Prescott/Barro Gordon

A large number of private agents play against a government.

The government solves

$$\min L = \left\{ (y - y^*)^2 + \beta \pi^2 \right\}$$

subject to the Philips curve

$$y = \overline{y} + \alpha(\pi - \pi_e) - \varepsilon.$$

The shock  $\varepsilon$  is distributed uniform over [-e, +e]

The agents solve

$$\min \mathbb{E} \left( \pi_e - \pi \right)^2$$

The agents move first, setting  $\pi^e$  without observing  $\varepsilon$ . Nature then draws  $\varepsilon$ . The government moves last, setting  $\pi$  after observing  $\varepsilon$  and taking  $\pi^e$  as given.

The best response of the government is given by

$$\pi = g(\pi_e, \varepsilon) = \frac{\alpha \left(y^* - \overline{y} + \varepsilon\right) + \alpha^2 \pi_e}{\alpha^2 + \beta}.$$

Note that g(0,0) > 0 and  $g_{\pi}(\pi,\varepsilon) = \frac{\alpha^2}{\alpha^2 + \beta} \in (0,1)$ . The best response for the agents is

$$\pi_e = \mathbb{E}\pi$$

Hence, in equilibrium,

$$\pi_e = \mathbb{E}g(\pi_e, \varepsilon) = \frac{\alpha \left(y^* - \overline{y}\right) + \alpha^2 \pi_e}{\alpha^2 + \beta} \equiv G(\pi_e).$$

By the properties of g, we have G(0) and  $G' \in (0, 1)$ . Hence, there is a unique fixed point with  $\pi_e > 0$ . Indeed, this is given by

$$\pi_e = \frac{\alpha \left(y^* - \overline{y}\right)}{\beta}.$$

It follows that equilibrium inflation is

$$\pi = \pi_e + \frac{\alpha}{\alpha^2 + \beta}\varepsilon$$

and equilibrium output is

$$y = \overline{y} + \frac{\alpha^2}{\alpha^2 + \beta}\varepsilon.$$

## 2 Obstfeld (1994)

We now reinterpret  $\pi$  as the rate of devaluation. We also modify the preferences of the government so that the government solves

$$\min L = \left\{ (y - y^*)^2 + \beta \pi^2 + \theta R \right\}$$

subject to the Philips curve, where R is an indicator that takes the value 1 if  $\pi \neq 0$ and 0 if  $\pi = 0$ . The variable  $\theta$  represents the value of maintaining the peg. The timing is the same. The government chooses  $\pi$  after observing  $\varepsilon$  and after agents have set  $\pi_e$ .

If the government sets  $\pi = g(\pi_e, \varepsilon)$ , then welfare losses are given by

$$L = L_{flex}(\pi_e, \varepsilon) = \frac{\beta}{\alpha^2 + \beta} \left( y^* - \overline{y} + \varepsilon + \alpha \pi^e \right)^2 + \theta.$$

If instead the government sets  $\pi = 0$ , then welfare losses are given by

$$L = L_{fixed}(\pi_e, \varepsilon) = (y^* - \overline{y} + \varepsilon + \alpha \pi^e)^2$$

Define  $\underline{\varepsilon} = \underline{\varepsilon}(\pi_e)$  and  $\overline{\varepsilon} = \overline{\varepsilon}(\pi_e)$  as the lowest and highest solution to

$$L_{flex}(\pi_e,\varepsilon) = L_{fixed}(\pi_e,\varepsilon)$$

Whenever  $\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]$ , the government finds it optimal to set  $\pi = 0$  (fixed). Whenever  $\varepsilon \notin [\underline{\varepsilon}, \overline{\varepsilon}]$ , the government prefers to set  $\pi = g(\pi_e)$  (flexibility).

Now consider the equilibrium. In equilibrium,

$$\pi^{e} = \mathbb{E}\pi = 0 \cdot \Pr\left(\varepsilon \in [\underline{\varepsilon}, \overline{\varepsilon}]\right) + \mathbb{E}g(\pi_{e}, \varepsilon) \cdot \Pr\left(\varepsilon \notin [\underline{\varepsilon}, \overline{\varepsilon}]\right) \equiv G(\pi_{e})$$

Note that as  $\pi_e$  increases, the interval  $[\underline{\varepsilon}, \overline{\varepsilon}]$  shifts down. It can be shown that G(0) > 0, G' > 0 and, over some range, G' > 1. Hence, G may possibly have either a unique of multiple fixed points, depending on the value of  $\theta$ .

The graph of G is illustrated in Figure 1. For  $\theta$  either small enough or large enough, the unique equilibrium is unique. But for intermediate value of  $\theta$ , there are multiple equilibria. In particular, for intermediate  $\theta$ , there are three equilibria, represented by points A, B, and C in the figure. In point C, the peg is always abandoned. In point A, the peg is abandoned only for very extreme shocks. The equilibrium in C thus represents an "expectations trap". On the other hand, only the "bad" equilibrium (C) survives for  $\theta$  small enough, whereas only the "good" equilibrium (A') survives for  $\theta$  high enough.

Discuss the role of coordination and the role of commitment.

Discuss Albanesi, Chari and Christiano (2000).

Discuss Fisher (19??).



Figure 1: