A…Rational Model, no Social Security

Simplest case – no uncertainty, no utility discounting, zero interest rate, intertemporally additive preferences, no age variation in preferences except in additive disutility of work, work a zero-one decision.

Notation

V	Lifetime utility
$u[x_z]-a_z$	flow utility at age z if working, a nondecreasing
x_z	consumption when working
$v[c_z]$	flow utility at age z if not working
w_z	wage at age z, assumed to be nondecreasing, but more slowly than a
L	age at which early liquidity constraints stop
R	retirement age
T	age at death
E	early entitlement age for social security benefits
t,b	social security taxes and benefits
I	lump sum income – often set to zero

Without social security, the problem is:

$$
\begin{aligned}
V = \text{Maximize } & \int_0^R \{u[x_z]-a_z\}dz + \int_R^T \{v[c_z]\}dz \\
\text{s. t. } & \int_0^R \{x_z - w_z\}dz + \int_R^T \{c_z\}dz \le I \\
& \int_0^Z \{x_z - w_z\}dz \le I \quad \forall \mathrm{Z} \le \mathrm{R} \\
& \int_0^R \{x_z - w_z\}dz + \int_R^Z \{c_z\}dz \le I \quad \forall \mathrm{Z} \ge \mathrm{R}
\end{aligned}
\tag{1}
$$

<u>If the liquidity constraints don't bind</u>, then consumption is equalized while working and while retired, with marginal utilities equalized between the two. That is, the FOC with respect to x and c are:

$$
\begin{aligned}
u'[x_z] &= \lambda \\
v'[c_z] &= \lambda
\end{aligned}
\tag{2}
$$

The FOC with respect to the retirement age, assumed to have a unique solution:

$$\begin{aligned} u[x_R]-a_R-v[c_R] &= \lambda(x_R-w_R-c_R) \\ &= u'[x_R](x_R-w_R-c_R) = v'[c_R](x_R-w_R-c_R) \end{aligned} \tag{3}$$

Note that if $u[x]=v[x]$, using the FOC above, this becomes:

$$w_R u'[x_R] = a_R \tag{4}$$

<u>If liquidity constraints do bind when young but not when old</u>, i. e., for all $Z \le L \le R$, for the case I=0, we have

$$\begin{aligned} & x_z = w_z, 0 \le z \le L \\ & x_z = x, L \le z \le R, w_L = x_L \\ & c_z = c, R \le z \le T \\ & u'[x] = v'[c] \\ & u[x_R]-a_R-v[c_R] = u'[x_R](x_R-w_R-c_R) \\ & (R-L)x+(T-R)c = \int_L^R w_z dz \end{aligned} \tag{5}$$

B…Rational Model, Social Security with no earnings test

Now add a mandate to pay tw_z when working and to receive b when past the early entitlement age – there is no need to retire to start receiving benefits. Assume that the optimum involves work from 0 to R and retirement thereafter. Assume that the system is "actuarially fair" in the sense that the system is breakeven and seen to be breakeven:

$$t\int_0^R w_z dz = (T-E)b \tag{6}$$

With this condition, the PDV of consumption equals the PDV of gross-of-tax wages plus lump sum income.
We stay with the case I=0.

This can have no effect – the previous optimum may still be feasible. This can only happen if the early liquidity effects do not bind and the tax rate is not "too large." It also requires that the EEA not be "too late."

It can tighten early liquidity constraints and so raise the sum of private wealth plus social security wealth at retirement age, leading to earlier retirement by an income effect (if EEA is not too late). Then the decline in wealth at any age is less than the accumulated amount of taxes. Check that you understand why this is so.

And it can lead to later retirement (before or at the EEA) because of late liquidity effects. In the case of retirement before EEA, assuming early liquidity constraints do not bind, the optimum has

$$
\begin{aligned}
& x_z = x, 0 \le z \le R \\
& c_z = c', R \le z \le E, \\
& c_z = c, E \le z \le T \\
& u'[x] = v'[c'] \\
& u[x_R] - a_R - v[c'_R] = v'[c'_R]\left(x_R - (1-t)\, w_R - c'_R\right) + tv'[c_R]\, w_R \\
& Rx + (E-R)c' = (1-t)\int_0^R \{w_z\}\, dz \\
& (T-E)c = t\int_0^R \{w_z\}\, dz
\end{aligned}
\qquad (7)
$$

Since there is higher consumption after E than before, the decline in wealth at any age is less than the accumulated taxes net of benefits.

Thus social security can lead to no change or earlier or later retirement. If the taxes are saved to finance retirement benefits aggregate wealth at each age is unchanged or rises.