

**A...Kaplow in Gale, Hines, Slemrod**

Notation

$x_i$	consumption in period $i$
$y$	labor supply
$z$	net-of-tax earnings
$w$	wage
$g$	net-of-tax gift at end of life
$u[x, y, g]$	utility
$f_i$	number of workers of type $i$
$R$	discount factor

Social welfare maximization assuming full nonlinear taxation:

$$\begin{aligned}
 &\text{Maximize}_{x,y,g} \quad \sum f_i u^i [x_1^i, x_2^i, y^i, g^i] \\
 &\text{subject to:} \quad E + \sum f_i (x_1^i + R^{-1}x_2^i + R^{-2}g^i - w_i y^i) \leq 0 \\
 &\quad \quad \quad u^i [x_1^i, x_2^i, y^i, g^i] \geq u^i [x_1^j, x_2^j, y^j w_j / w_i, g^j x_j] \text{ for all } i \text{ and } j
 \end{aligned} \tag{1}$$

Kaplow places the bequest during period 2 instead of period 3. He assumes that preferences are all the same. He examines the case of a nonlinear earnings tax and linear taxes on consumption and gifts. This is an additional constraint on allowable  $(x, y, g)$  vectors.

To proceed with this problem, we use a mixed direct-indirect utility function reflecting a tax on interest income and a tax on bequests. Let  $T$  be 1 plus the tax on bequests:

$$\begin{aligned}
 v^i [y^i, z^i, Q, T] = &\text{Maximize}_{x,g} \quad u^i [x_1^i, x_2^i, y^i, g^i] \\
 &\text{subject to:} \quad x_1^i + Q^{-1}x_2^i + Q^{-2}Tg^i = z^i
 \end{aligned} \tag{2}$$

Using this function, we can write SWF maximization:

$$\begin{aligned}
 & \text{Maximize}_{y,z,Q,T} \quad \sum f_i v^i [y^i, z^i, Q, T] \\
 & \text{subject to:} \quad E + \sum f_i \left( x_1^i [y^i, z^i, Q, T] + R^{-1} x_2^i [y^i, z^i, Q, T] + R^{-2} g^i [y^i, z^i, Q, T] - w_i y^i \right) \leq 0 \\
 & \quad v^i [y^i, z^i, Q, T] \geq v^j [y^j w_j / w_i, z^j, Q, T] \quad \text{for all } i \text{ and } j \\
 & \quad (3)
 \end{aligned}$$

The Atkinson-Stiglitz result is that the optimal  $Q$  is equal to  $R$  and optimal  $T$  is one with suitable separability when all sub-utility functions are the same.

From the Kaplow starting place, the fact that bequests are inherited, not resources used up, strengthens the case against the estate tax. Counterarguments would be from nonseparability, different preferences (as in Saez) and effects from uncertainty that make bequests accidental rather than part of the utility function being maximized.

### B...Inheritances

Kaplow starts with utilities of those leaving bequests, ignoring receipt to begin with. Instead, let us start with those receiving bequests, ignoring the utilities of those leaving the bequests. Let us consider a one-parameter family of bequest taxes, with parameter  $T$ . Assume that person  $i$  receives a gross-of-tax bequest  $G^i [T]$ , net-of-tax bequest of  $g^i [T]$ , with aggregate gross-of-tax bequests denoted  $G [T]$ . For notational convenience, we incorporate this into preferences. We assume that income taxation is just about earnings and drop savings. Note that with an expenditure tax, taxes would be paid when spending an inheritance.

$$\begin{aligned}
 & \text{Maximize}_{x,y,T} \quad \sum f_i u^i [x_1^i + g^i [T], y^i] \\
 & \text{subject to:} \quad E + \sum f_i \left( x_1^i + g^i [T] - w_i y^i \right) \leq G [T] \\
 & \quad u^i [x_1^i + g^i [T], y^i] \geq u^j [x_1^j + g^j [T], y^j w_j / w_i] \quad \text{for all } i \text{ and } j \\
 & \quad (4)
 \end{aligned}$$

Assume two types. Assume the only binding moral hazard constraint is type 1 considering imitating type 2.

$$\begin{aligned} \text{Maximize}_{x,y,T} \quad & \sum f_i u^i [x_1^i + g^i [T], y^i] \\ \text{subject to:} \quad & E + \sum f_i (x_1^i + g^i [T] - w_i y^i) \leq G[T] \\ & u^1 [x_1^1 + g^1 [T], y^1] \geq u^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1] \end{aligned} \quad (5)$$

FOC:

$$f_1 u_x^1 [x_1^1 + g^1 [T], y^1] - \lambda (f_1) = -\mu (u_x^1 [x_1^1 + g^1 [T], y^1]) \quad (6)$$

$$f_1 u_y^1 [x_1^1 + g^1 [T], y^1] + \lambda (f_1 w_1) = -\mu (u_y^1 [x_1^1 + g^1 [T], y^1]) \quad (7)$$

$$f_2 u_x^2 [x_1^2 + g^2 [T], y^2] - \lambda (f_2) = \mu (u_x^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1]) \quad (8)$$

$$f_2 u_y^2 [x_1^2 + g^2 [T], y^2] + \lambda (f_2 w_2) = \mu (u_y^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1]) w_2 / w_1 \quad (9)$$

Note that we are ignoring the nonnegativity constraint on x. Without inheritances an infinite marginal utility of consumption at zero consumption is sufficient to rule out this corner. But now the story is different.

Remaining FOC:

$$\begin{aligned} & \sum f_i u_x^i [x_1^i + g^i [T], y^i] g^{i'} [T] - \lambda (\sum f_i g^{i'} [T] - G' [T]) \\ & + \mu (u_x^1 [x_1^1 + g^1 [T], y^1] - u_x^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1]) g^1 [T] = 0 \end{aligned} \quad (10)$$

To focus on the difference between estate and income taxation, let us assume that only type 1 receives bequests.

Then the FOC becomes:

$$f_1 u_x^1 [x_1^1 + g^1 [T], y^1] g^{1'} [T] - \lambda (f_1 g^{1'} [T] - G' [T]) + \mu (u_x^1 [x_1^1 + g^1 [T], y^1] - u_x^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1]) g^{1'} [T] = 0 \quad (11)$$

Rearranging terms:

$$(f_1 u_x^1 [x_1^1 + g^1 [T], y^1] - \lambda (f_1) + \mu (u_x^1 [x_1^1 + g^1 [T], y^1] - u_x^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1])) g^{1'} [T] + \lambda (G' [T]) = 0 \quad (12)$$

Using the FOC for x:

$$(\mu (-u_x^1 [x_1^2 + g^1 [T], y^2 w_2 / w_1])) g^{1'} [T] + \lambda (G' [T]) = 0 \quad (13)$$

Taxing estates is different from further taxation of the earnings of the high type in two ways. One is that it has a different impact on incentive compatibility. The other is that it can affect aggregate bequests. Thus there may be a corner at  $T=0$  if the drop in aggregate bequests is large enough.

In a first-best world, the goal would be to maximize gross inheritances, which would give  $T=0$ , assuming that  $G$  is decreasing in  $T$  (which is not necessarily the case).

Note that if bequests are unaffected by taxes, then the solution to this FOC may tax away all of inheritances. This would be clearer if we had a linear tax on bequests.

From this starting place, adding the utility of consumption of donors (but not the utility gain from bequests) would strengthen the case for estate taxation since lowering bequests raises consumption, adding to social welfare.

### C...Inheritances and Bequests

In putting together the two sections (in a 2 generation model, although OLG would bring in more issues), the critical issue is whether the utility of bequests is part of the SWF or is viewed as double counting.