14.471: Fall 2012: PS4

Due on Friday December 7

November 25, 2012

1. Optimal Income Taxation - Numerical Exploration

This question asks you to solve the Mirrlees (1971) model numerically. Assume that an individual's utility is given by

$$\tilde{U}(c,l) = c - \frac{l^2}{2}$$

where $y = \theta l$ and θ is the skill level. Evaluate candidate allocations using the social welfare function $W(v) = \log(v)$.

(a) Find the skill distribution such that the distribution of income when individuals face a flat tax $T(y) = 0.3y$ is pareto with $h(y) = ky^{-k-1}\underline{y}^k$.

(b) Solve for the optimum numerically ignoring the monotonicity condition. Use $\underline{y} = 2$ and $k = 4$ and truncate your distribution at the top x percentile for some small x. Compare your results to Saez's.

2. Pareto-optimal income taxation

This problem asks you to evaluate the Pareto Efficiency of a tax schedule. Assume the income elasticitity of labor supply is zero. Let ϵ_w^* denote the compensated elasticity of labor supply with respect to the real wage. Let the distribution of income generated by the current tax system be Pareto

$$h(Y) = k(Y)^{-k-1}\underline{Y}^k \, for \, Y \geq \underline{Y} \, and \, k > 0$$

(a) Suppose there is a linear flat tax

$$T(Y) = T + \tau Y$$

with marginal tax rate τ and intercept T. Suppose ϵ_w^* does not vary across individuals (at all income levels). Note that this would be true if the utility function is $U(c,Y,\theta) \equiv c - \theta Y^\alpha$. Starting from the general test for Pareto efficiency derive an inequality for τ , ϵ_w^* and k . Consider some empirically plausible values.

(b) How would an elasticity ϵ_w^* that varies across individuals, and is higher for individuals with higher income affect your analysis? (Gruber-Saez paper may be useful to think of plausible numbers). How would progressivity of the tax schedule (convexity of $T(Y)$) affect the analysis?

3. Linear and Nonlinear Tax Implementation

Suppose we have preferences

$$u^0(c_0) + \beta\mathbb{E}\left[u^1(c_1, y_2, \theta_1)\right]$$

where $u^0(\cdot)$ and $u^1(\cdot,\cdot,\theta_1)$ are differentiable and concave functions for each θ_1. We start by making no assumptions on the distribution of shocks θ_1.

(a) Suppose the government has set a tax on labor $T(y_2)$ that is twice differentiable, strictly increasing and strictly convex, so that $T' > 0$ and $T'' > 0$. This tax schedule may or may not be optimal. The government has forbidden savings; in period 1 agents solve

$$\max_{y} \quad u^1(y - T(y), y, \theta_1).$$

Let $y^*(\theta_1)$ denote the solution to this problem and $c_1^*(\theta_1) \equiv y^*(\theta_1) - T(y^*(\theta_1))$ the associated consumption. The government also hands out some initial transfer, equal to initial consumption c_0.

The government wants advice on whether it can allow agents to choose their level of savings. Show that if the technological gross rate of return is R, it can allow agents to save at some distorted interest rate $R^\star$. That is, that a linear tax on savings can implement the allocation in which savings is forbidden. [Hint: set up the savings problem faced by agents and argue that it is convex.]

(b) Show that the same allocation can be achieved with positive savings if we change both the labor income tax schedule and the period-0 transfer by a constant.

(c) Assume now that θ_1 is continuously distributed on a bounded interval and u^1 is additively separable between c_1 and (y_1, θ_1). Suppose T is chosen to maximize

$$\mathbb{E}\left[u^1(c_1, y_2, \theta_1)\right]$$

as in the static Mirrlees model, for some given total amount of net resources dedicated to period $t = 1$. Set up this planning problem. Does the solution to this problem yield a convex tax schedule T?

(d) After solving the problem above for T the government has chosen c_0 to ensure that the Inverse Euler equation is satisfied:

$$\frac{1}{u_c^0(c_0)} = \frac{1}{\beta R}\mathbb{E}\left[\frac{1}{u_c^1(c_1^*(\theta_1), y^*(\theta_1), \theta_1)}\right]$$

(recall that utility u^1 is additively separable, although I have not incorporated that information in the Inverse Euler equation). Argue that the allocation constructed by this procedure is efficient, i.e. it solves a two-period planning problem.

(e) Now, again, the government wants advice on whether it can allow savings. Under what conditions can a linear tax on savings implement the same allocation with zero savings? If a linear tax does not work, is there a nonlinear tax on savings that will do the job? How does the answer depend on θ_1 having a continuous distribution?