14.662 (Spring 2015): Problem Set 1

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1. **Wage Density Decomposition Methods**

This exercise is designed to give you some practice with decomposition methods discussed in class and recitation.[1] *Some of the data and code you'll need can be found in a .zip file on the course website. Please hand in any additional code you write. As usual with empirical work, you'll need to make some subjective decisions in applying these methods (e.g. price deflators, kernels, bandwidths, etc.). Please be sure to mention these in your responses.*

(a) [5 points] *The data set men7988.dta contains information on male earnings in 1979 and 1988. Begin by plotting a kernel-based estimate of the density of log hourly wages in each of these years.*[2] *Draw a line on your graph to denote the minimum wage level in each year. Your results should look similar to Figure 4a in DiNardo, Fortin, and Lemieux (1996).*
See Figure 1 and attached code. To match the DFL (1996) figure, I used a deflating factor of 0.63 for 1988 wages and a gaussian kernel evaluated at 200 equal-width points with a bandwidth of 0.06.

(b) [5 points] *Group the data into cells of education and potential experience as follows:*

i. *Education: high school dropout, high school graduate, some college, college or more*
ii. *Potential experience: 0-9 years, 10-19 years, 20-29 years, 30+ years*

Let X_{it} denote a vector of 12 dummies for each education $\times$ experience cell, omitting the high school dropout category. Estimate quantile regressions of the form:

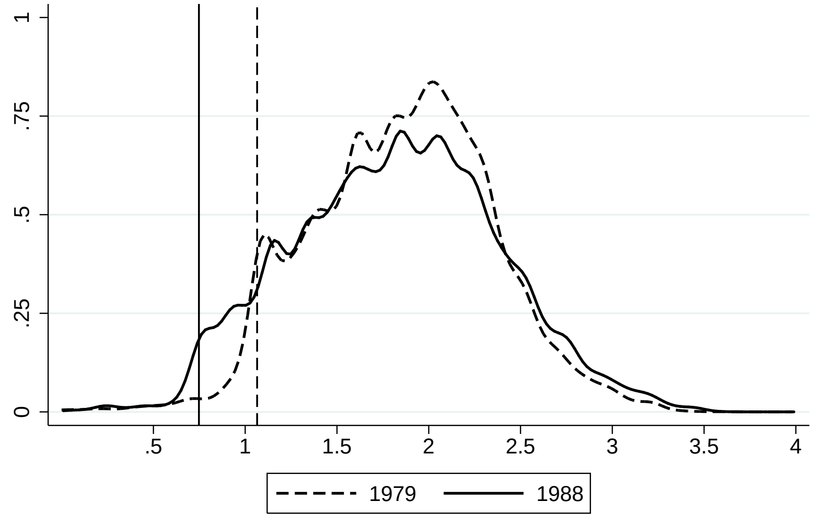
$$Q_{\ln w_{it}}(\tau|X_i) = \alpha^\tau + X_{it}'\beta_t^\tau$$

for each decile τ and each $t \in \{1979, 1988\}$. Plot your estimated $\widehat{\beta}_t^\tau$ in two sets (one for each year) of four graphs (one for each experience group), each with three series (one for each education group, relative to the omitted category) that consist of nine data points (one for each τ). Discuss your plots.
See Figures 2a and 2b and attached code. Most of you chose to run a fully-saturated model for education/experience cells, so that's what I did too. Returns to education are increasing in education in all experience cells and years. Returns are roughly constant at all quantiles of the residual wage distribution for workers with at least 10 years of experience, but are increasing for higher quantiles for workers with less experience. This could indicate that low-wage, low-experience workers have low-skill jobs for which additional schooling has little effect on productivity. Returns are similar across experience groups within years. As we compare across years, note how the y-axes differ. The 1988 estimates are always larger than the 1979 estimates, which is consistent with rising returns to education. In particular, the gap between the college and high school plots grows during this decade, especially among low-experience workers.

(c) [7 points] *Use the DFL re-weighting method to simulate the wage distribution that would have prevailed in 1988 if the distribution of education and experience had not changed since 1979. Graph this counterfactual wage distribution along with the actual distribution from part (a). Interpret your results in light of your*

[1] *Some of these questions are adopted from a replication exercise developed by Nicole Fortin, which can be found on her UBC website.*
[2] *Be sure to think carefully about how you weight each observation. See the original DFL paper for guidance.*

Figure 1: Kernel density estimates of real log wages



Note: vertical lines indicate real 1979/1988 minimum wages

plots in part (b) and your knowledge of trends in educational attainment during this period.
See Figure 3 and attached code. The counterfactual 1988 wage distribution lies somewhat to the left of the observed distribution. Since educational attainment is rising during this period, we would expect 1979's work force to be paid less at 1988 prices than 1988's labor force.

(d) [6 points] *Discuss the limitations of the quantile regression analysis in part (b) and explain why they motivate the development of unconditional quantile regression methods.*
We are interested in estimating the causal effect of education on the shape of the log wage distribution. As with any observational estimate of causal effects, we need to worry about omitted-variables bias, and would like to control for enough covariates such that education is plausibly as good as randomly assigned to workers conditional on these covariates. For average wages, we can model the conditional expectation as a linear function and interpret the coefficient(s) on education as saying something about the average unconditional effect of schooling (by the linearity of expectations). When we model conditional quantiles as a linear function of education and controls, however, the coefficients need not describe unconditional effects, since quantile functions are not linear in general. We thus turn to other methods that allow us to describe the unconditional effect of education while controlling for confounding variables, such as FFL.

(e) [5 points] *Install the Firpo, Fortin, and Lemieux (2009) RIF regression package $rifreg$.ado and use it to estimate the unconditional quantile effects from the model in part (b). Plot these estimates and compare them with the plots from part (b).*
See Figures 4a and 4b and attached code. The RIF regression coefficients are quite different than the quantile regression coefficients of Figures 2a and 2b, most strikingly in that they reveal very heterogeneous effects of increased educational attainment on lower vs. higher quantiles of the unconditional log wage distribution. For workers with less than 10 years of experience, the marginal impact of increased education is larger for lower quantiles, but that the opposite is true among workers with more experience.

(f) [8 points] *Use your density estimates from part (a) and the procedure outlined in recitation to manually construct RIF regression estimates of the effect of education on the unconditional deciles of the wage distribution in 1988. Compare these results to those you obtain from the canned $rifreg$ command. Are these wage data well-suited for the FFL procedure? Why or why not?*
See Figure 5 and attached code. Manual RIF regression estimates that divide estimated effects of each education/experience group on the probability of exceeding a given decile by the estimated unconditional density of log wages at that decile are nearly identical to those obtained by the $rifreg$ package in Figure 4b. In general smoother distributions are better-suited for the FFL procedure, as distributional clumping may make the estimates more sensitive to the parameters of the kernel density estimation. Based on Figure 1 the log wage distribution looks fairly smooth at most, though not all, quantiles.

Figure 2a: QR estimates of the effect of education on real log wages, 1979

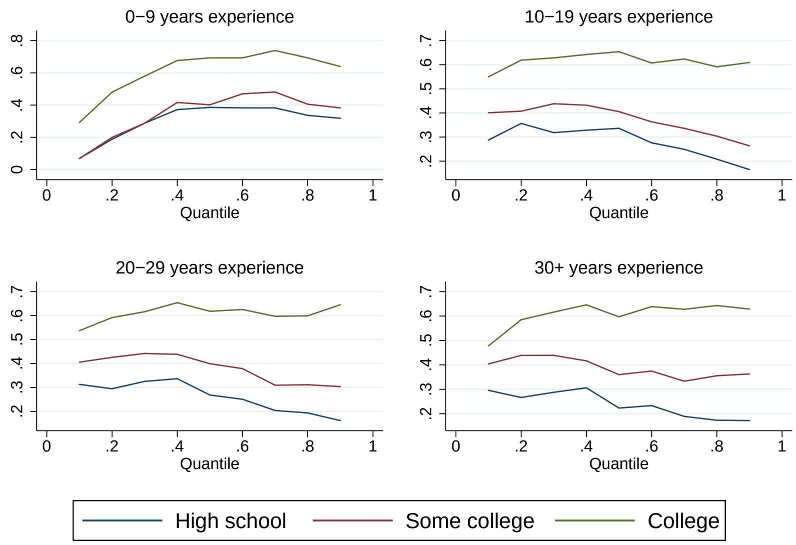


Figure 2b: QR estimates of the effect of education on real log wages, 1988

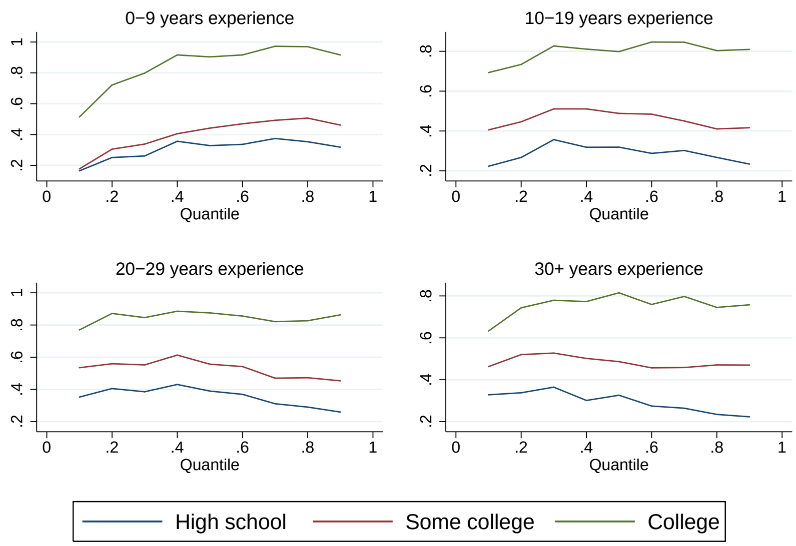


Figure 3: Counterfactual kernel density estimate of real log wages, 1988

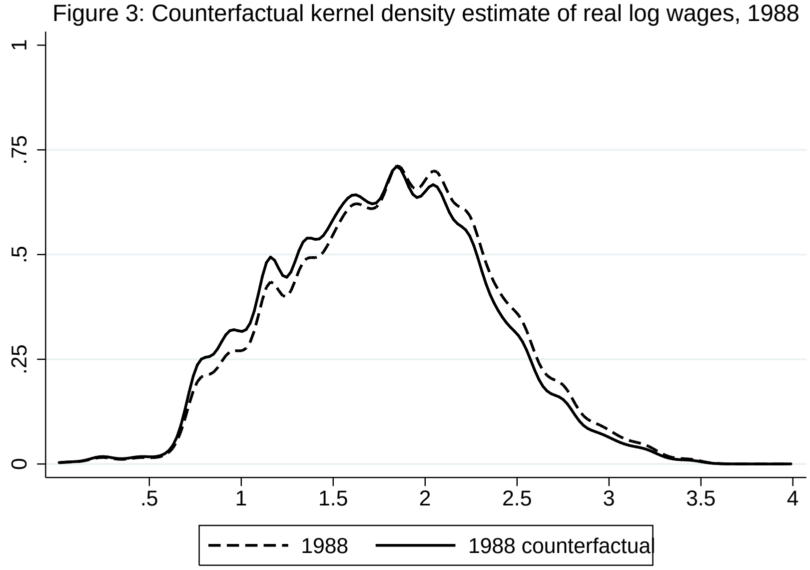


Figure 4a: RIF estimates of the effect of education on real log wages, 1979

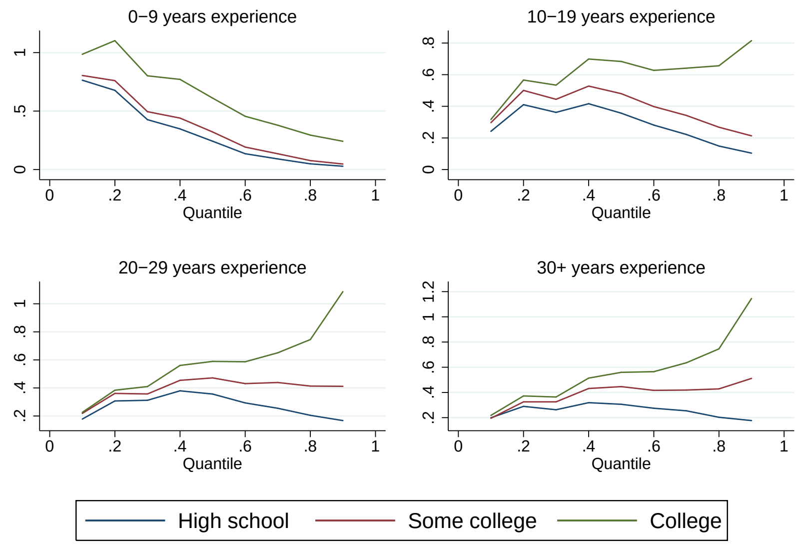


Figure 4b: RIF estimates of the effect of education on real log wages, 1988

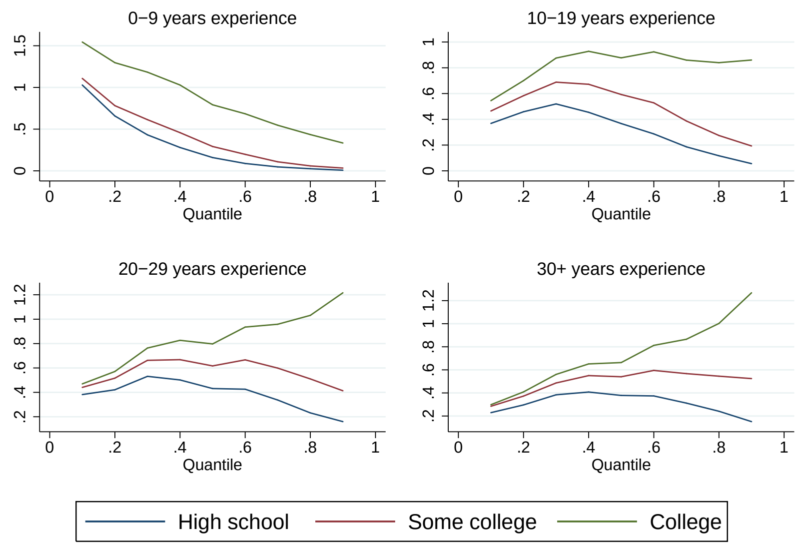
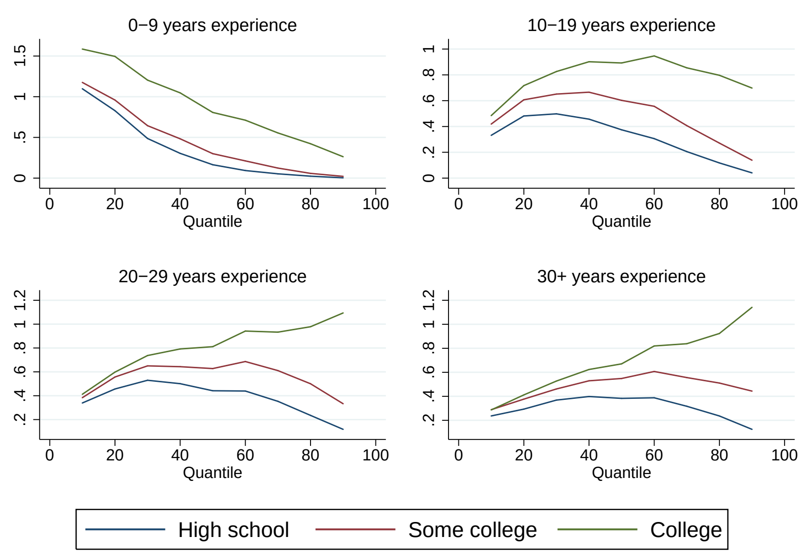


Figure 5: Manual RIF estimates of the effect of education on real log wages, 1988



2. **Skill-biased Technical Change**

Many have argued that recent technological advances (for example, computers) complement skilled labor, such that the rising demand for skill can be partly explained by a decline in these technologies prices. We'd like a model of skill-biased technical change with implications that we can test to check this story.

Consider an economy in which aggregate manufacturing output is given by

$$Y = \left((K^\theta + L^\theta)^{1/\theta}\right)^\alpha H^{1-\alpha}$$

for $\theta \leq 1$ and $\alpha \in (0,1)$. Here H and L are the aggregate supply of high- and low-skilled labor, respectively, and K is the aggregate stock of machinery that automates certain manufacturing tasks. Assume H and L are supplied inelastically. K is supplied perfectly elastically at rental rate r.

(a) [5 points] *Write the first-order condition for K under perfect competition.*
We have

$$\begin{aligned}\frac{\partial Y}{\partial K} &= \alpha K^{\theta-1} H^{1-\alpha}(K^\theta + L^\theta)^{(\alpha-\theta)/\theta} \\ &= \alpha \frac{Y}{K}\frac{K^\theta}{K^\theta + L^\theta} \\ &= r\end{aligned}$$

(b) [8 points] *Defining $\omega \equiv rK/\alpha Y$ as capital's share of the K and L share of production, derive an expression for how $\ln K$ responds to changes in $\ln H$ and $\ln L$.*
Note first that by the first-order condition:

$$\begin{aligned}\omega \frac{\alpha Y}{K} &= r \\ \implies \omega &= \frac{K^\theta}{K^\theta + L^\theta}\end{aligned}$$

Taking logs of both sides of the FOC and differentiating gives

$$\begin{aligned}\ln r &= \alpha + (\theta - 1)\ln K + (1-\alpha)\ln H + \frac{\alpha - \theta}{\theta}\ln(K^\theta + L^\theta) \\ 0 &= (1-\alpha)d\ln H + (\theta-1)d\ln K + (\alpha-\theta)\frac{K^\theta d\ln K + L^\theta d\ln L}{K^\theta + L^\theta} \\ &= (1-\alpha)d\ln H + (\theta-1)d\ln K + (\alpha-\theta)\left(\omega d\ln K + (1-\omega)d\ln L\right) \\ d\ln K &= \frac{(1-\alpha)}{(\theta-\alpha)\omega + (1-\theta)}d\ln H - \frac{(\theta-\alpha)(1-\omega)}{(\theta-\alpha)\omega + (1-\theta)}d\ln L\end{aligned}$$

(c) [5 points] *Under what condition is K increasing in H and decreasing in L? Provide some intuition for this result.*
In this model $d\ln K/d\ln H > 0$ and $d\ln K/d\ln L < 0$ if and only if $\theta > \alpha$. When L increases it raises the return to capital (as K and L are q-complements in the K and L aggregate), but lowers the return by decreasing the ratio of H to the K and L aggregate. When L and K are close substitutes ($\theta \to 1$) or the output share of H is large ($\alpha \to 0$) this second effect dominates and we have capital-skill complementarity.

(d) [7 points] *Now consider an alternative aggregate production function given by*

$$Y = \left(K^\rho + \left((H^\nu + L^\nu)^{1/\nu}\right)^\rho\right)^{1/\rho}$$

for $\rho, \nu \leq 1$. What does this functional form imply about capital-skill substitution patterns that's different from the model in (a)? How does K respond to changes in the relative supply of high skill labor (holding $H + L$ fixed) when the skill wage premium is positive?
Capital is skill-neutral in this model: the elasticity of substitution between K and the aggregate labor

supply Q is $1/(1-\rho)$ and high- and low-skilled labor enter Q symmetrically. The new first-order condition for capital is

$$r = K^{\rho-1}(K^\rho + Q^\rho)^{1/\rho-1}$$

When the skill wage premium is positive, a relative shift towards high-skilled labor will increase Q and thus increase K because of their q-complementarity. Conversely increases in L holding $H + L$ fixed will decrease K. Thus, despite the skill-neutrality of this model, we have the same empirical predictions on the increased use of automation capital from skill mix shocks as we had in (c).

(e) [7 points] *How does the capital/output ratio $W \equiv rK/Y$ respond to changes in the relative supply of high-skilled labor in the model in (d)? What about in the model in (a)? How might we use this to test for skill-biased technical change?*
In the model in (d) we have

$$\begin{aligned} r &= K^{\rho-1}(K^\rho + (H^\nu + L^\nu)^{\rho/\nu})^{1/\rho-1} \\ &= \frac{Y}{K}K^\rho \quad K^\rho + (H^\nu + L^\nu)^{\rho/\nu} \quad {}^{-1} \\ &= \frac{Y}{K}r^{\rho/(\rho-1)} \\ \implies \frac{rK}{Y} &= r^{\rho/(\rho-1)} \end{aligned}$$

so that the capital/output ratio is independent of relative skill supplies. However, in part (a) we have

$$\begin{aligned} r &= \alpha \frac{Y}{K}\frac{K^\theta}{K^\theta + L^\theta} \\ \frac{rK}{Y} &= \alpha \frac{K^\theta}{K^\theta + L^\theta} \end{aligned}$$

which clearly depends on the relative share of high-skill labor. We can thus differentiate between the two models (and by doing so test for skill-biased technical change) by looking at the effect of exogenous increases in the relative supply of low-skilled labor on the rate of adoption of the technology.

3. **Liquidity-Constrained Job Search**

Consider a simplified discrete-time search model where jobs last indefinitely once found. Individuals have a subjective time discount rate of δ and flow utility of $u(c_t) - \psi(s_t)$ where c_t denotes consumption in period t, s_t represents the individual's search effort (normalized as the probability of finding a job) and u and ψ are strictly concave and convex, respectively. Agents are initially unemployed at $t = 0$ with assets A_0. An agent who enters period t without a job first chooses s_t and learns whether or not she has become employed. If she has, she begins working in period t at the exogenous wage of w_t and remains employed at all future periods. If she fails to find a job she receieves an unemployment benefit $b_t < w_t$ and the problem repeats. Agents can save or borrow in each period at the fixed interest rate r, but are potentially constrained by a lower bound on assets L.

(a) [5 points] *Write the value function of assets A_t for an employed individual in recursive form.*
Since an employed individual faces no risk of job loss, she sets $s_t = 0$. We thus have

$$V_t(A_t) = \max_{A_{t+1} \geq L} u\left(A_t + w_t - \frac{A_{t+1}}{1+r}\right) + \frac{1}{1+\delta}\left(V_{t+1}(A_{t+1}) - \psi(0)\right)$$

This is the employed agent's Bellman equation.

(b) [6 points] *Write the maximization problem characterizing optimal search for an individual entering period t with assets A_t and no job. Use this to write the value function for an individual who fails to find a job at the start of period t in recursive form.*
An unemployed agent solves

$$\max_{s_t} s_t V_t(A_t) + (1 - s_t)U_t(A_t) - \psi(s_t)$$

where $U_t(A_t)$ is the value function of an unemployed agent. The optimum of this problem is the expected value of entering period t without a job, so

$$U_t(A_t) = \max_{A_{t+1} \geq L} u\left(A_t + b_t - \frac{A_{t+1}}{1+r}\right) + \frac{1}{1+\delta}\left(\max_{s_{t+1}} s_{t+1}V_{t+1}(A_{t+1}) + (1 - s_{t+1})U_{t+1}(A_{t+1}) - \psi(s_{t+1})\right)$$

is the unemployed agent's Bellman equation.

(c) [6 points] *Derive and interpret the first-order condition of the optimal search problem. How is optimal search intensity affected by an exogenous increase in A_t (such as a windfall cash grant)? What does testing $\partial s_t^*/\partial A_t = 0$ tell us?*
The first-order condition is

$$\psi'(s_t^*) = V_t(A_t) - U_t(A_t)$$

Intuitively, the agent chooses s_t such that the marginal cost of search effort $\psi(s_t^*)$ equals the marginal benefit of search, in this case the difference in the (optimal) values of employment and unemployment. Invoking the implicit function theorem to write $s_t^* = s_t^*(A_t)$ we have

$$\begin{aligned}
\psi''(s_t^*)\frac{\partial s_t^*}{\partial A_t} &= V_t'(A_t) - U_t'(A_t) \\
&= u'\left(A_t + w_t - \frac{A_{t+1}^{*e}}{1+r}\right) - u'\left(A_t + b_t - \frac{A_{t+1}^{*u}}{1+r}\right) \\
\frac{\partial s_t^*}{\partial A_t} &= \frac{u'(c_t^e) - u'(c_t^u)}{\psi''(s_t^*)}
\end{aligned}$$

where A_{t+1}^{*e} and A_{t+1}^{*u} are the optimal choices of $t+1$ asset levels when the agent is employed and unemployed, respectively, and c_t^e and c_t^u are the corresponding consumption levels in period t. By assumption $\psi'' > 0$ and $u' > 0$, so the sign of $\partial s_t^*/\partial A_t$ depends on whether $c_t^e \lesseqgtr c_t^u$. We know $c_t^e \geq c_t^u$, since an unemployed agent could always set $s_t = 0$ to recreate the employed agent's problem but with $b_t < w_t$ in place of w_t and thus weakly lower consumption due to the asset constraint; increasing search can only decrease optimal consumption further. If $c_t^e = c_t^u$ (the agent is able to perfectly smooth consumption), optimal job search does not respond to cash grants. However if the unemployed agent is liquidity constrained, $c_t^u < c_t^e$ and $\partial s_t^*/\partial A_t < 0$. Thus testing whether agents decrease their job search in response to cash grants reveals the degree to which they cannot intertemporally smooth their consumption.

(d) [7 points] *How is optimal search intensity affected by an exogenous increase in the benefit level b_t? By an exogenous increase in wages w_t? Write an expression linking $\partial s_t^*/\partial A_t$, $\partial s_t^*/\partial b_t$, and $\partial s_t^*/\partial w_t$ and interpret.*
Similar to (c), we can differentiate the first-order condition for s_t^* with respect to b_t:

$$\begin{aligned}\psi''(s_t^*)\frac{\partial s_t^*}{\partial b_t} &= \frac{\partial V_t}{\partial b_t} - \frac{\partial U_t}{\partial b_t} \\ &= -u'\left(A_t + b_t - \frac{A_{t+1}^{*u}}{1+r}\right) \\ \frac{\partial s_t^*}{\partial b_t} &= \frac{-u'(c_t^u)}{\psi''(s_t^*)} < 0\end{aligned}$$

and to w_t:

$$\begin{aligned}\psi''(s_t^*)\frac{\partial s_t^*}{\partial w_t} &= \frac{\partial V_t}{\partial w_t} - \frac{\partial U_t}{\partial w_t} \\ &= u'\left(A_t + w_t - \frac{A_{t+1}^{*e}}{1+r}\right) \\ \frac{\partial s_t^*}{\partial w_t} &= \frac{u'(c_t^e)}{\psi''(s_t^*)} > 0\end{aligned}$$

Thus

$$\frac{\partial s_t^*}{\partial b_t} = \frac{\partial s_t^*}{\partial A_t} - \frac{\partial s_t^*}{\partial w_t}$$

Intuitively, the decrease in search intensity due to an increase in unemployment benefits can be equated to the change in search intensity due to a pure wealth effect ($\partial s_t^*/\partial A_t$) and a substitution effect ($\partial s_t^*/\partial w_t$) from a change in the price of leisure. The weath effect, if nonzero, reflects a welfare-improving response due to the correction of a market failure (liquidity constraints), while the substitution effect can be thought of as a moral hazard response due to the subsidization of unemployment.

(e) [8 points] *How is optimal search intensity affected by an exogenous increase in a future benefit level b_{t+j}? What does testing $\partial s_t^*/\partial b_{t+j} = 0$ tell us?*
Now we have

$$\begin{aligned}\psi''(s_t^*)\frac{\partial s_t^*}{\partial b_{t+j}} &= \frac{\partial V_t}{\partial b_{t+j}} - \frac{\partial U_t}{\partial b_{t+j}} \\ &= -(1-s_{t+1}^*)(1-s_{t+2}^*)...(1-s_{t+j}^*)\frac{u'(c_{t+j}^u)}{(1+\delta)^j} \\ \frac{\partial s_t^*}{\partial b_{t+j}} &= -\frac{\rho_{t,j}^* u'(c_{t+j}^u)}{(1+\delta)^j \psi''(s_t^*)}\end{aligned}$$

where $\rho_{t,j}^*$ is the probability that an individual is still unemployed in period $t+j$ conditional on being unemployed in period t. As with current benefits, the effect of increasing future benefits on current search behavior is negative provided the agent is not completely myopic (that is, provided $\delta < \infty$). Testing whether $\partial s_t^*/\partial b_{t+j} = 0$ is thus a test of myopic search behavior.