14.472
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Changing debt in the presence of interest income taxes
Inelastic labor supply

T lump sum tax on young
t interest income tax
g debt per young person

Individual budget constraint

1. $c_1 + c_2/\left(1 + r\left(1 - t\right)\right) = w - T$

First period consumption $c^*\left[w - T, r\left(1 - t\right)\right]$.
Assume normality: $0 < c^*_Y < 1$

Government budget balance

2. $$T = (r - n)\,g \quad -tr\,(k + g)$$
$$= (r\,(1 - t) - n)\,g - rtk$$

Market clearance

3. $(1 + n)\,k = w - T - c^*\left[w - T, r\left(1 - t\right)\right] - g\left(1 + n\right)$

Increase debt, **w, r given**. Note

4. $dT/dg = (r\,(1 - t) - n) - rt\frac{dk}{dg} = (r - n) - rt\left(1 + \frac{dk}{dg}\right)$

Substituting in market clearance

5. $(1 + n)\,k = w - (r\,(1 - t) - n)\,g + rtk - c^*\left[w - (r\,(1 - t) - n)\,g + rtk, r\,(1 - t)\right] - g\,(1 + n)$

Differentiating,

6. $dk/dg = -\{1 + n + (1 - c^*_Y)\,(r\,(1 - t) - n)\}/\{1 + n - (1 - c^*_Y)\,rt\}$

For $n > r\,(1 - t)$ this can be larger or smaller than minus one.
For $n < r\,(1 - t)$ $dk/dg < -1$ - a bigger effect.
For $dk/dg = -1$ $dT/dg = r - n$

Alternative technology: **k given** - fixed coefficients

Instead of r and w fixed, k endogenous, w and r satisfy

7. $y = w + rk$

This can be used to eliminate w from the analysis.
Differentiating 2, we have

8. $dT/dg = (r(1-t) - n) + \{(1-t)g - tk\} \frac{dr}{dg}$

Note that g can be larger or smaller than $t(k+g)$. We will give a sufficient condition for $\frac{dr}{dg} < 0$

Market clearance

9. $(1+n)(k+g) = y - rk - T - c^* [y - rk - T, r(1-t)]$

Note that

10. $y - rk - T = y - r(1-t)k - (r(1-t) - n)g$

Differentiating market clearance:

11. $dr/dg = -\frac{1+n+(r(1-t)-n)(1-c_Y^*)}{(k+g)(1-t)\left(1-c_Y^*\right)+(1-t)c_i^*}$

If $c_i^* = 0$

12. $dr/dg = -\{1 + nc_Y^* + r(1-t)(1-c_Y^*)\}/\{(1-t)(k+g)(1-c_Y^*)\}$
< 0

To get more savings the wage must go up, lowering r