

Question 1

Assume that a consumer's utility function is given by:

$$u(x_1, x_2, x_3) = \beta_1 \log(x_1 - \alpha_1) + \beta_2 \log(x_2 - \alpha_2) + \beta_3 \log x_3$$

and that the consumer faces consumer prices q_1 and q_2 , with the price of good three normalized to unity. The consumer's endowment (y) is measured in units of good three.

- (a) Find the indirect utility function and expenditure function corresponding to this set of preferences. This is the famous Stone-Geary utility function, the basis for the linear expenditure system in demand analysis.
- (b) Use your results from (a) to find an analytic expression for the compensating variation. In the special case of $\alpha_1 = \alpha_2 = .50$ and $\beta_1 = \beta_2 = .40$, find the CV associated with a tax reform that changes (q_1, q_2, y) from $(1, 1, 5)$, a setting with no taxes, to $(2.0, 1.5, 5)$. Find the efficiency cost of the tax reform, defined as CV minus the compensated revenue associated with this reform.

Question 2

Assume that cross-sectional data on household demand for gasoline (g_i), with households located in different states and facing different gasoline tax burdens, suggests a demand curve of the form:

$$\ln g_i = -1.50 - 0.40 * \ln q_i + 0.80 \ln y_i$$

(0.20)	(0.07)	(0.20)
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where q_i denotes the tax-inclusive price of gasoline facing household i and y_i denotes household i 's income in dollars. Assume that the producer price of gasoline is \$3.00.

- (a) Do the parameter estimates make sense? What units might g_i be measured in?
- (b) For a household with income of \$50,000, find the Harberger triangle estimate of the deadweight loss from a specific tax of \$1.00/gallon of gasoline.
- (c) Using the indirect utility function and expenditure function corresponding to this demand curve, find the exact welfare loss (CV - Hicksian Revenue) associated with a \$1/gallon for a household with an income of \$50,000. How does this compare with the triangle estimate in (b)?
- (d) Without doing any calculations, describe briefly how you would compute the standard error of the estimate of exact welfare loss in (c). Besides the standard errors listed above, what other information would you need for this calculation?

Question 3

Consider a consumer endowed with an exogenous income y . If there is an initial price vector p^0 and a post-reform price vector p^1 , we know that both the compensating variation $CV(p^0, p^1, y)$ and the equivalent variation $EV(p^0, p^1, y)$ provide a meaningful welfare ranking of p^0 and p^1 . Suppose now, however, that the status quo p^0 is being compared with two possible price vectors p^1 and p^2 . For instance, the government considers which goods to tax, starting from a no tax situation.

- (a) Show that the consumer is better off under p^1 than under p^2 if and only if $EV(p^0, p^1, y) < EV(p^0, p^2, y)$. Thus, the measures $EV(p^0, p^1, y)$ and $EV(p^0, p^2, y)$ can be used not only to compare these two price vectors with p^0 , but also to determine which of them is better for the consumer.
- (b) By constructing an example, show that a comparison of $CV(p^0, p^1, y)$ and $CV(p^0, p^2, y)$ does not necessarily rank the consumer's welfare under p^1 and p^2 correctly. Interpret this result.
- (c) Assume a linear production technology so that the producer price vector p is fixed. In this setting, the resource cost of providing the consumer with a welfare level u_0 is equal to $e_0 = \sum_i p_i h_i(p, u_0)$, where $h_i(\cdot)$ denotes the consumer's compensated demand function for good i . If we distort consumer prices to a new price vector q (for instance by imposing a tax vector $\tau = q - p$), the resource cost necessary to provide the consumer with utility u_0 becomes $e_1 = \sum_i p_i h_i(q, u_0)$. The deadweight loss of moving from consumer prices p to consumer prices q is given by

$$e_1 - e_0 = CV(p, q, u_0) - \sum_i (q_i - p_i) h_i(q, u_0).$$

This simply states that the deadweight loss can be interpreted as the additional resource cost, relative to that provided when the price vector is p , that is needed to provide utility u_0 to the consumer under the distorted consumer price vector q . How would you extend the analysis to incorporate a population of consumers with heterogeneous preferences?

- (d) Continuing with the linear technology case, suppose that the government does not impose taxes, but instead imposes quantity restrictions, as in the quotas and rations imposed during war time. How would you compute the deadweight loss? How would heterogeneous consumers affect your analysis? Consider separately the cases where they can trade their quotas or rations and a situation where they cannot.

Question 4

Consider a world economy consisting of N identical countries, each endowed with one unit of land. The world contains one unit of capital, which is freely mobile between countries. Land, in contrast, is immobile. All countries have identical production technologies, $Y = K^{.25}L^{.75}$ where K and L denote capital and land, respectively. The output price is normalized to unity.

- (a) Find the equilibrium interest rate and total income received by capitalists and landowners when none of the jurisdictions tax either capital or land. Evaluate these quantities for $N = 2$ and $N = 100$. Note that the $N = 100$ economy has more total land than the $N=2$ economy.
- (b) Now consider the impact of a tax at rate θ on capital income in country 1. Assume that revenues are used to purchase good Y , and that the government's purchases do not affect the production technology in country 1. The after-tax rate of return to capital invested in country 1 is now $(1-\theta)F_K$. Find new expressions for the after-tax interest rate, total landowner and capitalist income, and government revenue in country 1 as functions of N and θ .
- (c) For $\theta = .25$, find the change in total capital and total land income, and the revenue raised in country 1, if $N = 2$ and $N = 100$. What happens to the pretax marginal product of capital in the countries without taxes? How do landowners in country 1 fare as a result of the tax? What do these examples suggest about the usefulness of the "small open economy" assumption that world interest rates are fixed, so capital taxes are shifted to land?

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