1.3 Nominal rigidities

- two period economy
- households of consumers-producers
- monopolistic competition, price-setting
- uncertainty about productivity

- preferences

$$\sum_{t=1}^{2} \beta^t \left(\log C_{it} - \frac{\kappa}{1+\eta} N_{it}^{1+\eta} \right),$$

C_{it} is the CES aggregate

$$C_{it} = \left(\int_0^1 C_{ijt}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}},$$

with $\sigma > 1$

- Technology

$$Y_{it} = A_t N_{it}.$$

- productivity shocks A_t

$$A_t = e^{a_t}$$

$$\begin{aligned} a_1 &= x + \epsilon_1, \\ a_2 &= x + \epsilon_2 \end{aligned}$$

- x and ϵ_t mean-zero, i.i.d., normal

- A signal about long-run productivity

$$s = x + e$$

- nominal balances with central bank at nominal rate R

- household set P_{it} then consumers buy

- intertemporal BC

$$(P_2 C_{i2} - P_{i2} Y_{i2}) + R \cdot (P_1 C_{i1} - P_{i1} Y_{i1}) \leq 0,$$

- P_t is the price index

$$P_t = \left(\int P_{it}^{1-\sigma} di \right)^{\frac{1}{1-\sigma}}.$$

Flexible price equilibrium

- period 2. Optimality for price-setting,

$$(1-\sigma)\frac{1}{P_t C_{it}}\frac{P_{it}Y_{it}}{P_{it}} + \kappa\sigma\frac{1}{A_t}\frac{Y_{it}}{P_{it}}N_{it}^{\eta} = 0.$$

- symmetric equilibrium, $Y_t = A_t N_t$, this condition gives

$$N_t = \left(\frac{\sigma - 1}{\kappa\sigma}\right)^{\frac{1}{1+\eta}} = 1$$

(normalization of κ).

- quantities

$$C_t = Y_t = A_t.$$

- what about consumers' decisions?

- consumer Euler equation

$$\frac{1}{C_1} = RE\left[\frac{P_1}{P_2}\frac{1}{C_2}|a_1, s\right]$$

- $C_t = A_t$ log-normal

$$r + p_1 - E\left[p_2|a_1, s\right] = E\left[a_2|a_1, s\right] - a_1 - \frac{1}{2}Var\left[a_2|a_1, s\right].$$

- all changes in $E\left[y_2\right]$ go to the real interest rate

- notice role of p_1: neutralizes r

Fixed prices in period 1

- price-setting before any shock observed

$$E\left[(1-\sigma)\frac{1}{P_1 C_{i1}}\frac{P_{i1}Y_{i1}}{P_{i1}}+\kappa\sigma\frac{1}{A_2}N_{i1}^{\eta}\frac{Y_{i1}}{P_{i1}}\right]=0.$$

- rearranging this gives

$$E\left[N_1^{1+\eta}\right]=1$$

- this will pin down averages but not responses to shocks

- quantities: equilibrium in period 2 identical

- in period 1 now Euler equation (set $p_2 = 0$)

$$c_1 = E\left[a_2|a_1, s\right] - \frac{1}{2}Var\left[a_2|a_1, s\right] - r - p_1.$$

- suppose r fixed, p_1 fixed by assumption

- now “sentiment shocks” affect consumption

Figure 8: RBC and Simple Monetary Model
Expectation of Technology Shock in Period 13 Not Realized

Image removed due to copyright restrictions.

- pin down p_1

$$E\left[N_1^{1+\eta}\right] = E\left[e^{(1+\eta)(y_1 - a_1)}\right] = 1,$$

- thanks to log-normality this equation can be solved explicitly and gives

$$-r - p_1 - \frac{1}{2}Var\left[a_2|a_1, s\right] + \frac{1}{2}(1+\eta)(\beta+\delta-1)^2\sigma_x^2 +$$
$$+\frac{1}{2}(1+\eta)(\beta-1)^2\sigma_\epsilon^2 + \frac{1}{2}(1+\eta)\delta^2\sigma_e^2 = 0$$

- where

$$E\left[a_2|a_1, s\right] = \beta a_1 + \delta s$$

- simple implication anticipated changes in r are neutral

- if instead we follow rule, e.g.

$$r = \alpha_0 + \alpha_1 y_1$$

  then economy response changes

- we'll go back to monetary policy

What about demand shocks?

- here price response is absent

- need a bit more flexibility

  – sticky prices

  – imperfect information

1.4 Lucas-Phelps islands

- Lucas 1972

- Overlapping generations

- Agents work at date t consume at date $t+1$

- Preferences

$$E\left[C_{i,t+1} - \frac{1}{2}N_{i,t}^2\right]$$

- money

x_t proportional subsidy from gov't

agents work, accumulate money, spend, die

$$\begin{aligned} Y_{i,t} &= N_{i,t} \\ M_{i,t+1} &= P_{i,t} Y_{i,t} (1 + x_{t+1}) \\ P_{j,t+1} C_{i,t+1} &= M_{i,t+1} \end{aligned}$$

at date $t + 1$ agent i consumes the output of agent j

- continuum of islands, $i \in [0, 1]$

- unit mass of agents on each

- old agents receive proportional transfer x_t from govt'

- they travel to one island where they spend all their money

- prices $P_{i,t}$ determined in walrasian equilibrium

- young agents decide their labor supply only observe $P_{i,t}$

- old agents in island i are representative sample

- but different mass $\phi_{i,t}$

- nominal demand demand in island i is

$$\phi_{i,t} \int_0^1 M_{i,t} di = \phi_{i,t} M_t$$

- $\phi_{i,t}$ log-normal with

$$\int_0^1 \phi_{i,t} di = 1$$

- Idiosyncratic demand shock

$$\log \phi_{i,t} = u_{i,t}$$

- Monetary shocks log-normal

$$\epsilon_t = \log(1 + x_t)$$

- total nominal demand is

$$D_{i,t} = \phi_{i,t}(1 + x_t) M_{t-1}$$

in logs

$$d_{i,t} = \epsilon_t + u_{i,t} + m_{t-1}$$

Market clearing

$$P_{i,t} N_{i,t} = \phi_{i,t} \left(1 + x_t\right) M_{t-1}$$

Information structure

- all agents observe $\{M_{t-1}, M_{t-2}, ...\}$
- old agents observe $x_t, P_{j,t}$ (j is the good they buy)
- young agents observe $P_{i,t}$

observing $P_{i,t}$ and M_{t-1}, and knowing their own $N_{i,t}$ young agents can infer

$$\phi_{i,t}(1 + x_t)$$

Labor supply

Agents solve

$$\max_{N_{i,t},C_{i,t+1}} \quad E\left[C_{i,t+1} - \frac{1}{2}N_{i,t}^2 | P_{i,t}, M_{t-1}\right]$$
$$s.t. \quad P_{j,t+1}C_{i,t+1} = P_{i,t}N_{i,t}\left(1 + x_{t+1}\right)$$

Substitute $C_{i,t+1}$ and obtain

FOC

$$E\left[\frac{P_{i,t}}{P_{j,t+1}}\left(1 + x_{t+1}\right) - N_{i,t} | P_{i,t}, M_{t-1}\right] = 0$$

interpetation

$$\underbrace{N_{i,t}}_{\text{labor supply}} = \quad E[\underbrace{\frac{P_{i,t}}{P_{j,t+1}}}_{\text{exp.infl.}} (1 + x_{t+1}) \,|P_{i,t}, M_{t-1}]$$

Equilibrium prices

guess:

$$P_{i,t} = g\left(\phi_{i,t}\left(1 + x_t\right)\right) M_{t-1}$$

Because $\phi_{i,t}$ and x_t are i.i.d. the distribution of $\phi_{j,t+1}\left(1 + x_{t+1}\right)$ is given at date t.

Decompose

$$\begin{aligned} N_{i,t} &= E[\frac{P_{i,t}}{P_{j,t+1}}\left(1 + x_{t+1}\right)|P_{i,t}, M_{t-1}] = \\ &= E_{i,t}\left[\frac{P_{i,t}}{M_t}\right] E_{i,t}\left[\frac{M_t}{P_{j,t+1}}\left(1 + x_{t+1}\right)\right] \end{aligned}$$

then

$$E\left[\frac{1+x_{t+1}}{g\left(\phi_{j,t+1}\left(1+x_{t+1}\right)\right)}|\phi_{j,t+1}\left(1+x_{t+1}\right)\right]=\xi$$

is a constant independent of today's shocks

$$N_{i,t}=\xi E_{i,t}\left[\frac{P_{i,t}}{M_{t-1}}\frac{1}{1+x_t}\right]$$

From equilibrium condition we obtain

$$\phi_{i,t}\frac{M_t}{P_{i,t}} = N_{i,t} = \xi E_{i,t}\left[\frac{P_{i,t}}{M_t}\right]$$

in logs,

$$m_t - p_{i,t} + u_{i,t} = (...) - E_{i,t}\left[m_t - p_{i,t}\right]$$

(constant terms in (...), depend on variances)

We obtain

$$p_{i,t} = \bar{p} + \frac{1}{2}\left(m_t + u_{it}\right) + \frac{1}{2}E_{i,t}\left[m_t\right]$$

Agents observe

$$m_t + u_{i,t} = m_{t-1} + \epsilon_t + u_{i,t}$$

Define

$$\overline{E}_t [m_t] = \int_0^1 E \left[m_t | m_{t-1}, \epsilon_t + u_{i,t} \right] di$$

Then, averaging, we have

$$p_t = \bar{p} + \frac{1}{2} m_t + \frac{1}{2} \overline{E}_t [m_t]$$

Imperfect information

$$\overline{E}_t [m_t] \neq m_t$$

in particular

$$E\left[m_t | m_{t-1}, \epsilon_t + u_{i,t}\right] = m_{t-1} + \beta (\epsilon_t + u_{it})$$

where

$$\beta = \frac{\sigma_m^2}{\sigma_m^2 + \sigma_u^2}$$

so

$$\overline{E}_t [m_t] = m_{t-1} + \beta \epsilon_t \neq m_{t-1} + \epsilon_t$$

We have

$$p_t = \bar{p} + m_{t-1} + \frac{1}{2}(1+\beta)\,\epsilon_t$$

and output is

$$\begin{aligned} y_t &= m_t - p_t \\ &= \bar{y} + \frac{1}{2}(1-\beta)\,\epsilon_t \end{aligned}$$

- larger $\frac{\sigma_m^2}{\sigma_u^2}$ implies smaller real effects of monetary policy

- Phillips curve depends on the monetary regime

Wrapping up

- with partially revealing prices
  - first order expect. $m_t \neq \overline{E}_t [m_t]$
- this can explain short-run non-neutrality
- prices adjust less than 1:1 with imp. info
- policy regime affects inference and thus effects of shocks