Static Labor Supply: Theory

Josh Angrist

MIT 14.661 (FALL 2024)

A Framework

As with most things micro, the decision about whether and how much to work is determined by a trade-off. In this case, the trade-off is between consumption and leisure, both of which generate utility. Specifically, utility is a function of consumption (x) and leisure (l), where $h = T - l$ is hours worked and T is the constraint on our time (verily, there are not enough hours in the day). The value of time, w, determines the consumption-leisure budget set:

$$px = w(T - l) + y,$$

where y is unearned income (perhaps a Bar Mitzvah bond that matures when you start work) and p is the price of a Hicksian composite commodity. Rewrite the budget set as:

$$px + wl = wT + y$$

This *full income* representation highlights the core economic idea that *time is money* (don't waste it!)

- Draw the consumption-leisure choice problem
  - visualize the first-order conditions (FOCs)
- *Uncompensated* (Marshallian) commodity demand and labor supply are defined by:

$$\begin{aligned} \{x(p,w,y), l(p,w,y)\} &= \arg\max U(x,l) \\ s.t. \quad px &= w(T-l)+y \end{aligned}$$

  These two first-order conditions (FOCs):

$$\frac{U_l}{U_x} = \frac{w}{p}$$
$$MRS = real\ wage,$$

  plus the budget line makes 3 equations in 3 unknowns

- Solving these generates uncompensated labor supply as a function of prices, wages, and unearned income:

$$h(p, w, y) = T - l(p, w, y).$$

  - Derivatives and elasticities of labor-most interest:

$$\frac{\partial h}{\partial w} = \textit{uncompensated labor supply response}$$
$$\frac{\partial h}{\partial w}\frac{w}{h} = \textit{uncompensated labor supply elasticity}$$

  - Can these be signed?
  - What are the key economic assumptions in this framework? What's consumer-chosen and what's parametric (given)?
  - We might also study income effects on hours worked, $\frac{\partial h}{\partial y}$; price effects on commodity demand, $\frac{\partial x}{\partial p}$; and the consumption consequences of income changes, $\frac{\partial x_j}{\partial y}$, for specific goods, x_j
  - The relationship between consumption and income is called an *Engel curve* (after statistician E. Engel, not Karl Marx's pal, F. Engels)

- *Compensated* (Hicksian) labor supply is a function of wages, prices and utility:

$$\begin{aligned} \{x^c(p, w, \bar{u}), l^c(p, w, \bar{u})\} &= \arg\min\ wl + px \\ s.t.\quad \bar{u} &= U(x, l) \end{aligned}$$

  Instead of maximizing utility subject to a budget constraint, this *dual problem* minimizes cost (expenditure) subject to a utility constraint.

- Draw the cost-minimization problem
  - Same FOCs! (well, almost: we're now constrained to be on indifference curve $\bar{u}$)

- The dual solution generates compensated labor supply functions:

$$h^c(p, w, \bar{u}) = T - l^c(p, w, \bar{u})$$

  - Compensated derivatives and elasticities of interest:

$$\frac{\partial h^c}{\partial w} = \textit{compensated labor supply response}$$
$$\frac{\partial h^c}{\partial w}\frac{w}{h} = \textit{compensated labor supply elasticity}$$

  - The derivative of the compensated labor supply function is the *substitution effect*
  - Draw a decomposition of the consequences of a wage increase into *income and substitution effects*

The *excess expenditure function*

- A consumer *spends* this much to get to $\bar{u}$ (cf. *full income*, above):

$$\begin{aligned} E[p,w,\bar{u}] &= px^c(p,w,\bar{u}) + wl^c(p,w,\bar{u}) \\ &= px^c(p,w,\bar{u}) + w(T - h^c(p,w,\bar{u})) \end{aligned}$$

- My time is worth something, so I need this much *cash* to get to $\bar{u}$:

$$E^*[p,w,\bar{u}] = E[p,w,\bar{u}] - wT = px^c(p,w,\bar{u}) - wh^c(p,w,\bar{u})$$

  Viewed as a function of prices, wages, and my utility target, $\bar{u}$,this is called the *excess expenditure function* (see

- The excess expenditure function has two important properties (proved in recitation):

  1. Shephard's Lemma

  $$\frac{\partial E^*[p,w,\bar{u}]}{\partial w} = \frac{\partial E[p,w,\bar{u}]}{\partial w} - T = l^c(p,w,\bar{u}) - T = -h^c(p,w,\bar{u})$$

     This is the envelope theorem in action.

  2. Concavity

  $$\frac{\partial^2 E^*[p,w,\bar{u}]}{\partial w^2} = \frac{\partial^2 E[p,w,\bar{u}]}{\partial w^2} = \frac{\partial l^c(p,w,\bar{u})}{\partial w} < 0$$

     Why *should* the expenditure function be concave in prices? A matter of economics–not calculus! When the price or any good rises, consumers reallocate away from the newly expensive item, whether it's consumption or leisure. By virtue of this reallocation, expenditure (cost) increases less than linearly in prices (linear increase means consumption is unchanged). Mathematically, therefore

  $$\frac{\partial h^c(p,w,\bar{u})}{\partial w} = -\frac{\partial l^c(p,w,\bar{u})}{\partial w} = -\frac{\partial^2 E^*[p,w,\bar{u}]}{\partial w^2} > 0$$

     From concavity and Shephard's Lemma, we deduce that the substitution effect of a wage increase on hours worked is positive.

Slutsky for hours (done in minutes)

- Compensated and uncompensated labor supply are related by an identity:

$$h^c(p,w,\bar{u}) = h(p,w,E^*[p,w,\bar{u}]). \tag{1}$$

  The compensated hours function *compensates* you (up or down) by changing E^* so as to hold you on $\bar{u}$ when your wage changes.

- Differentiate both sides of (1)

$$\begin{aligned}\frac{\partial h^c(p,w,\bar{u})}{\partial w} &= \frac{\partial h(p,w,E[p,w,\bar{u}]-wT)}{\partial w} + \frac{\partial h(p,w,E[p,w,\bar{u}]-wT)}{\partial y}\left[\frac{\partial E[p,w,\bar{u}]}{\partial w} - T\right] \\ \frac{\partial h^c}{\partial w} &= \frac{\partial h}{\partial w} + \frac{\partial h}{\partial y}[-h^c(p,w,\bar{u})]\end{aligned}$$

- Re-arrange to get the Slutsky equation for hours worked:

$$\frac{\partial h}{\partial w} = \underbrace{\frac{\partial h^c}{\partial w}}_{subs.} + \underbrace{\frac{\partial h}{\partial y}h}_{inc.}$$

  - $\frac{\partial h}{\partial w}$ is the sum of a positive *substitution effect* and a negative *income effect* (assuming leisure is a normal good)
  - We can imagine groups of workers or employment scenarios with known or presumed income effects, thereby signing the theoretical labor supply consequences of policy changes and other sources of pay variation
  - For one sort of person, at least, the theoretical labor supply response is surely positive (draw this)

B Labor Supply and Transfer Programs

Most developed-country transfer programs (social insurance, welfare, guaranteed minimum income, earned income tax credit, social credit, in-kind benefits like food stamps) can be described as a type of *negative income tax*.

- A stylized negative income tax (NIT) provides a subsidy of G, reduced by amount t for every dollar a worker earns. Actual NIT programs first emerged in the US in the 1970s as an alternative to traditional in-kind benefit programs that provide food stamps and public housing
  - Assuming (as is typical) the program taxes earnings (wh) and ignores unearned income (y), the *program subsidy* is

$$S = G - twh$$

    when

$$wh < \frac{G}{t} = B$$

    and zero otherwise
  - Assuming $t > 0$, earnings level B is the *program breakeven,* the earnings level above which I no longer qualify for a subsidy

- This is a model for any transfer
  - t may vary with earnings (if so, it's usually increasing)
  - t can be negative, at least for an initial range of earnings, though not forever (as in the EITC)
  - t can be (and often is) 100% or even effectively infinite (as in some states' AFDC programs, TANF's predecessor)
  - Assets might be taxed; Piketty notwithstanding, this is rare
  - Program eligibility might depend on a work requirement
  - Beneficiaries may face lifetime caps or time limits
- Budget set bonanza (from Moffitt 2002)

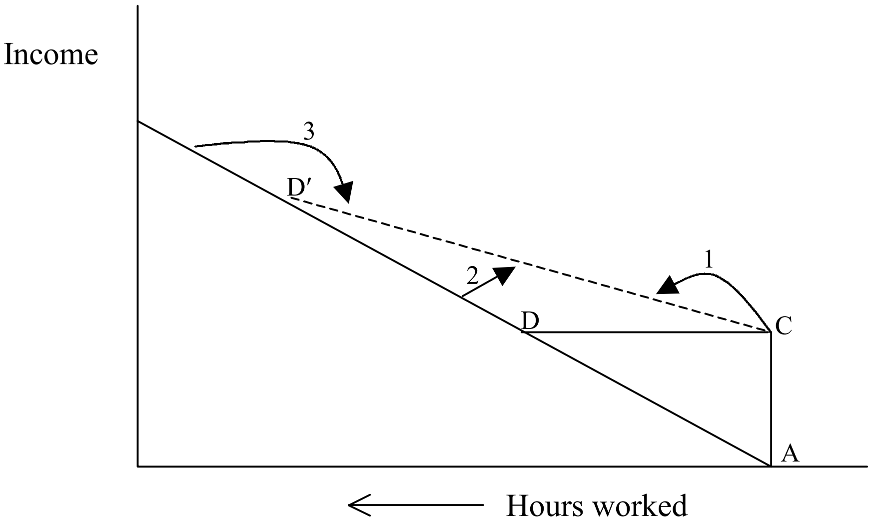

Figure 3. Effect of a Decrease in t

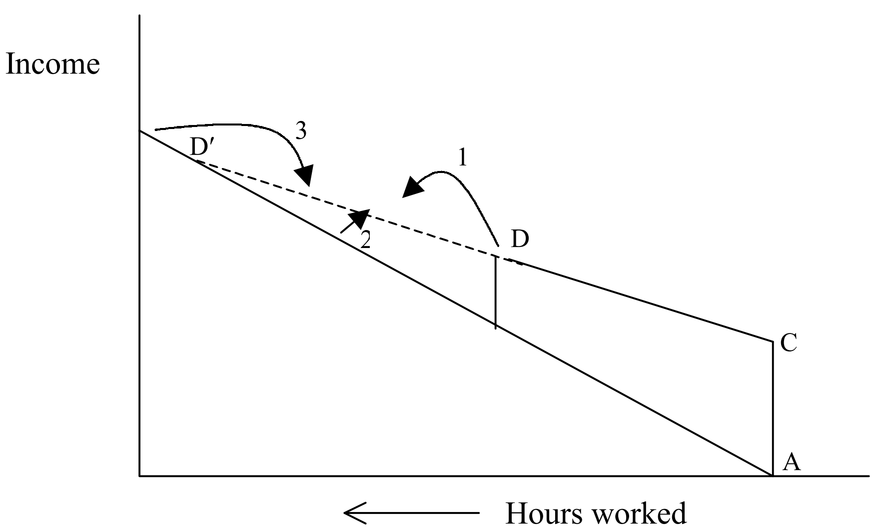

Figure 4. Welfare Program with a Notch

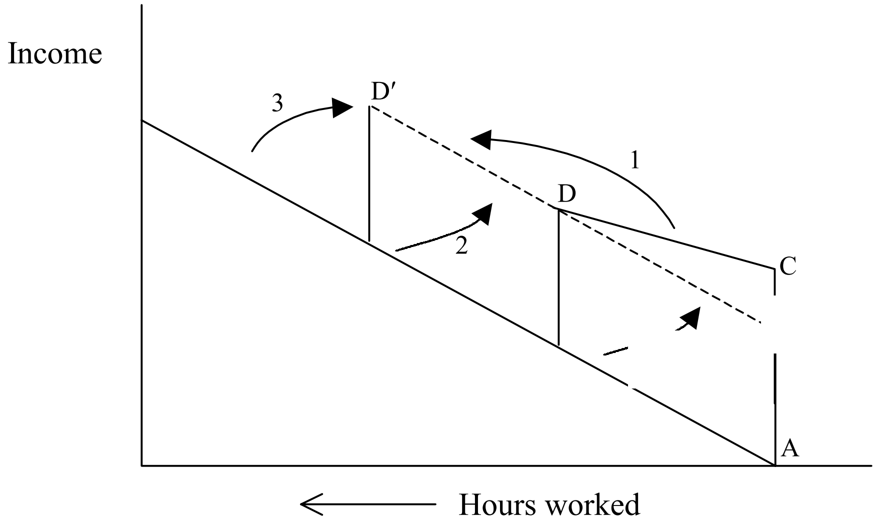

Figure 5. Medicaid Expansion

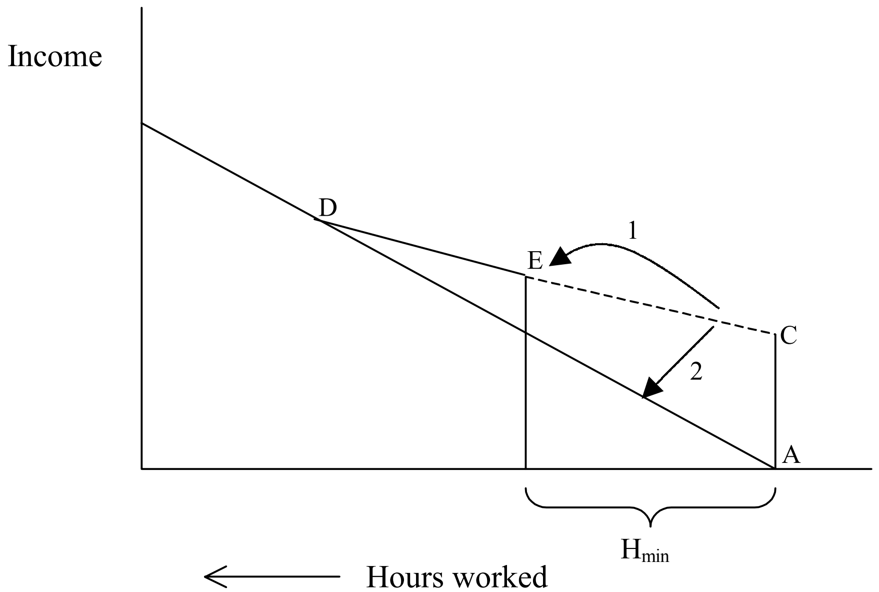

Figure 6. Work Requirement

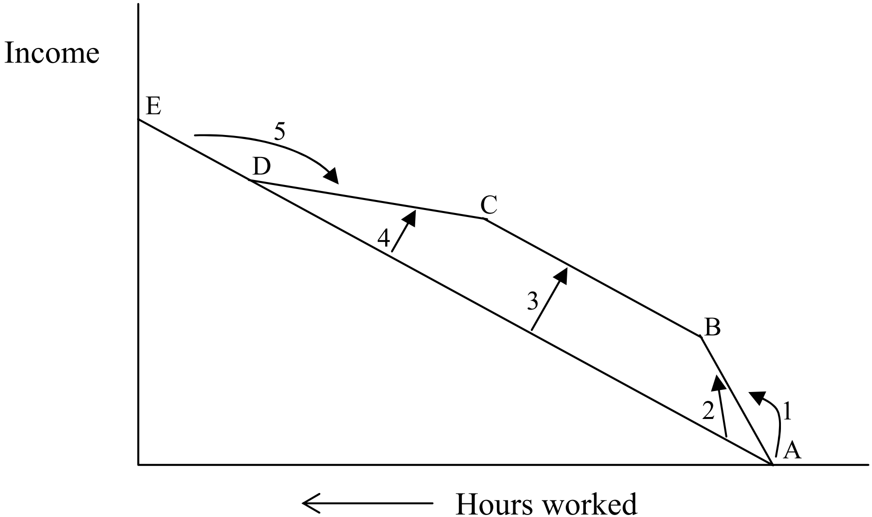

Figure 4. Earned Income Tax Credit

Table 1

Characteristics of Major Means-Tested Transfer Programs in the U.S.

Program	Main Eligible Population[a]	Form of Assistance	Annual Expenditures[b] (FY2000)	Average Monthly Expenditure for Family of 3 (FY2000)	Marginal Tax Rate on Earnings
Temporary Assistance for Needy Families	Mostly single mother families	Cash	14,490	600	Ranges across states from 0 to 100 percent
Supplemental Security Income	Aged, blind, and disabled individuals and families	Cash	35,066	1326	50%
Earned Income Tax Credit	Individuals with positive earnings	Cash	30,000[c]	135[e]	Ranges from -40% to 21%
Food Stamps	All individuals and families	Food coupons	20,341	279	30%
Medicaid	Families with dependent children, disabled, elderly	Health care services	207,195	2,238[f]	0% or >100%

Table 1 (continued)

Program	Main Eligible Population[a]	Form of Assistance	Annual Expenditures[b] (FY2000)	Average Monthly Expenditure for Family of 3 (FY2000)	Marginal Tax Rate on Earnings
Subsidized Housing[d]	All individuals and families	Housing units	22,498	422[g]	Ranges from 20% to 30%
Child Care					
Child Care Block Grant	Working parents of children under 13	Child care assistance	6,934	861[f]	Sliding fee scale set by states (can be zero)
Dependent Care Tax Credit	Working parents of children under 13	Nonrefundable credit in federal income tax	2,200	75[h]	Credit is 20% to 30% of eligible expenses

Notes:

[a] In addition to low income and assets
[b] Combined federal and state and local; in millions
[c] Includes tax reduction as well as refundable portion
[d] Combined Section 8 and public housing
[e] Per filing unit, tax year 2000
[f] FY1999
[g] Family or dwelling unit
[h] FY1998, for 2 children in child care

Sources: Blau (forthcoming), Burke (2001), Rowe and Roberts (2002), U.S. House of Representatives (2000)

Table 2

Multiple Benefit Receipt by Nonelderly Single Mother Households, 1997
(percent distribution)

No Program	48.0
AFDC, Food Stamps, Medicaid, and another program	10.4
AFDC, Food Stamps, and Medicaid only	6.0
AFDC, Medicaid, and another program	1.1
AFDC and Medicaid only	0.7
Medicaid only	6.2
Food Stamps only	1.8
Other cash transfers only	4.9
Other	18.0
Total	100.0

Notes:

Source: Tabulations from the Survey of Income and Program Participation by Kara Levine, University of Wisconsin.

Table 3

Cumulative Marginal Tax Rates for Recipients of
TANF and Food Stamps in 12 States, 1997
(percent)

	From No Work to Part Time Work at Minimum Wage		From Part Time Work to Full Time Work Minimum Wage		From Minimum Wage to $9 Hourly Wage at Full Time Work	
	Without EITC	With EITC	Without EITC	With EITC	Without EITC	With EITC
Alabama	46	6	33	9	24	58
California	50	9	67	33	67	89
Colorado	57	17	71	39	29	59
Florida	46	6	59	28	35	63
Massachusetts	57	13	64	28	64	87
Michigan	63	23	84	47	35	63
Minnesota	55	8	65	27	69	89
Mississippi	34	-6	32	7	24	55
New Jersey	64	23	62	30	41	67
New York	65	16	67	27	55	84
Texas	50	10	24	0	25	57
Washington	71	30	67	33	50	76

Notes:

Income includes earnings, TANF and Food Stamp benefits, federal and state EITC amounts, less employee payroll and federal and state income taxes. Minimum Wage is $5.15 per hour. Family size of three assumed.

Source: Coe et al. (1998, Tables 4,5).

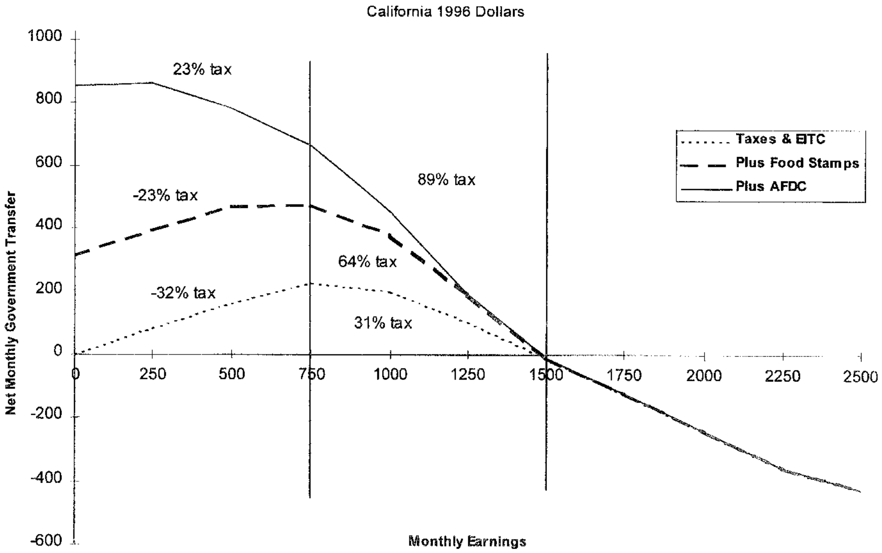

Fig. 1. Net transfers/taxes for California in 1996.

Blundell-MaCurdy (1999)

Theoretical NIT effects

- Recall:

$$\begin{aligned} h(p,w,y) &= \textit{uncompensated .l.s} \\ h^c(p,w,\bar{u}) &= \textit{compensated .l.s} \end{aligned}$$

  Uncompensated total differential:

$$dh = \frac{\partial h}{\partial w}dw + \frac{\partial h}{\partial y}dy$$

  Substitute using Slutsky:

$$dh = \left[\frac{\partial h^c}{\partial w} + \frac{\partial h}{\partial y}h\right]dw + \frac{\partial h}{\partial y}dy \tag{2}$$

  - collect terms:

$$dh = \frac{\partial h^c}{\partial w}dw + \frac{\partial h}{\partial y}[hdw + dy]$$

- What does an NIT do? For participants, the program ...
  - raises unearned income

$$dy = G$$

  - lowers wages

$$dw = -tw$$

- Rearranging and inserting program parameters:

$$\begin{aligned} dh &= \frac{\partial h^c}{\partial w}dw + \frac{\partial h}{\partial y}[hdw + dy] \\ &= \frac{\partial h^c}{\partial w}(-tw) + \frac{\partial h}{\partial y}[-twh + G] \\ &= \frac{\partial h^c}{\partial w}(-tw) + \frac{\partial h}{\partial y}S \end{aligned}$$

  Divide by h to get a nice elasticity equation:

$$dlnh = \eta_c(-t) + \eta_y\frac{S}{y} \tag{3}$$

  - offsetting income and substitution effects notwithstanding, a*n individual NIT (theoretically) reduces labor supply*
  - Ashenfelter (1978) treats (3) as an empirical model for the effects of an experimental NIT on earnings

- Labor supply can be hard to measure; earnings and NIT payments might be available in administrative data. This motivates Ashenfelter and Plant (1990) to compare predicted and actual payments in the legendary Seattle-Denver Income Maintenance Experiment (SIME/DIME) NIT RCT;

- SIME/DIME and other early NIT experimental findings are distressingly ambiguous, partly because attrition is highly correlated with program generosity. Some NIT studies also relied on self-reported rather than administrative earnings; program participants clearly under-report (see, e.g., Greenberg and Halsey, 1983).

C Predicting Program Participation: Cylons and Colonists on Welfare (details done in recitation)

- Ashenfelter (1983) shows that NIT program program participation identifies substitution elasticities. I use Battlestar Gallactica to interpret the theory.

- Cylons (robots) are mechanical; they get welfare because they qualify for it:

$$D = 1\left(wh < \frac{G}{t}\right)$$

  The probability Cylons get welfare is therefore

$$P[D=1] = P\left(wh < \frac{G}{t}\right)$$

- The earnings distribution alone determines how many Cylons get welfare. Suppose:

$$\ln wh \sim N(\mu, \sigma^2)$$

  Then

$$\begin{aligned} P\left(wh < \frac{G}{t}\right) &= \Phi\left[\frac{\ln(\frac{G}{t}) - \mu}{\sigma}\right] \\ &= \Phi\left[\frac{1}{\sigma}\ln(\frac{G}{t}) - \frac{\mu}{\sigma}\right] \end{aligned}$$

  and

$$\Phi^{-1}\left[P\left(wh < \frac{G}{t}\right)\right] = \frac{1}{\sigma}\ln(\frac{G}{t}) - \frac{\mu}{\sigma}$$

- The *Probit* of the Cylon participation fraction should vary linearly with log breakeven. In fact, variation in participation across NIT programs identifies μ and σ with no data on earnings!

Back on Caprica

- Colonists (human consumers) *choose* consumption (x) and leisure (l) given wages (w), prices (1), and unearned income (z)

$$\max u(x,l) \quad s.t. \quad x = w(T-l) + z$$

- Recall the *excess expenditure fn:*

$$\min wl + x - wT \quad s.t. \quad u(x,l) = \bar{u}$$

  Call this
$$E^*[w,\bar{u}] \equiv wl^c(w,\bar{u}) + x^c(w,\bar{u}) - wT$$

- Who among the humans on Caprica gets welfare?
  - A non-participating colonist needs cash in amount $E^*[w,\bar{u}]$ to hit $\bar{u}$
  - A participant colonist needs unearned income in amount $E[(1-t)w,\bar{u}]$ to hit $\bar{u}$ because welfare programs impose a tax
  - Participants *opt-in* if G is enough to close the gap generated by taxes:

$$D = 1\left(G > E^*[(1-t)w,\bar{u}] - E^*[w,\bar{u}]\right) \tag{4}$$

Simplification

- Second-order Taylor expansion of $E^*[(1-t)w,\bar{u}]$ around $E^*[w,\bar{u}]$:

$$\begin{aligned} E^*[(1-t)w,\bar{u}] &\simeq E^*[w,\bar{u}] + \frac{\partial E^*}{\partial w}(-tw) + \frac{1}{2}\frac{\partial^2 E^*}{\partial w^2}(-tw)^2 \\ &= E^*[w,\bar{u}] + (l^c - T)(-tw) + \frac{1}{2}\frac{\partial l^c}{\partial w}(-tw)^2 \\ &= E^*[w,\bar{u}] + twh - \frac{1}{2}\frac{\partial h^c}{\partial w}(tw)^2, \end{aligned}$$

  The behavioral rule for welfare receipt implied by (4) is therefore

$$G > twh - \frac{1}{2}\frac{\partial h^c}{\partial w}(tw)^2 \tag{5}$$

  or

$$G - twh > -\frac{1}{2}\frac{\partial h^c}{\partial w}(tw)^2 \tag{6}$$

- The RHS is negative, so some human consumers who would otherwise have earnings above breakeven get welfare: these people *opt in* by reducing their labor supply and hence their earnings enough to qualify

- Manipulate to solve for the earnings level below which people opt-in:

$$\begin{aligned} G+\frac{1}{2}\frac{\partial h^c}{\partial w}(tw)^2 &> twh \\ \frac{G}{t}+\frac{1}{2}\left(\frac{\partial h^c}{\partial w}\frac{w}{h}\right)twh &> wh \\ \frac{G}{t}+\frac{1}{2}etwh &> wh \end{aligned}$$

where e is the compensated supply elasticity. So opting-in earnings is

$$wh < \frac{G}{t}(1-.5et)^{-1}$$

This is above the program breakeven unless $e = 0$

  - example: $t = e = .5$, then $1 - .5et = .875$, and $\frac{1}{.875} \cong 1.14$
  - In general, flatter indifference curves ⇒ higher compensated elasticities ⇒ more opt in

Estimation

- Write opt-in as

$$y_0 = \frac{G}{t}(1-.5et)^{-1}$$

Note that

$$\begin{aligned} \ln y_0 &= \ln(\frac{G}{t}) - \ln(1-.5et) \\ &\simeq \ln(\frac{G}{t}) + .5et \end{aligned}$$

Assume, as we did at the outset, that earnings are log normal. Then:

$$\begin{aligned} P[D &= 1] = P\left(wh < y_0\right) \\ &= \Phi\left[\frac{\ln y_0 - \mu}{\sigma}\right] \\ &= \Phi\left[\frac{\ln(\frac{G}{t}) + .5et - \mu}{\sigma}\right] \\ &= \Phi\left[\frac{1}{\sigma}\ln(\frac{G}{t}) + \left(\frac{e}{\sigma}\right)\frac{t}{2} - \frac{\mu}{\sigma}\right] \qquad (7) \end{aligned}$$

  - Participation choices in the treated sample identify σ, μ, e (via Probit likelihood)
  - Colonists' (economic) behavior nests cylon (mechanical) behavior
  - μ and σ are over-identified (hint: what's the control likelihood?)

- Add covariates by modeling μ as a linear fn of X

- A non-parametric test for $H_0 : e = 0$ asks whether treated households get welfare at rates higher than predicted by the control earnings distribution
  - Cylons of the world unite!

Table 1. Data on Experimental and Control Families in the Second Year of the Seattle/Denver Income Maintenance Experiment

Program Earnings Breakeven for Experimental Families $	The Proportion of Experimental Families Participating in the Receipt of Payments: Unadjusted for Truncation (1)	The Proportion of Experimental Families Participating in the Receipt of Payments: Adjusted for Truncation (2)	Income Ordinate for Control Families $	The Cumulative Earnings Distribution for Control Group Families: Unadjusted for Truncation (3)	The Cumulative Earnings Distribution for Control Group Families: Adjusted for Truncation (4)
5,430–6,850	.328	.29	6,850	.315	.28
7,366–7,600	.319	.29	7,600	.383	.34
8,000	.521	.45	8,000	.433	.39
8,001–8,821	.558	.48	8,821	.527	.46
9,600	.673	.57	9,600	.609	.53
11,200	.734	.64	11,200	.774	.66

components of (3.3) it is clear that the parameters β and σ are common to all parts of the likelihood function. It is a relatively straightforward matter to maximize (3.3) with respect to these parameters and *e* by numerical methods and to compare the maximized value of (3.3) against the sum of the unconstrained values of (3.1) and (3.2) by a likelihood ratio test. The economic significance of this test is that the null hypothesis that $\beta - \beta^* = \beta$ and $\sigma = \hat{\sigma}$ is consistent with a model in which program participation is not influenced by "welfare stigma" or other nonpecuniary participation costs. If this null hypothesis is accepted it offers the opportunity of pooling the data from the control and experimental groups so as to increase the efficiency of the estimation of the effect of economic incentives on program participation by increasing the precision with which the parameter *e* may be estimated. If this null hypothesis is rejected, however, then these nonpecuniary participation costs must be judged important determinants of program participation and this raises questions about whether the results ought to be generalized to other populations without further investigation.

4. EMPIRICAL RESULTS

An important message from (2.5) is that it should be fitted to data on groups that come from homogeneous populations. Particularly, groups with differences in mean earnings may be pooled using dummy variables to account for these differences, but this is not sufficient to handle differences in variances because the value of σ affects every coefficient in (2.5).

As a preliminary effort, therefore, the data for the control group were stratified by location of experiment (Se-

Table 2. Estimates of the Determinants of Payment Receipt for White and Chicano Two-Parent Families (estimated standard errors in parentheses)

	Controls (Regression)	Experimentals (Probit)	Combined	Combined
	(1)	(2)	(3)	(4)
Estimates of:				
$1/\sigma$ (or $1/\hat{\sigma}$)	1.332	1.562	1.328	1.351
	(.038)	(.187)	(.036)	(.037)
e	——	.232	.134	.190
		(.295)	(.278)	(.314)
Coefficient of:				
Pre-Experimental Earnings	.395	.187	.309	.421
	(.046)	(.046)	(.035)	(.043)
3-Year Treatment	——	.108	.134	.118
		(.067)	(.057)	(.076)
Denver Location	.004	−.102	−.050	−.039
	(.061)	(.065)	(.047)	(.048)
*E*0	−.383	−.224	−.352	−.339
	(.243)	(.269)	(.193)	(.189)
*E*1	−.085	.130	.163	.021
	(.539)	(.376)	(.343)	(.335)
*E*2	−.467	−.155	−.361	−.343
	(.145)	(.156)	(.115)	(.113)
*E*3	−.242	−.011	−.141	−.128
	(.101)	(.099)	(.074)	(.074)
*E*4	−.061	.079	−.006	.003
	(.081)	(.087)	(.068)	(.061)
*E*6	.033	.122	.061	.061
	(.086)	(.103)	(.070)	(.069)
Constant	5.493	7.295	6.220	5.243
		(.432)	(.312)	(.385)
Pre-Experimental Earnings*				.171
				(.037)
Constant*				7.420
				(.471)
Log-Likelihood	−701.7	−479.5	−1193.5	−1184.7
Number of Observations	624	800	1424	1424

NOTE: Coefficients with a star are for experimentals.

Welfare dependence

- Families on welfare tend to stay on welfare - we therefore worry about "welfare dependence"
- Yet, even cyclons stay on welfare! Why?
- Plant (1984) distinguishes human from cylon behavior using a dynamic model
- Card and Hyslop (2005) extends this to a dynamic labor supply, applied to a time-varying EITC-type work bonuses program called SSP

That's it for classic labor supply ... someone call me an Uber!