Handout on taxing savings

E. Saez, The Desirability of Commodity Taxation Under Non-Linear Income Taxation and Heterogeneous Tastes" Journal of Public Economics, 83, 2002, 217-230

Notation

$x_i$ consumption in period 1 of household i

$c_i$ consumption in period 2 of household i

$z_i$ earnings of household i

$n_i$ skill of household i

$\delta_i$ discount factor of household i

$U^i$ utility of household i - concave

$f_i$ number of workes of type i

$w$ wage per unit of skill, set equal to 1

$R$ 1 plus the return to capital

Utility

We assume a simple additive structure:

$$U^i\left[x, c, z/n_i\right] = u\left[x\right] + \delta_i u\left[c\right] - v\left[z/n_i\right] \tag{1}$$

Full nonlinear taxation (that is not just repeated annual income taxation):

For notational convenience, assume the real return on capital is zero.

$$\text{Maximize}_{x,c,z} \quad \sum f_i \left(u\left[x_i\right] + \delta_i u\left[c_i\right] - v\left[z_i/n_i\right]\right)$$

$$\text{subject to:} \quad E + \sum f_i \left(x_i + R^{-1}c_i - z_i\right) \leq 0$$

$$u\left[x_i\right] + \delta_i u\left[c_i\right] - v\left[z_i/n_i\right] \geq u\left[x_j\right] + \delta_i u\left[c_j\right] - v\left[z_j/n_i\right]$$

for all i and j

(2)

Assume two types. Assume the only binding moral hazard constraint is type 1 considering imitating type 2.

$$\text{Maximize}_{x,c,z} \quad f_1\left(u\left[x_1\right] + \delta_1 u\left[c_1\right] - v\left[z_1/n_1\right]\right) + f_2\left(u\left[x_2\right] + \delta_2 u\left[c_2\right] - v\left[z_2/n_2\right]\right)$$

$$\text{subject to:} \quad E + \sum f_i \left(x_i + R^{-1}c_i - z_i\right) \leq 0$$

$$u\left[x_1\right] + \delta_1 u\left[c_1\right] - v\left[z_1/n_1\right] \geq u\left[x_2\right] + \delta_1 u\left[c_2\right] - v\left[z_2/n_1\right]$$

(3)

FOC:

$$
\begin{aligned}
f_1 u'[x_1] - \lambda f_1 + \mu u'[x_1] &= 0 \qquad (4)\\
f_1 \delta_1 u'[c_1] - \lambda f_1 R^{-1} + \mu \delta_1 u'[c_1] &= 0 \qquad (5)\\
-f_1 v'[z_1/n_1]/n_1 + \lambda f_1 - \mu v'[z_1/n_1]/n_1 &= 0 \qquad (6)\\
f_2 u'[x_2] - \lambda f_2 - \mu u'[x_2] &= 0 \qquad (7)\\
f_2 \delta_2 u'[c_2] - \lambda f_2 R^{-1} - \mu \delta_1 u'[c_2] &= 0 \qquad (8)\\
-f_2 v'[z_2/n_2]/n_2 + \lambda f_2 + \mu v'[z_2/n_1]/n_1 &= 0 \qquad (9)
\end{aligned}
$$

First let us review the familiar result that there is no marginal taxation of earnings at the top of the earnings distribution. From the FOC for first-period earnings and consumption, we have:

$$(f_1 + \mu)\, u'[x_1] = \lambda f_1 = (f_1 + \mu)\, v'[z_1/n_1]/n_1 \qquad (10)$$

Similarly, from the FOC for first- and second-period consumption, we have:

$$(f_1 + \mu)\, u'[x_1] = \lambda f_1 = (f_1 + \mu)\, \delta_1 R u'[c_1] \qquad (11)$$

This implies no taxation of savings for type 1. This is the familiar no-taxation condition at the very top of the earnings distribution.

Now let us turn to type 2. First, the marginal taxation of work:

$$
\begin{aligned}
(f_2 - \mu) u'[x_2] &= \lambda f_2 = f_2 v'[z_2/n_2]/n_2 - \mu v'[z_2/n_1]/n_1 &\quad (12)\\
&= (f_2 - \mu) v'[z_2/n_2]/n_2 + \mu\left(v'[z_2/n_2]/n_2 - v'[z_2/n_1]/n_1\right)
\end{aligned}
$$

With $v$ convex and $n_1 > n_2$, we have $v'[z_2/n_2]/n_2 > v'[z_2/n_1]/n_1$. Thus we have $u'[x_2] > v'[z_2/n_2]/n_2$. This implies marginal taxation of earnings for type-2 workers. The intuition is that type-1 workers imitating type-2 workers find it easier to earn than do type-2 workers, so we tax that. It is similar to the analysis of the deviation from the Samuelson rule for public goods.

Turning to savings for type 2:

$$
\begin{aligned}
(f_2 - \mu) u'[x_2] &= \lambda f_2 = f_2 \delta_2 R u'[c_2] - \mu \delta_1 R u'[c_2] &\quad (13)\\
&= (f_2 - \mu) \delta_2 R u'[c_2] + \mu (\delta_2 - \delta_1) R u'[c_2] &\quad (14)
\end{aligned}
$$

The plausible case is that high earners value have a lower discount rate, resulting in a higher multiplicative factor on future consumption: implying $\delta_2 < \delta_1$. Therefore (with $f_2 - \mu > 0$) we have

$$u'[x_2] < \delta_2 R u'[c_2] \tag{15}$$

That is, type-2 would save if that were possible at zero taxation of savings, so there is marginal taxation of savings.

If and only if $\delta_2 = \delta_1$ does this imply no taxation of savings for type 2.

Saez considers linear taxation of savings. He concludes that since higher earners have higher savings rates, taxing savings is part of the optimum.