## 1. Unemployment

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## A framework to think about unemployment

From the facts:

- Large job creation/destruction. Flows of workers
- Decentralized market. At any point of time, workers looking for jobs, jobs looking for workers.
- Bargaining power. Depends on technology, state of the market.

So: Matching/bargaining/flow models. Built on search and bargaining theory. Mortensen. Pissarides. Diamond. So DMP. What they leave out:

- Internal wage structure. Firms as sets of jobs and workers.
- Collective bargaining
- Many interesting questions at these two connections.

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1. The matching function

- Treating the matching process as a black box. The matching function (parallel to production function): $u$ unemployment, $v$ vacancies, $h$ hires.

$$
h=m(u, v) \quad m_{u}>0, m_{v}>0, m_{v u}>0
$$

- Constant returns? Could think of congestion, or increasing returns. Evidence suggests roughly CRS. So can write:

$$
\frac{h}{u}=m\left(1, \frac{v}{u}\right), \quad \frac{h}{v}=m\left(\frac{u}{v}, 1\right)
$$

$h / u$ : exit rate from unemployment. (in steady state, inverse of average duration of unemployment). same for vacancies.

- Convenient to define $\theta \equiv v / u$, and the function $q$ :

$$
\frac{h}{v}=q(\theta), q^{\prime}<0 \text { so } \frac{h}{u}=\frac{h}{v} \frac{v}{u}=\theta q(\theta)
$$

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- $\theta$ index of labor market tightness. The tighter the market, the lower the rate at which vacancies are filled, the higher the exit rate from unemployment.
- Matching function and the data. Measurement issues:

Equivalent of $u$ : From previous lecture, many searchers in employment and not in labor force. Distinguishing between short and long-term unemployed because of different search intensity?
Equivalent of $v$. For the US, help wanted index, with many problems at medium/low frequencies. Since 2001, index from JOLTS (BLS, establishment survey).
Plot $h / u$ versus $u / v$ for the US (from Shimer's notes).

- A good, and convenient, approximation:

$$
h=m \sqrt{u v}, \text { so } h / u=m \sqrt{\frac{v}{u}}, h / v=m \sqrt{\frac{u}{v}}
$$

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Reduced-Form Matching Function. V-u ratio and Job finding probability versus year. p. 37. Shimer, R. Lecture notes for a course on "Labor Markets and Macroeconomics."

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Reduced-Form Matching Function. p. 38.
Shimer, R. Lecture notes for a course on "Labor Markets and Macroeconomics."
2. First pass at comparative statics and dynamics

Dynamics of unemployment. Normalize labor force to 1 , so:

$$
\dot{u}=s(1-u)-m(u, v)
$$

- $s$ constant separation rate. Counterfactual, will relax and endogenize later.
- All movements between employment and unemployment. Counterfactual also.

Dynamics of vacancies? Assume:

$$
\dot{v}=f\left(\frac{v}{u}, x\right), f_{v / u}<0, f_{x}>0
$$

where $x$ shifts underlying labor demand (profitability).
Behind the equation: The tighter the labor market, the higher the wage, the lower profit, and the lower the rate of job creation. Later on: Look at wage setting, and job creation.

[^0]In $u, v$ space (Beveridge curve space):

$$
\dot{u}=0 \Rightarrow s(1-u)=m(u, v)
$$

Downward sloping. Higher unemployment increases hires and decreases separations. To maintain equilibrium, vacancies must decrease.
(Another way: To maintain a given flow of hires, need either high $u$. low $v$, or low $u$ and high v.)

$$
\dot{v}=0 \Rightarrow f\left(\frac{v}{u}, x\right)=0
$$

Ray from the origin. There is a unique ratio of vacancies to unemployment such that the profit of firms is consistent with zero vacancy creation.

Equilibrium is stable. Counterclockwise movements.

## The Beveridge space



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Comparative statics.

- Increase in profitability $x: \quad \dot{v}=0$ locus rotates up. Lower unemployment. Higher vacancies. Roughly constant flows. Higher exit rate.
- Improvement in matching (shift of the matching function inwards). Shift inwards of $\dot{u}=0$ locus. Lower unemployment, and lower vacancies. Roughly same flows. Higher exit rate.
- Decrease in separation rate, $s$. Shift inwards of $\dot{u}=0$ locus. Lower unemployment, lower vacancies. Lower flows. Unchanged exit rate.

Dynamics

- Shifts in profitability. Counterclockwise loops around downward sloping $\dot{u}=0$ locus
- Shifts in reallocation efficiency or intensity. Counterclockwise loops around ray $\dot{v}=0$.

Empirical evidence. The Beveridge curve, and what it tells us.

- Movements in vacancies and unemployment in the United States
- Movements along the $\dot{u}=0$ clearly dominate at high frequency. Interpretation?
- Roughly: Movements along: bargaining. Shifts: matching.
- Some shifts right and left at lower frequency. Measurement issues. More relevant in analyzing the increase in unemployment in Europe.
- Some evidence on European Beveridge curves (Nickell et al 2002). Shifts in $\dot{u}=0$, or flat $\dot{u}=0$ at high unemployment? Look at France, Germany, Netherlands.


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Figure 8. Beveridge Curve, 1952-88. p. 39.
Blanchard, O., and P. Diamond. "The Beveridge curve." Brookings Papers on Economic Activity 1989, no. 1 (1989): 1-60.

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## Image removed due to copyright restrictions.

## Vacancies p. 35.

Shimer, R. Lecture notes for a course on "Labor Markets and Macroeconomics."

## Image removed due to copyright restrictions.

Beveridge Curve Plots for Various Countries.

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Beveridge Curve Plots for Various Countries.

## 3. Wage determination

Let $U, E$ be the values associated with being currently unemployed, currently employed. These follow:

$$
\begin{gathered}
r U=b+\left(\frac{h}{u}\right)(E-U)+\dot{U} \\
r E=w+s(U-E)+\dot{E}
\end{gathered}
$$

Note in particular:

- Workers have linear preferences. Incorporate more standard preferences later.
- $b$ : wage equivalent of being unemployed, including unemployment benefits, leisure.
- Could distinguish between $w_{i}$ and $w\left(E_{i}\right.$ and $\left.E\right)$. No need to, as we look at symmetric equilibrium.

Let $V, J$ be the values associated with being currently vacant, currently filled. These follow:

$$
\begin{aligned}
r V & =-c+\left(\frac{h}{v}\right)(J-V)+\dot{V} \\
r J & =y-w+s(V-J)+\dot{J}
\end{aligned}
$$

- Firm owners have linear preferences
- $c$ is the flow cost of posting a vacancy. Likely to be small. Return to this when discussing free entry condition later.
- Separations come from matches, not jobs, becoming unproductive. If, instead, it were jobs, then the last two equations would become:

$$
\begin{gathered}
r V=c-s V+\left(\frac{h}{v}\right) J+\dot{V} \\
r J=y-w-s J+\dot{J}
\end{gathered}
$$

The wage band

Wage that makes workers indifferent to working or not:

$$
E=U \Rightarrow w=b
$$

Wage that makes firms indifferent to hiring or not:

$$
J=V \Rightarrow w=y
$$

So, within this wage band, both sides will be willing to produce.

Wage could stay constant within the band. Reasonable to think that bargaining will determine the outcome. Convenient (but not more than that) assumption: Nash bargaining.

$$
S \equiv(J-V+E-U) \quad J-V=(1-\beta) S,(E-U)=\beta S
$$

Various ways of characterizing the wage (with additional assumptions on entry, can be made simpler. More below). Using the equations above:

$$
\begin{gathered}
\left(r+s+\frac{h}{u}\right)(E-U)=w-b+(\dot{E}-\dot{U}) \\
\left(r+s+\frac{h}{v}\right)(J-V)=y-w+c+(\dot{J}-\dot{V})
\end{gathered}
$$

Or:

$$
\begin{gather*}
\left(r+s+\frac{h}{u}\right) \beta S=w-b+\beta \dot{S}  \tag{1}\\
\left(r+s+\frac{h}{v}\right)(1-\beta) S=y-w+c+(1-\beta) \dot{S} \tag{2}
\end{gather*}
$$

Eliminating the surplus between the two equations:

$$
\left[r+s+(1-\beta) \frac{h}{v}+\beta \frac{h}{u}\right] w=(1-\beta)\left(r+s+\frac{h}{v}\right) b+\beta\left(r+s+\frac{h}{u}\right)(y+c)+\beta(1-\beta)\left(\frac{h}{u}-\frac{h}{v}\right) \dot{S}
$$

where

$$
\left[r+s+\frac{h}{v}\right] S=2(y-w+c)+\dot{S}
$$

- Ignore the last term in $\dot{S}$ in the wage equation. The wage is a weighted average of $b$ and $y$, with weights which reflect:
- The bargaining power of each side. As $\beta \rightarrow 0, w \rightarrow b$. As $\beta \rightarrow 1$, $w \rightarrow y+c$.
- The state of labor market. As the market is more depressed, as $h / u \rightarrow 0$ and $h / v \rightarrow \infty, w \rightarrow b$. As the market is tighter, $w \rightarrow y+c$.


## Shortcuts and useful approximations

- Depends on the future with ambiguous sign, depending on $h / u$ versus $h / v$.
- $h / u$ and $h / v$ much larger than $r$ and $s$. So

$$
w \approx\left(\frac{(1-\beta) u}{(1-\beta) u+\beta v}\right) b+\left(\frac{\beta v}{(1-\beta) u+\beta v}\right) y
$$

- If symmetric Nash bargaining, so $\beta=1 / 2$, and we can rewrite:

$$
w \approx\left(\frac{1}{1+\theta}\right) b+\left(\frac{\theta}{1+\theta}\right) y
$$

- All these useful for intuition and back of the enveloppe.

Some alternative approaches to wage setting

- Posted wages. Directed search. Trade-off between wage and probability of getting a job.
Very different efficiency properties. No bargaining after meeting.
Why do we see some firms post wages, and others not? (Look at announcements in newspapers.
- Efficiency wages. $E-U \geq B$. Role of $h / u$. Often easier. (ShapiroStiglitz)

4. Job creation and equilibrium

- Can think of the cost of creating a vacancy as the cost of creating a job. Buying a machine, cost of which is then sunk. So $V=p_{K}$

Then, cost of adjustment from installation costs, or production of new machines.

- Tradition in this class of models: Capital is rented, and needs to be installed only when worker is recruited. In which case $y$ is output net of user cost. Then $V=0$, and only cost is flow cost of posting, $c$.
- Second route much less convincing (size of $c$ ? )

Avoids important hold up issues. (Caballero book). Implausible short run dynamics ( $v$ can adjust freely).

- Shall—reluctantly—follow tradition here, for ease of comparison.

Solving for the equilibrium: MC of creating vacancy equals MB

- Free entry ties down the value of a filled job.

$$
V=0 \Rightarrow \dot{V}=0 \Rightarrow J=\frac{c}{q(\theta)}
$$

Interpretation. Now solve for the wage and implied $J$ :

- Surplus of a match.

$$
J-V=(1-\beta) S \Rightarrow S=\frac{c}{(1-\beta)} q(\theta)
$$

- Replace in the wage equations from Nash bargaining, (1) and (2):

$$
\begin{gather*}
(r+s+\theta q(\theta)) \frac{\beta}{1-\beta} \frac{c}{q(\theta)}=w-b+\beta \dot{S}  \tag{3}\\
(r+s+q(\theta)) \frac{c}{q(\theta)}=y-w+c+(1-\beta) \dot{S} \tag{4}
\end{gather*}
$$

Eliminating $\dot{S}$ between the two equations gives the wage as a function of labor market tightness (note time derivatives are gone)

$$
w=(1-\beta) b+\beta y+\beta \theta c
$$

Note the simplicity of the relation. The tighter the market, the higher the wage. (note also that the effect of tightness is additive, not through the relative weights on $b$ and $y$

Next step is to determine labor market tightness. From $V=0$, and the equation of $J$ :

$$
(r+s) J=y-w+\dot{J}
$$

Using $J=c / q(\theta)\left(\right.$ which implies $\left.\dot{J}=-c q^{\prime}(\theta) /\left(q(\theta)^{2} \dot{\theta}\right)\right)$ :

$$
(r+s) \frac{c}{q(\theta)}=(1-\beta)(y-b)-\beta \theta c-c \frac{q^{\prime}(\theta)}{q(\theta)^{2}} \dot{\theta}
$$

Interpretation.
This is an unstable differential equation in $\theta$. So for ct parameters and exogenous variables, equilibrium labor market tightness $\theta$ is given by:

$$
\frac{r+s}{q(\theta)}+\beta \theta=(1-\beta) \frac{y-b}{c}
$$

Finally, unemployment is given by:

$$
\dot{u}=(1-u) s-u \theta q(\theta)
$$

For the (constant) equilibrium value of $\theta$, unemployment (and by implication, vacancies) adjusts over time to:

$$
u=\frac{s}{s+\theta q(\theta)}
$$

$$
\frac{r+s}{q(\theta)}+\beta \theta=(1-\beta) \frac{y-b}{c}
$$

Go back to the comparative exercises we went through earlier:

- Increase in productivity $y$ : tighter labor market, lower u duration, lower unemployment.
- Increase in bargaining power of workers, $\beta$ : looser labor market, higher u duration, higher unemployment.
- Better matching $q($.$) : tighter labor market, lower u duration, lower$ unemployment.
- Lower separations $s$ : lower $J$, higher u duration. Effect on unemployment ambiguous.


## 5. Efficiency?

Is the decentralized equilibrium efficient? To answer the question, write the corresponding central planning problem:

$$
\max _{\left\{v_{t}\right\}} \int_{0}^{\infty}\left[\left(1-u_{t}\right) y+u_{t} b-c v_{t}\right] e^{-r t} d t
$$

subject to:

$$
\dot{u_{t}}=s\left(1-u_{t}\right)-m\left(u_{t}, v_{t}\right)
$$

(This assumes that $b$ stands for production by the unemployed, rather than unemployment benefits paid by the state, and financed by taxes. No justification for unemployment benefits within the model, as preferences are linear.) Rewrite in terms of $\theta$ :

$$
\max _{\left\{\theta_{t}\right\}} \int_{0}^{\infty}\left[\left(1-u_{t}\right) y+u_{t} b-c \theta_{t} u_{t}\right] e^{-r t} d t
$$

subject to:

$$
\dot{u_{t}}=s\left(1-u_{t}\right)-u_{t} \theta_{t} q\left(\theta_{t}\right)
$$

Use the maximum principle to solve. The Hamiltonian is given by:

$$
H_{t}=\left(1-u_{t}\right) y+u_{t} b-c \theta_{t} u_{t}+\mu_{t}\left(s\left(1-u_{t}\right)-u_{t} \theta_{t} q\left(\theta_{t}\right)\right)
$$

So:

$$
\begin{gathered}
H_{\theta}=0 \Rightarrow c=-\mu_{t}\left(\theta_{t} q^{\prime}\left(\theta_{t}\right)+q\left(\theta_{t}\right)\right) \\
\dot{\mu}-r \mu=-H_{u} \Rightarrow \dot{\mu}_{t}=\left(r+s-\theta_{t}^{2} q^{\prime}\left(\theta_{t}\right)\right) \mu_{t}+(y-b)
\end{gathered}
$$

The two equations give a differential equation for $\mu$, which is unstable. Thus, the solution is given by $\dot{\mu}=0$, which implies;

$$
\frac{r+s-\theta^{2} q^{\prime}(\theta)}{q(\theta)+\theta q^{\prime}(\theta)}=\frac{y-b}{c}
$$

Compare it to the characterization of the decentralized equilibrium:

$$
\frac{r+s}{q(\theta)}+\beta \theta=(1-\beta) \frac{y-b}{c}
$$

The two are the same iff $\beta=-\theta q^{\prime}(\theta) / q(\theta)$. Known as the Mortensen/Hosios condition.

- Interpretation. Share going to labor has to be equal to its contribution to matching, measured by the elasticity of $h / v$ with respect to $u / v$
- In the Cobb Douglas case , $h=m u^{\alpha} v^{1-\alpha}$. Then the condition is $\beta=\alpha$.
- No particular reason for this condition to be satisfied. $\alpha$ around 0.5 , most estimates of $\beta$ around or below 0.2 . So too low a level of unemployment.

An application: Hartz reforms and the matching function. (Fahr Sunde)

- Hartz reforms in Germany. Focus on better matching. Better job by employment agency. More incentives for the unemployed to take jobs.
- Hartz I and II: January 1, 2003. Hartz III: January 1, 2004: reorganization of agency. Hartz IV: January 1, 2005: new rules for unemployment assistance. (Last outside sample)
- Diff in diff. Estimate a matching function for 40 occupational groups, using monthly data from 2000 to 2004.
- Hartz reform dummies, alone and interacted with group dummy, reflecting role of employment agency for that group. The less involved, the smaller the predicted effect of the reforms.
$\log h_{i t}=a+D_{m}+D_{i}+Z_{t} \beta+a_{U} U_{i t}+a_{V} V_{i t}+a_{u} u_{i t}+a_{v} v_{i t}+a_{h} H_{t}+\sum_{j} a_{j} H_{t} D_{j}$
where
- $i$ is the occupational group, $t$ is the year.
- $D_{m}, D_{i}$ is a set of month and occupational dummies, $Z_{t}$ are exogenous variables (business cycle measures)
- $U, V$ stock of unemployed and vacancies, $u, v$, newly unemployed and new vacancies,
- $H_{t}$ is a Hartz dummy, 1 after reform. Separate samples for Hartz I and II, and Hartz III. (?)
- $D_{j}$ is a set of dummies for one of four occupational groups, by decreasing order of use of employment agency.

Results.

Tables 1 and 5.

- Matching function itself. A bit strange.

Zero coefficient on $V$. (Plausible? Measurement of vacancies?)
Negative coefficient on $u$ ? Coefficient on $U$ in the second sample?

- Interaction effects look plausible. Bigger effect on group 1 than on group 2 and so on.

Not a triumph: serious data issues. But an indication of how we can get at some of the parameters of the basic model.

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Table 1. The Effects of Hartz I/II Reforms on the Speed of Matching by Broad Occupations. p. 18.
Fahr, R., and U. Sunde. "Did the Hartz Reforms Speed Up Job Creation? A Macro-evaluation Using Empirical Matching Functions." IZA Discussion Paper No. 2470, November 2006. pp. 1-34.

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Table 1. The Effects of Hartz III Reforms on the Speed of Matching by Broad Occupations. p. 25.

Fahr, R., and U. Sunde. "Did the Hartz Reforms Speed Up Job Creation? A Macro-evaluation Using Empirical Matching Functions." IZA Discussion Paper No. 2470, November 2006. pp. 1-34.
6. Tentative conclusions

- A very useful framework. Flows. Matching. Bargaining. Creation/detruction.
- Diagnostic tools: Matching function, Beveridge curve, wage curve?

Problem: Wages largely a-cyclical. Nominal rigidities. So Phillips curve as tool. But specific form?

- The set of factors affecting unemployment.

Some factors affect the Beveridge curve, some affect the matching function, some affect the wage curve.

Unemployment benefits. Through search, affect matching and Beveridge curve. Through reservation wage, affect wage.

- Can relate to and use parameters from micro studies (with care).

Can/must be extended in many dimensions:

- Endogenous destruction. Link to reallocation. Cycles and reallocation. Growth and reallocation.
- Alternative wage setting. Firm as collection of workers. Internal labor market. Efficiency wages.
- Asymmetric information and bargaining.
- Concave preferences, risk aversion. Saving, unemployment insurance.
- Nominal rigidities. Pressure on wages, or pressure on inflation. Integration in standard BC models.


[^0]:    Nr. 7

